Journal of the Nigerian Association of Mathematical Physics
Volume 65, (October 2022 – August 2023 Issue), **pp63 – 70**

NUMERICAL SOLUTION AND SIMULATION OF A SINGLE-PHASE, ONE-DIMENSIONAL, SLIGHTLY COMPRESSIBLE FLUID FLOW IN A PETROLEUM RESERVOIR

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Abstract

Predicting pressure distribution in a petroleum reservoir is principal to the reservoir's evaluation and maintenance, as pressure changes with space and time. A convenient approach to effectively achieve this task is to formulate fluid flow equations based on the reservoir characteristics and solve them numerically. Numerical method provides solutions to mathematical fluid flow models developed in a reservoir simulation. This study provides numerical solutions, using finite difference implicit method, to a mathematical model by developing MATLAB codes to ascertain the pressure distribution for a single phase, one-dimensional, slightly compressible fluid flow in a petroleum reservoir. Series of numerical simulations were carried out during the first year of production using timestep sizes of 1, 2 and 3 days, respectively. The implicit method gave a quite satisfactory results for all timesteps, and including less than 1 day, confirming the robustness and unconditionally stable nature of the implicit method. This study provides insights to reservoir's pressure profile during hydrocarbon recovery beforehand so that efficient pressure maintenance decisions can be made to achieve economic hydrocarbon recovery throughout the life of the reservoir.

Keywords: Reservoir simulation, single phase, implicit formulation, Porous media

1. Introduction

The study of fluid flow through porous media consists of solving the conservation of mass and the balance of momentum on a representative elementary volume (REV). This is essential to numerous environmental, biological, and industrial systems such as the movement of contaminants in the subsurface and their remediation, geologic nuclear waste disposal, medical application such as brain and liver cancer treatment and most notably in oil recovery from petroleum reservoirs [1]. In petroleum reservoirs, the inherent heterogeneity of subsurface porous media, as well as the complexity involved in the multiphase physics, highlights some of the most important technological challenges of our time [2, 3, 4]. Describing the flow of fluid through porous media is extremely complex compared to that of flow through pipes or conduits [5]. Unlike flow in pipes or conduits, there are no definite flow paths in porous media thereby making porous media flow capacity as a function of pressure difficult to estimate. Due to the complex nature of multiphase flow, nonlinearity of their governing equations and reservoir intricacies, finding analytical solutions to practical fluid flow problems is extremely difficult and discouraging. Therefore, the only means by which such models can be solved is by using numerical methods such as finite difference, finite volume or finite element, [6] among others. According to [7], reservoir simulation is a skill of developing a tool to forecast the performance of hydrocarbon reservoir under different operating settings by the combination of physics, mathematics, reservoir engineering and computer programming. Generally, reservoir simulation is used to predict the performance of reservoirs so that intelligent decisions can be made to enhance the economic recovery of hydrocarbons from the reservoir. This makes the description of fluid flow and the pressure distribution in a petroleum reservoir of great importance, as pressure varies with time and location, [8]. In view of this, [9] did a comparative study of finite difference methods for solving one-dimensional transport equation with an initial boundary value discontinuity. Their study revealed

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that the implicit method has an advantage in terms of lower CPU times over the explicit methods provided a desired level of time steps is used. [8] also studied the pressure distribution in a one-dimensional reservoir system by providing numerical solutions using explicit finite difference method. In related research, [10], carried out a comparative study between finite difference explicit and implicit method for predicting pressure distribution in a petroleum reservoir. They observed that the implicit formulation which is unconditionally stable provides a more physically realistic results when compared to the explicit formulation. This paper focuses on providing numerical solution to a single-phase, one-dimensional, slightly compressible fluid flow in a petroleum reservoir using backward-difference implicit method to predict the pressure distribution in a petroleum reservoir. We begin by modelling single phase flow equation through a porous medium from first principle.

