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A PRODUCTION INVENTORY MODEL WITH CONSTANT PRODUCTION RATE, LINEAR LEVEL DEPENDENT DEMAND AND CONSTANT HOLDING COST

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Abstract

In this paper, a production inventory model with a constant holding cost is considered. The demand is level dependent linear with time. The model determines the total average optimal inventory cost and optimal time cycle. The production starts with a buffer stock reaching the desired level inventory and begins to deplete due to demand and deterioration. The cost function has been shown to be convex and a numerical example is given to show the application of the model. A sensitivity analysis is then carried out on the example to see the effect of parameter change.

Keywords: Production Inventory, economic order quantity, Deteriorating items, Depreciation

1. Introduction

Inventory handling is an important part of manufacturing, retail and distribution infrastructure.

It is normal that a large quantity of goods on shelves in a superstore will lead the costumer to buy more goods and that situation creates greater demand of the goods. It motivates the retailers to increase their order quantities in manufacturing companies. When items are internally produced instead of being obtained from an outside supplier, the Economic Production Quantity (EPQ) model is often employed to determine the optimal production size that minimizes overall production inventory costs. Sing and Mishra [1] developed an EPQ model for perishable items by considering demand as a power form function of the inventory level. Deterioration increases with time in case of fast deteriorating items like tomatoes, meat, vegetables, fruits and so on. Behrouz and Babak [2], developed an EPQ model by considering both the depreciation cost of stored items and process quality cost. Depreciation cost and process quality cost were assumed to be continuous functions of holding time and of production run. Ata *et al* [3] discussed an EPQ model for multi products, single machine inventory with discrete delivery. Taleizadeh *et al* [4] focused on EPQ model with production capacity limitation and a random defective production rate. Gede and Hui [5] analyze an EPQ model for deteriorating items with stochastic machine unavailability and price dependent demand. Jinn *et al.,* [6] used time varying demand and cost to analyze an EPQ model and characterize the influences of both demand and cost over the length of production run time and Economic Production Quantity.

Disruption in production system is a common phenomenon. Khedleker [7] attempted to establish exponential demand in disrupted production system and determined production time before and after disruption. Kapik and Navin [8] developed solution procedure which helps, to take decision whether to rent a warehouse or not. Demand consideredwas exponential, time dependent and for deteriorating items. Andrak and Borade [9] developed an EPQ model with inventory dependent demand and deterioration. Maragatham and Palani [10] studied on an inventory model for deteriorating items with lead time, price dependent demand and shortages.

Ghare and Schrader [11] were the pioneers to use the concept of deterioration, in developing an inventory model with a constant rate of deterioration. This was followed by Convert and Philip [12] who formulated a model considering rate of deterioration to be a two parameter Weibull distribution. Nahmias [13]studied the problem of determining suitable ordering policies for fixed life perishable inventory, subject to continuous exponential decay. Mishara [14] analyzed an inventory model with a variable rate of deterioration, finite rate of replenishment and no shortages, but only a special case of the model was solved under very restrictive assumptions.

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In this paper a production and inventory model is presented with a linear level dependent demand. The demand during production is assumed to be less than the demand after production. There is a small amount of decay during and after production and a constant holding cost h is considered. The difference between this work and that of Shirajul Islam and Sharafuddin [15] is the fact in Shirajul Islam and Sharafuddin [15] the demand rate is the same during and after production whereas in this paper the demand rate during production is different from the demand rate after production.

Assumptions

Production rate β is always constant and greater than the demand rate at all times.

The rate of decay μ is constant and small.

The demand rate during production at any instant t is given by $a+bI_1(t)$, where a and b are constants and satisfying the condition that $\beta > a+bI_1(t)$.

The demand rate after production is $\gamma + dI_1(t)$ and assumed to be greater than demand during production at any instant t where d and γ are constants.

Production starts with buffer inventory.

Inventory level is highest at the end of production and after this point, the inventory depletes due to demand and deterioration.

Shortages are not allowed.