2 Modelling of single-phase flow equations in a porous medium

2.1 Governing Equations and their specifications (Mass conservation and Darcy's Law)

The principle of conservation of mass discusses the balance between the rate of mass change in an arbitrary volume and the inflow of mass through the boundary surface area. In integral form, this can be expressed as follows:

$$\frac{\partial}{\partial t}\iiint \rho\phi dV + \iint \rho\mathbf{u}\cdot\mathbf{n}dS = \iiint q dV \qquad (2.1)$$

The double and triple integrals in (2.1) are taken over the surface and volume respectively while the parameters □,□,**u**,**n**, and q represent the fluid density, the porosity the medium, the velocity vector, the unit outward normal vector and the external mass flow rate respectively. The second term of equation (2.1) can be converted into a volume integral form by using the Gauss' divergence theorem such as:

$$\iint \rho\mathbf{u}\cdot\mathbf{n}dS = \iiint \nabla\cdot(\rho\mathbf{u})dV \qquad (2.2)$$

Using equation (2.2) in (2.1) and for a fixed control volume, the integral form of the conservation law results to

$$\iiint\left[\frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot(\rho\mathbf{u}) - q\right]dV = 0 \qquad (2.3)$$

since $dV \neq 0$ (i.e the control volume), it implies that

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot(\rho\mathbf{u}) - q = 0 \text{ or}$$

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = q \qquad (2.4)$$

where Δ is the del operator defined as

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$$

Equation (2.4) is known as the mass conservation equation.

We remark that q by convention is negative for sinks and positive for sources. Equation (2.4) can also be presented in terms of the formation volume factor B as:

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) = -\nabla\cdot\left(\frac{1}{B}\mathbf{u}\right) + \frac{q}{\rho_s} \qquad (2.5)$$

With $B = \frac{\rho_s}{\rho} \Rightarrow \rho = \frac{\rho_s}{B}$ where ρ_s is the fluid density at standard conditions.

In Darcy's empirically observed Law, we see that the flow rate of a single-phase fluid through a horizontal homogeneous porous medium is proportional to the pressure gradient across the medium and inversely proportional to the viscosity of the fluid [11]. That is:

$$Q = \frac{KA}{\mu}\frac{\nabla P}{L} \qquad (2.6)$$

where Q and K are the volumetric flow rate and permeability of the porous medium respectively, μ is the viscosity of the fluid, ∇P is the pressure gradient across the medium while A and L are the cross sectional area and length of the system respectively. The differential form of Darcy law is given as:

$$\mathbf{u} = \frac{Q}{A} = -\frac{K}{\mu}\frac{\partial P}{\partial x} \qquad (2.7)$$

where **u** is the superficial Darcy velocity and the negative sign signifies that the fluid flows in the direction of decreasing pressure. For multidimensional flow, we could restate Darcy law as:

$$\mathbf{u} = -\frac{K}{\mu}(\nabla P - \rho g \nabla D) \quad (2.8)$$

where $\mathbf{u}$ is the fluid flow velocity, P, the fluid pressure is the unknown function to be determined by the flow model, K is the absolute permeability tensor and a parameter of the solid matrix only and may depend on position. □ is the dynamic viscosity of the given fluid and is taken either as a constant or as a function of pressure. g is the gravitational vector, ρ is the fluid density and $D$is the physical depth. Darcy's law is valid for slow flow of a Newtonian fluid through porous medium with rigid solid matrix, [12].

By substituting equation (2.8) into equation (2.4) we obtain

$$\frac{\partial(\phi\rho)}{\partial t} = \nabla\cdot\left(\frac{\rho K}{\mu}(\nabla P - \rho g \nabla D)\right) + q \quad (2.9)$$

where $\phi, \rho, \mu, K, P, g, D, q$, as earlier defined represent the porosity, density, viscosity, permeability, pressure, gravity, physical depth and external mass flow rate respectively. Equation (2.9) is a single-phase flow equation in porous media. In most practical applications, substituting equation (2.8) into equation (2.5) we have an alternative form of the single-phase flow equation as