Notations

$I_1(t)$ = Inventory level at any instance t

L_1 = Un decayed inventory for the period from 0 *to* t_1

L_2 = Un decayed inventory for the period from t_1 *to* T_1

R_1 = Deteriorated Inventory for the period from 0 *to* t_1

R_2 = Deteriorated Inventory for the period from t_1 *to* T_1

Z_1 and Z are the inventory levels at time $t=0,\ t_1$ and T_1 respectively. Here Z is the buffer stock.

dt = Very small portion of instance t

A_o = Set up cost

ρ = Holding cost per unit

$TC(T_I)$ = Total average inventory cost in a unit time.

t_I = Time when inventory gets to the maximum level

T_I = Total cycle time

Z_1^* = Optimal order quantity

t_I^* = Optimal time for the maximum inventory

T_I^* = Optimal Order Interval

$TC(T_1)^*$ = Optimal average inventory cost per unit time

Model formulation

At the beginning, while $t=0$, the production starts with a buffer stock where the production rate β is constant for the entire period during production.

The inventory increases at the rate of $\beta - a - bI_1(t) - \mu I_1(t)$ between $t=0$ *to* t_1. The market demand is $a+bI_1(t)$ and $\mu I_1(t)$ is the decay of $I_1(t)$ inventory at an instance t. By using the above facts, we formulate the differential equation of the problem as below:

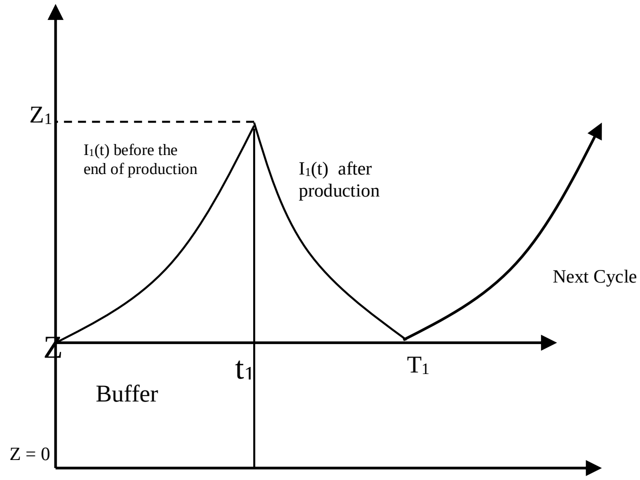

Figure 1: Inventory situation before and after production

$I_1(t+dt) = I_1(t) + \{(\beta - a - bI_1(t))dt - \mu I_1(t)dt\}$ or

$I_1(t+dt) - I_1(t) = \{\beta - a - bI_1(t)dt - \mu I_1 t\}dt$ or

$\lim_{dt\to 0} \frac{I_1(t+dt) - I_1(t)}{dt} = \beta - a - bI_1(t) - \mu I_1(t)$ or

$\frac{d}{dt} I_1(t) + \mu I_1(t) = \beta - a - bI_1(t)$

$$\therefore I_1(t) = \frac{\beta - a}{\mu + b} + Ke^{-(\mu+b)t} \quad (1)$$

which is the general solution of the differential equation

Applying the following initial condition, $I_1(t) = Z$ at $t = 0$ we get

$$\therefore K = Z - \frac{\beta - a}{\mu + b} \quad (2)$$

$$I_1(t) = \frac{\beta - a}{\mu + b} + \left(Z - \frac{\beta - a}{\mu + b}\right)e^{-(\mu+b)t} \quad (3)$$