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) = \nabla\cdot\left(\frac{K}{\mu B}(\nabla P - \rho g \nabla D)\right) + \frac{q}{\rho_s} \quad (2.10)$$

3. Numerical Modelling Of Single-Phase Flow Equation

Consider the single-phase flow equation (2.10) given as

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) = \nabla\cdot\left(\frac{K}{\mu B}(\nabla P - \rho g \nabla D)\right) + \frac{q}{\rho_s} \quad (3.1)$$

The discretization is based on the following physical considerations:

- the reservoir is a block centred grid, having dimensions as shown in Figure (3.1); with impermeable external boundaries but has an internal boundary in the form of a production well which is located in the grid block 4 with a production rate of 100 STB/Day. The rock and fluid properties for physical problem are presented in Table (3.1).

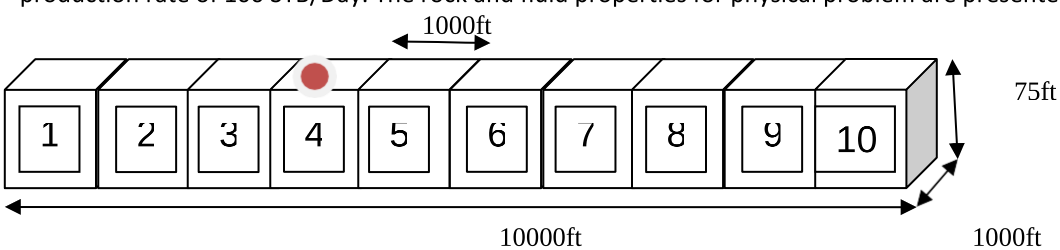

Figure 3.1: Reservoir grid blocks

Table 3.1: Reservoir rock and fluid properties

Grid block dimension (x, y, z)	$\Delta x = 100\,ft; \Delta y = 50\,ft; \Delta z = 75\,ft$
Permeability (x - direction), k_x	0.015 Darcy
Oil viscosity μ	10 Cp
Oil formation volume factor B	1.0 RB/STB
Porosity ϕ	0.19
Total compressibility C_t	0.0000035 ps^{-1}
Initial reservoir pressure, P_i	6000 Psia
Production rate, q	100 Day

3.1 Discretization Technique

In this model, we apply a slightly modified Implicit Backward-Difference Formulation. The implicit backward difference approximation to slightly compressible porous media flow results in an implicit calculation procedure for the new time level pressure. Hence it is used to find solutions by solving an equation involving both the current state of the system and in the later time. The procedure of implicit finite-difference formulation is illustrated as follows:
By expanding the first term on the right hand side (RHS) of equation (3.1) gives

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)=\frac{\partial}{\partial x}\left(\frac{k_x}{\mu B}\frac{\partial\Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y}{\mu B}\frac{\partial\Phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{k_z}{\mu B}\frac{\partial\Phi}{\partial z}\right)+\frac{q}{\rho_s} \quad (3.2)$$

where $\Phi = P\text{-}\rho gD$. By letting V_b be the volume of each grid block, then $A_x\Delta x$, $A_y\Delta y$ and $A_z\Delta z$ are the volumes of the grid block in the x , y and z directions respectively. Thus equation (3.2) becomes

$$V_b\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)=\frac{\partial}{\partial x}\left(\frac{A_xk_x}{\mu B}\frac{\partial\Phi}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\frac{A_yk_y}{\mu B}\frac{\partial\Phi}{\partial y}\right)\Delta y+\frac{\partial}{\partial z}\left(\frac{A_zk_z}{\mu B}\frac{\partial\Phi}{\partial z}\right)\Delta z+\frac{q}{\rho_s} \quad (3.3)$$

Since we are considering horizontal flow only; the y and z components as well as the gravitational force term are ignored. As a result, equation (3.3) reduces to

$$V_b\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)=\frac{\partial}{\partial x}\left(\frac{A_xk_x}{\mu B}\frac{\partial P}{\partial x}\right)\Delta x+\frac{q}{\rho_s} \quad (3.4)$$

discretizing the first term on the right-hand side of equation (3.4) results to

$$\begin{aligned}\frac{\partial}{\partial x}\left(\frac{A_xk_x}{\mu B}\frac{\partial P}{\partial x}\right)\Delta x &\approx \left(\frac{A_xk_x}{\mu B}\frac{\partial P}{\partial x}\right)_{i+\frac{1}{2}}-\left(\frac{A_xk_x}{\mu B}\frac{\partial P}{\partial x}\right)_{i-\frac{1}{2}} \\ &=\left(\frac{A_xk_x}{\mu B}\right)_{i+\frac{1}{2}}\left(\frac{\partial P}{\partial x}\right)_{i+\frac{1}{2}}-\left(\frac{A_xk_x}{\mu B}\right)_{i-\frac{1}{2}}\left(\frac{\partial P}{\partial x}\right)_{i-\frac{1}{2}}\end{aligned} \quad (3.5)$$

Equation (3.5) in fully implicit form gives

$$\begin{aligned}\frac{\partial}{\partial x}\left(\frac{A_xk_x}{\mu B}\frac{\partial P}{\partial x}\right)\Delta x &\approx \left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i+\frac{1}{2}}P_{i+1}^{n+1}-\left[\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i+\frac{1}{2}}+\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i-\frac{1}{2}}\right]P_i^{n+1} \\ &+\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i-\frac{1}{2}}P_{i-1}^{n+1}\end{aligned} \quad (3.6)$$

Since our reservoir is slightly compressible, the porosity ϕ must be a function of pressure; that is $\phi = \phi(P)$. With this information, the (RHS) of equation (3.4) becomes:

$$V_b\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)=\frac{V_b}{B}\frac{\partial\phi}{\partial t}=\frac{V_b}{B}\frac{\partial\phi}{\partial P}\frac{\partial P}{\partial t} \quad (3.7)$$

By defining total compressibility $C_t=\frac{1}{\phi}\frac{\partial\phi}{\partial P}$ implies

$$\phi C_t=\frac{\partial\phi}{\partial P} \quad (38)$$

$$V_b\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)=\frac{\phi V_bC_t}{B}\frac{\partial P}{\partial t} \quad (3.9)$$

Now expressing the change in pressure term over time in equation (3.9) in implicit form gives:

$$\frac{\partial P}{\partial t}=\frac{P_{i+1}^{n+1}-P_i^n}{\Delta t} \quad (3.10)$$

Substituting equation (3.6) and equation (3.10) into equation (3.4) results to:

$$\begin{aligned}\frac{\phi V_bC_t}{\Delta t}\left(P_{i+1}^{n+1}-P_i^n\right)&=\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i+\frac{1}{2}}P_{i+1}^{n+1}-\left[\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i+\frac{1}{2}}+\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i-\frac{1}{2}}\right]P_i^{n+1} \\ &+\left(\frac{A_xk_x}{\mu B\Delta x}\right)_{i-\frac{1}{2}}P_{i-1}^{n+1}+\frac{q}{\rho_s}\end{aligned} \quad (3.11)$$

Rearranging equation (3.11) gives:

$$\left(-\frac{A_x k_x}{\mu B \Delta x}\right)_{i-\frac{1}{2}} P_{i-1}^{n+1} + \left[\left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i+\frac{1}{2}} + \left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i-\frac{1}{2}}\right] P_i^{n+1} + \left[\frac{\phi V_b C_t}{\Delta t} - \left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i+\frac{1}{2}}\right] P_{i+1}^{n+1} \quad (3.12)$$
$$= \left(\frac{\phi V_b C_t}{\Delta t}\right)_{i+\frac{1}{2}} P_i^n + \frac{q}{\rho_s}$$