From the other boundary condition i.e. at $t = t_1, I_1(t) = Z_1$ taking up to the first degree of μ we get

$$Z_1 = \frac{\beta - a}{\mu + b} + \left\{Z - \frac{\beta - a}{\mu + b}\right\}e^{-(\mu+b)t_1} \quad (4)$$

$$Z_1 = \frac{\beta - a}{\mu + b} + \left\{Z - \frac{\beta - a}{\mu + b}\right\}\{1 - (\mu + b)t_1\}$$

$$= Z + \{-Z\mu - Zb + \beta - a\}t_1$$

$$= Z + \{\beta - a - Z\mu - Zb\}t_1 \quad (5)$$

Using equation (3) and considering up to the second degree of μ for convenience the total un decayed inventory in the period $t = 0$ *to* t_1 we get

$$L_1 = \int_0^{t_1} I_1(t)\,dt = \int_0^{t_1}\left[\frac{\beta - a}{\mu + b} + \left(Z - \frac{\beta - a}{\mu + b}\right)e^{-(\mu+b)t}\right]dt$$

$$= \left[\left(\frac{\beta - a}{\mu + b}\right)t\Big|_0^{t_1} + \left\{Z - \frac{\beta - a}{\mu + b}\right\}\frac{e^{-(\mu+b)t}}{-\mu + b}\Big|_0^{t_1}\right]$$

$$= \left(\frac{\beta - a}{\mu + b}\right)t_1 - \left\{Z - \frac{\beta - a}{\mu + b}\right\}\left\{\frac{e^{-(\mu+b)t_1} - 1}{\mu + b}\right\}$$

$$\therefore L_1 = \frac{(\beta - a)t_1}{\mu + b} - \left\{Z - \frac{\beta - a}{\mu + b}\right\}\left(\frac{1}{\mu + b}\right)\left\{1 - (\mu + b)t_1 + \frac{1}{2}(\mu + b)^2 t^2{}_1 - 1\right\}$$

$$= \frac{(\beta - a)t_1}{\mu + b} + Zt_1 - \frac{Z(\mu + b)t_1^2}{2} - \frac{(\beta - a)t_1}{\mu + b} + \frac{(\beta - a)(\mu + b)t_1^2}{2(\mu + b)}$$

$$= Zt_1 - \frac{1}{2}Z(\mu+b)t_1^2 + \frac{1}{2}(\beta-a)t_1^2 \qquad (6)$$

Now we calculate the deteriorated items as follows:

$$R_1 = \int_0^{t_1} \mu I_1(t)dt = \mu\int_0^{t_1}\left[\frac{\beta-a}{\mu-b} + \left\{Z - \frac{\beta-a}{\mu+b}\right\}e^{-(\mu+b)t}\right]dt$$

$$R_1 = Z\mu t_1 - \tfrac{1}{2}Z(\mu+b)\mu t_1^2 + \tfrac{1}{2}(\beta-a)\mu t_1^2 \qquad (7)$$

On the other hand, the inventory decreases at the rate of $\gamma + dI_1(t) + \mu I_1(t)$ during $t = t_1$ to T_1 since there is no production after time t_1.

The demand after production is assumed to be greater than the demand during production. The inventory depletes due to market demand and deterioration of the items. Similar approach is applied as before to obtain

$$\Rightarrow \frac{d}{dt}I_1(t) + \mu I_1(t) = -\gamma - dI_1(t)$$

$$I_1(t) = \frac{-\gamma}{\mu+d} + De^{-(\mu+d)t} \qquad (8)$$

which is the general solution of the differential equation.

Applying the boundary condition when $t = T_1, I_1(t) = Z$, we get

$$\Rightarrow D = \left(Z + \frac{\gamma}{\mu+d}\right)e^{(\mu+d)T_1}$$

$$I_I(t) = \frac{-\gamma}{\mu+d} + \left(Z + \frac{\gamma}{\mu+d}\right)e^{(\mu+d)(T_1-t)} \qquad (9a)$$

Now putting the boundary condition $I_1(t) = Z_1$ when $t = t_1$, taking up to the first degree of μ we get

$$Z_1 = \frac{-\gamma}{\mu+d} + \left(Z + \frac{\gamma}{\mu+d}\right)e^{(\mu+d)(T_1-t_1)} \qquad (9b)$$

$$Z_1 = \frac{-\gamma}{\mu+d} + \left\{Z + \frac{\gamma}{\mu+d}\right\}\left\{1 + (\mu+d)(T_1-t_1)\right\}$$