In equation (3.12), the quantities $P_{i+1}^{n+1}, P_i^{n+1} + P_{i-1}^{n+1}$ are all unknown parameters, and we cannot explicitly solve for P_i^{n+1} since P_{i+1}^{n+1} and P_{i-1}^{n+1} are also unknown. Subsequently, equation (3.12) can be solved for all the grid blocks and the unknowns simultaneously. In compact form, equation (3.12) is written as:

$$W_i P_{i-1}^{n+1} + C_i P_i^{n+1} + E_i P_{i+1}^{n+1} = Q_i \qquad (3.13)$$

Where $W_i = \left(-\frac{A_x k_x}{\mu B \Delta x}\right)_{i-\frac{1}{2}}$

$$C_i = \left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i+\frac{1}{2}} + \left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i-\frac{1}{2}}$$

$E_i = \frac{\phi V_b C_t}{\Delta t} - \left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i+\frac{1}{2}}$ and

$$Q_i = \left(\frac{\phi V_b C_t}{\Delta t}\right)_{i+\frac{1}{2}} P_i^n + \frac{q}{\rho_s}$$

Note that W_i, C_i and E_i are the coefficients of the pressures $P_{i-1}^{n+1}, P_i^{n+1} + P_{i+1}^{n+1}$ respectively with W and E being the directions, West and East respectively of grid cell i (i =1, 2, 3… 10) for grid-blocks 1 to 10 respectively) whose centre is C

3.2 INITIAL AND BOUNDARY CONDITIONS

Initial condition: $P(I,0)$ = 6000 psi (at time-level $n = 0$; *pressure* $P_i^n = 6000$ psi at all grid-blocks).

Boundary condition: $\frac{\partial P}{\partial x} = 0$ both at the left and right boundaries; that is grid-block 1 and 10 respectively). Writing equation (3.13) for each grid-block gives a tri-diagonal matrix representation of the form:

$$\begin{bmatrix} (C_1+W_1) & E_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_2 & C_2 & E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & W_3 & C_3 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & W_4 & C_4 & E_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W_5 & C_5 & E_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W_6 & C_6 & E_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & W_7 & C_7 & E_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & W_8 & C_8 & E_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_9 & C_9 & E_9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_{10} & (C_{10}+W_{10}) \end{bmatrix} \begin{bmatrix} P_1^{n+1} \\ P_2^{n+1} \\ P_3^{n+1} \\ P_4^{n+1} \\ P_5^{n+1} \\ P_6^{n+1} \\ P_7^{n+1} \\ P_8^{n+1} \\ P_9^{n+1} \\ P_{10}^{n+1} \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \end{bmatrix} \qquad (3.14)$$

Using MATLAB and applying iterative linear solver technique to solve for the unknown pressures in equation (3.14), the results of the unknown pressures produced in two-dimensional and three-dimensional plots are presented and discussed.

4. RESULTS AND DISCUSSION

4.1 NUMERICAL EXAMPLE OF SINGLE-PHASE FLOW

We have considered a horizontal, one dimensional fluid (oil) flow for the block centred grid as shown in Figure (3.1). The reservoir has impermeable external boundaries but has an internal boundary in the form of a production well which is located in the grid block 4 at production rate of 100 STB/Day with an initial pressure of 6000 psi.