$$= Z + \left\{\gamma + Z(\mu+d)\right\}(T_1-t_1) \qquad (10)$$

Using Eqn (9a), we get the un decayed inventory from $t = t_1$ to T_1 and considering up to the first degree of μ we get

$$L_2 = \int_{t_1}^{T_1} I_1(t)dt = \int_{t_1}^{T_1}\left[\frac{-\gamma}{\mu+d} + \left\{Z + \frac{\gamma}{\mu+d}\right\}e^{(\mu+d)T_1}e^{-(\mu+d)t}\right]dt$$

$$= \frac{(-\gamma)t}{\mu+d}\Big|_{t_1}^{T_1} + \left[\left(Z + \frac{\gamma}{\mu+d}\right)\frac{e^{(\mu+d)(T_1-t)}}{-(\mu+d)}\Big|_{t_1}^{T_1}\right]$$

$$= \frac{-\gamma(T_1-t_1)}{\mu+d} - \left[\left(Z + \frac{\gamma}{\mu+d}\right)\left(\frac{1}{\mu+d}\right)\left\{1 - e^{(\mu+d)(T_1-t_1)}\right\}\right]$$

$$= \frac{-\gamma}{\mu+d}(T_1-t_1) + \left(Z + \frac{\gamma}{\mu+d}\right)(T_1-t_1)$$

$$\therefore L_2 = Z(T_1-t_1) \qquad (11)$$

Considering the decay of the items, we calculate the deteriorated items during the period t_1 to T_1 as follows:

$$R_2 = \int_{t_1}^{T_1} \mu I_1(t)dt = \int_{t_1}^{T_1}\mu\left[\frac{-\gamma}{\mu+d} + \left\{Z + \frac{\gamma}{\mu+d}\right\}e^{(\mu+d)(T_1-t)}\right]dt$$

$$R_2 = Z\mu(T_1-t_1) \qquad (12)$$

Equating equations 5 and 10:

$$Z + \{\beta - a - Z\mu - Zb\}t_1 = Z + \{\gamma + Z(\mu+d)\}(T_1-t_1)$$

$$\Rightarrow t_1 = \frac{\{\gamma + Z(\mu+d)\}T_1}{\gamma - a + Z(-b+d) + \beta} \qquad (13)$$

Now let

$$\delta = \frac{\gamma + Z(\mu+d)}{\gamma - a + Z(-b+d) + \beta} \qquad (14)$$

$$\Rightarrow t_1 = \delta T_1 \tag{15}$$

$$TC(T_1) = \frac{A_o + \rho(L_1 + R_1 + L_2 + R_2)}{T_1} \tag{16}$$

The total cost function is given by substituting equations (6), (7), (11) and (12) into equation (16) to get

$$= \frac{1}{T_1}\left\{A_0 + \rho\left\{\begin{array}{l} Zt_1 - \frac{1}{2}Z(\mu+b)t_1^2 + \frac{1}{2}(\beta-a)t_1^2 + Z\mu t_1 - \frac{1}{2}Z(\mu+b)\mu t_1^2 + \frac{1}{2}(\beta-a)\mu t_1^2 + Z(T_1-t_1) \\ +Z\mu(T_1-t_1)\end{array}\right\}\right\}$$

$$\therefore TC(T_1) = \frac{A_0}{T_1} + \frac{\rho Z(1+\mu)t_1}{T_1} - \frac{\rho Z(\mu+b)(1+\mu)t_1^2}{2T_1} + \frac{\rho(\beta-a)(1+\mu)t_1^2}{2T_1} + \frac{\rho Z(T_1-t_1)(1+\mu)}{T_1}$$