Figure 4.1 depicts the results of simulations obtained from the implicit scheme with a time step size of 1 day. There is a rapid pressure decline at grid block 4 at day 1. Similar trend is observed at grid blocks 3 and 5. This is because there is a gradual decrease in pressure first at the adjacent blocks (blocks 3 and block 5) due to their closeness to the producing well (grid block 4) and then moved out to the nearby blocks, thereby affirming the statement, the nearer the grid block is to the producing well, the higher the pressure drop during fluid withdrawal. Grid blocks 3 and 5 displayed identical pressure drop values from day 1 up to day 70 because they are symmetrically located at grid block 4 before pressure drop variation was seen from day 71 to day 360. Similar trend is also seen for grid blocks 2 and 6 from day 1 up to day 40 due to the equi-distance positions; West and East of the grid block 4, then a pressure decline was seen from day 41 to the last day. For grid block 2 and 6, the same pressure values were recorded from day 1 to day 45. The perfect trend that was observed maybe due to their placement side-by-side with grid block 3 and 5 respectively, therefore less pressure disturbance due to fluid withdrawal. However, the change in their respective pressure values can be ascribed to the pressure disturbances occurring at the immediate grid blocks (grid block 1 and 7) due to fluid withdrawal. Comparatively, grid blocks 1 and 7 recorded the same trend (constant pressure values) as seen in grid blocks 2 and 6 from the start of the simulation until a decline, at day 25. The slight difference in their pressure values is because of the pressure transient reaching grid block 8 at day 25, hence causing a drop in pressure for the first time at grid block 8. On the other hand, a smooth pressure decline was also seen in grid blocks 8 and 9 due to their wide distance from the producing well. However, the grid block 10 which on the right boundary maintained a constant pressure from simulation time of 1 day until a decline was seen on day 50. We observed similar trend with time step of 2 and 3 days as depicted in (Figures 4.3 and 4.5) respectively. It is worth mentioning that the implicit formulation is unconditionally stable as observed by [13]. In other words, it is impossible for the formulation to exhibit any unstable behaviour regardless of the time step or grid block size. However, this condition is not always suitable as large time steps and block sizes may result in impractical approximations. Due to this unconditional stability of the implicit formulation, it is the widely used formulation in petroleum reservoir simulation, [14]. The 3D view of the pressure distribution along the horizontal grid blocks 1 to 10 with time step of 1, 2 and 3 days are displayed in (Figures 4.2, 4.4 and 4.6) respectively.

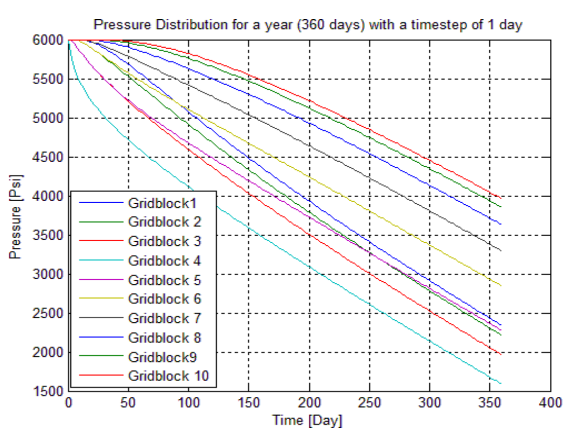

Figure 4.1: Pressure distribution for a year with a time step of 1 day.

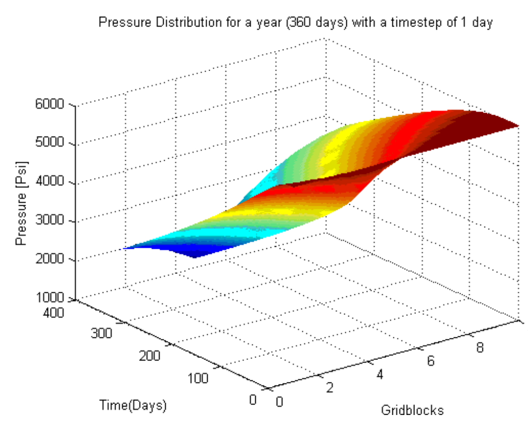

Figure 4.2. 3-D view of pressure distribution for a year with a time step of 1 day