Substituting $t_1 = \delta T_1$, the last equation becomes

$$TC(T_1) = \frac{A_0}{T_1} + \frac{\rho Z(1+\mu)T_1\delta}{T_1} - \frac{\rho Z(\mu+b)(1+\mu)\delta^2 T_1^2}{2T_1} + \frac{\rho(\beta-a)(1+\mu)\delta^2 T_1^2}{2T_1} + \frac{\rho Z(1+\mu)(1-\delta)T_1}{T_1}$$

$$= \frac{A_0}{T_1} + \rho Z(1+\mu)\delta - \frac{\rho Z(\mu+b)(1+\mu)\delta^2 T_1}{2} + \frac{\rho(\beta-a)(1+\mu)\delta^2 T_1}{2} \tag{17}$$

$$+\rho Z(1+\mu) - \rho Z(1+\mu)\delta$$

The main objective is to find the value of T_1 which gives the minimum variable cost per unit time. The necessary and sufficient condition to minimize $TC(T_1)$ are respectively

i. $\frac{dTC(T_1)}{dT_1} = 0$

ii. $\frac{d^2TC(T_1)}{dT_1^2} > 0$

To satisfy the necessary condition, we have to differentiate equation (17) with respect to T_1 as follows:

$$\frac{dTC(T_1)}{dT_1} = -\frac{A_0}{T_1^2} - \frac{\rho Z(\mu+b)(1+\mu)\delta^2}{2} + \frac{\rho(\beta-a)(1+\mu)\delta^2}{2} \tag{18}$$

Equating equation (18) to zero in order to determine the value of T_1 which minimizes the variable cost per unit time we obtain:

$$\frac{dTC(T_1)}{dT_1} = -\frac{A_0}{T_1^2} - \frac{\rho Z(\mu+b)(1+\mu)\delta^2}{2} + \frac{\rho(\beta-a)(1+\mu)\delta^2}{2} = 0$$

$$\Rightarrow \frac{A_0}{T_1^2} = -\frac{\rho Z(\mu+b)(1+\mu)\delta^2}{2} + \frac{\rho(\beta-a)(1+\mu)\delta^2}{2}$$

Substituting the value of δ from equation (14), i.e. $\delta = \frac{\gamma + Z(\mu+d)}{\gamma - a + Z(-b+d) + \beta}$ we get

$$\frac{A_0}{T_1^2} = -\frac{\rho Z(\mu+b)(1+\mu)}{2}\left\{\frac{[\gamma + Z(\mu+d)]}{[\gamma - a + Z(-b+d) + \beta]}\right\}^2 + \frac{\rho(\beta-a)(1+\mu)}{2}\left\{\frac{[\gamma + Z(\mu+d)]}{[\gamma - a + Z(-b+d) + \beta]}\right\}^2$$

$$T_1 = \sqrt{\frac{2A_0[\gamma - a + Z(-b+d) + \beta]^2}{\rho(-Z(\mu+b) + \beta - a)(1+\mu)\{\gamma + Z(\mu+d)\}^2}} \tag{19}$$

Now with the help of equation (15) and (19) we get the value of t_1 as below

$$t_1 = \delta T_1 = \frac{\gamma + Z(\mu+d)}{\gamma - a + Z(-b+d) + \beta}T_1$$

$$\therefore t_1 = \sqrt{\frac{2A_0}{\rho(-Z(\mu+b) + \beta - a)(1+\mu)}} \tag{20}$$

Theorem 1: The cost function $TC(T_1)$is convex.

Proof: From equation (18) we note that

$$\frac{dTC(T_1)}{dT_1} = -\frac{A_0}{T_1^2} - \frac{\rho Z(\mu+b)(1+\mu)\delta^2}{2} + \frac{\rho(\beta-a)(1+\mu)\delta^2}{2}$$

$$\therefore \frac{d^2TC(T_1)}{dT_1^2} = \frac{2A_0}{T_1^3} > 0 \tag{21}$$

Therefore, the convex property (ii) is satisfied i.e. $\frac{d^2TC(T_1)}{dT_1^2} > 0$ as A_0 and $T_1$are both positive. We conclude that the total cost function (17) is convex in T_1. Hence, there is optimal solution at T_1.

Numerical Example

To illustrate the model developed, an example is considered based on the following values of parameter A_0 = ₦ 100 set up cost. β = 50, Z =10, ρ =5, γ =5.5 b= 0.4 d =0.8 a= 5 and μ =0.01 per unit time. Substituting the above parameters into equations (5), (17), (19) and (20) and simplifying gives Z_1* = 50.26571,$TC(T_1)^*$ = ₦101.2183, T_1^*=3.945205 (1441days) and t_l^*= 0.984492 units.

Sensitivity analysis

We study the effect and changes of parameters A_0, β, Z, ρ, γ, a, b, d andμ on the optimal time for maximum inventory t_1^*,optimal cycle time T_1^*, optimal order quantity Z_1^* and the total average inventory cost per unit time $TC(T_1)^*$. We perform the sensitivity analysis by changing each of the parameters by 50%, 25%, 10%, 5% -5% -10% -25% and -50% taking one parameter at a time while keeping the other parameters unchanged. The details are shown in Table 1, below.

Table 1: Sensitivity Analysis on the numerical example to see changes in the values T_1^*, $TC(T_1)$*, Z_1^* and t_1^*as a result of changes in other parameters.

Parameter	**% change in parameter**	**t_1^***	**T_l^***	**Z_1^***	**$TC(T_1)^*$**
A_o	50%	1.20737	4.838356(1766 days)	59.38142	112.6171
	25%	1.100716	4.41095 (1611 days)	55.0193	107.2048
	10%	1.03234	4.13698(1511 days)	52.22307	103.6938
	5%	1.00842	4.041096(1476 days)	51.24439	102.4708
	0%	**0.984492**	**3.945205(1441 days)**	**50.26571**	**101.2183**
	-5%	0.959196	3.843836 (1404 days)	49.2311	99.93409
	-10%	0.9339	3.742466 (1367 days)	48.1965	98.61561
	-25%	0.852542	3.416438 (1248 days)	44.86898	94.42335
	-50%	0.695981	2.78904 (1019 days)	38.46562	86.36326
Z	50%	1.009916	3.232877 (1181 days)	54.23525	137.6303
	25%	0.997316	3.542466 (1294 days)	52.26797	119.6237
	10%	0.989503	3.769863 (1377 days)	51.06498	108.6294
	5%	0.986954	3.854795 (1408days)	50.6641	104.9321
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	0.981994	4.041096 (1476days)	49.86485	97.48781
	-10%	0.979337	4.142466(1513days)	49.45642	93.74049
	-25%	0.972116	4.4933151 (1641days)	48.25597	82.3959
	-50%	0.960357	5.279452(1928days)	46.24735	63.13666
β	50%	0.77567	4.534247 (1655days)	61.11664	94.6342
	25%	0.861435	4.243836(1550days)	56.00064	97.64055
	10%	0.929941	4.068493(1485 days)	52.65556	99.71407
	5%	0.955693	4.00519(1463days)	51.47709	100.4541
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	1.016059	3.884932(1419days)	49.0667	102.0065
	-10%	1.050815	3.824658 (1337days)	47.72426	102.8171
	-25%	1.181683	3.649315 (1333days)	43.5598	105.3416
	-50%	1.578825	3.424658(1252days)	35.10332	108.9221
ρ	50%	0.804685	3.221918(1177days)	42.88366	137.8671
	25%	0.880573	3.528767(1289days)	46.01544	119.8300
	10%	0.93865	3.76144(1375days)	48.39223	108.7439
	5%	0.963298	3.84731(1406days)	49.28703	104.99600
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	1.009788	4.046575(1478days)	51.30031	97.40909
	-10%	1.037818	4.158904(1518days)	52.44677	93.56562
	-25%	1.13627	4.553425(1663days)	56.47334	81.79835
	-50%	1.391962	5.578082(2037days)	66.93124	61.11327
μ	50%	0.982622	3.923288(1433days)	50.14009	101.7495
	25%	0.983562	3.934247(1437days)	50.20308	101.4839
	10%	0.983847	3.9397926(1439days)	50.22951	101.3246
	5%	0.98417	3.942466(1440days)	50.24762	101.2716
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	0.984713	3.94794(1442days)	50.281837	101.1546
	-10%	0.984851	3.947945(1443days)	50.29389	101.1122
	-25%	0.985412	3.956164 (1445 days)	50.32797	100.9528
	-50%	0.986321	3.967123(1449days)	50.38986	100.6875