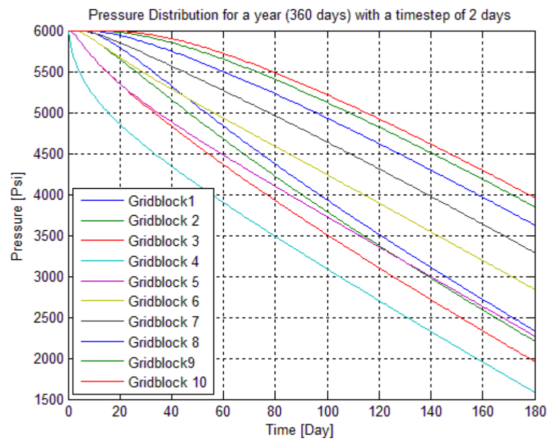

Figure 4.3: Pressure distribution for a year with a time step of 2 days

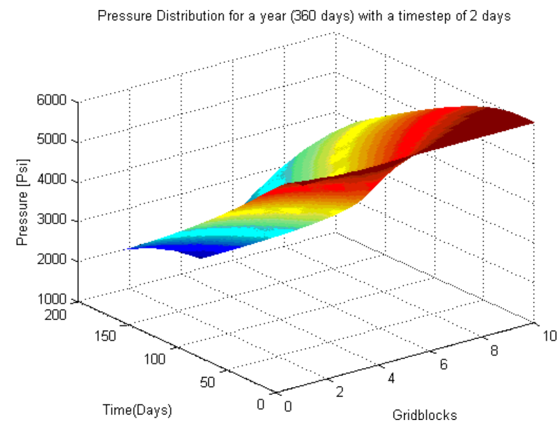

Figure 4.4: 3-D view of pressure distribution for a year with a time step of 2 days

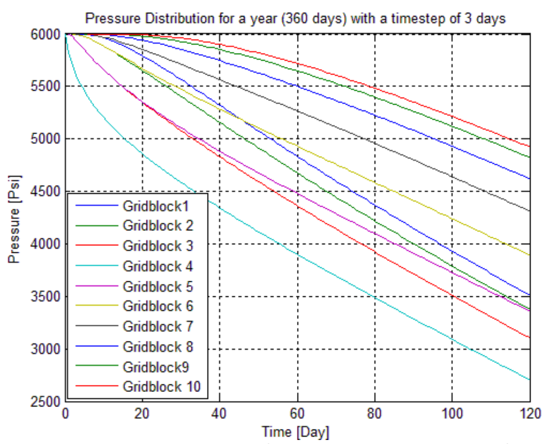

Figure 4.5: Pressure distribution for a year with a time step of 3 days

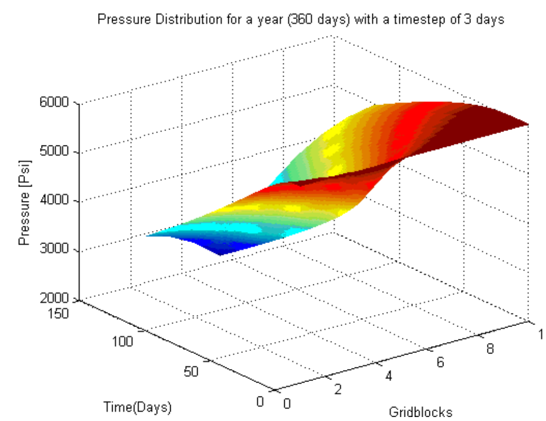

Figure 4.6: 3-D view of pressure distribution for a year with a time step of 3 days

4.2 CONCLUSION

We have developed a single-phase flow equation in a porous medium. We numerically solved and simulated the single-phase flow equation with MATLAB version 7b and investigated the pressure distribution both at the production well and adjacent grid blocks for time step 1, 2 and 3 days respectively within a period of one year. Our results are in line with what is obtainable in practical scenarios. Sufficient understanding of pressure distribution within and the vicinity of production wells would be of great asset to oil exploration practitioners. This will aid in the establishment of effective reservoir monitoring and pressure maintenance plans for improved ultimate recovery from the target reservoir and other reservoir systems alike.

Acknowledgments:

This work is part of the PhD thesis of the second author supervised by the first author. The first author is currently also partly supervising the third author who is working on extending compressible fluid flow in 3-dimensions. We are grateful to the Department of Mathematics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria for giving us the enabling platform for this research.

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