A	50%	1.016059	3.884932(1419days)	49.01667	102.0067
	25%	1.999905	3.915068(1430days)	49.6095	101.6096
	10%	0.990157	3.931507 (1436days)	50.0235	101.3742
	5%	0.987655	3.939726(1439days)	50.14817	101.2964
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	0.981357	3.95068591443days)	50.38286	101.1408
	-10%	0.978252	3.956164(1445days)	50.49962	101.0636
	-25%	0.96977	3.975342(1452days)	50.87579	100.8332
	-50%	0.955693	4.005479(1463days)	51.47709	100.4541
B	50%	1.009221	3.89589(1423days)	49.2587	101.8472
	25%	0.996625	3.920548(1434days)	49.76535	101.5309
	10%	0.989016	3.934247(1437days)	50.05514	101.3431
	5%	0.986745	3.939726(1439days)	50.16054	101.2806
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	0.982254	3.950685(1443days)	50.37066	101.1565
	-10%	0.979355	3.953425(144days)	50.44735	101.0945
	-25%	0.972795	3.969863(1450days)	50.76012	100.9099
	-50%	0.960853	3.991781(1458days)	51.2206	100.6050
γ	50%	0.984306	3.446575(1269days)	50.2581	108.5451
	25%	0.984658	3.673973(1342days)	50.27249	104.9719
	10%	0.984495	3.830137(1399days)	50.26584	102.7423
	5%	0.984088	3.884932(1419days)	50.2492	101.9842
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	0.984288	4.005479(1463days)	50.25738	100.4449
	-10%	0.98413	4.068493(1486days)	50.25093	99.66336
	-25%	0.984147	4.276712(1562days)	50.25161	97.27054
	-50%	0.984549	4.69589 (1715 days)	50.26805	93.11297
d	50%	0.984166	3.271233(1195days)	50.25238	111.6477
	25%	0.984148	3.56484 (1302days)	50.25167	106.6176
	10%	0.984518	3.780822(1381days)	50.26678	103.4252
	5%	0.984405	3.860274 (1410 days)	50.26217	102.3297
	0%	**0.984492**	**3.945205 (1441 days)**	**50.26571**	**101.2183**
	-5%	0.984661	4.035616 (1474 days)	50.22262	100.0907
	-10%	0.984138	4.128767 (1508 days)	50.25125	98.94601
	-25%	0.984297	4.45495 (1627days)	50.27777	95.40774
	-50%	0.984348	5.178082(1891days)	50.25985	89.13891

Discussion of Results

Observing Table 1 carefully, we make the following deductions:

i. With increase in the value of the parameter A_o (set up cost), the values of T_1*, t_1*, Z_1*, and $TC(T_1)$* all increase. This means that increase in set up cost will result in the increase of the optimal time for maximum inventory t_1*, optimal cycle time T_1*, optimal production quantity Z_1* and optimal total average inventory cost per unit time $TC(T_1)$*. This is clearly expected since excess stocking is encouraged as a result of higher set up cost. The total average inventory cost per unit time is therefore expected to increase due to increase in stocking cost. The values of T_1*, t_1* and Z_1*all increase due to high set up cost as well as stock holding cost.

ii. With increase in the value of Z (Buffer Stock), the values of t_1*, Z_1*, and $TC(T_1)$* increase while the value of T_1* decreases. This is because if Z increases the total average inventory cost increases due to increase in the value of the holding cost for buffer stock .The inventory produced takes shorter time to finish and this forces a reduction of optimal cycle time T_1*. The values of t_1* and Z_1* increase. Probably because Z increases.

iii. With increase in the value of the parameter β (production rate), there is decrease in the values of t_1*and $TC(T_1)$* but increase in the values of T_1*and Z_1*. The value of t_1* decreases due to increase in the production rate as it is seen in equation (20). The value of T_1* increases probably since much has been produced and so takes longer time to finish. The value of the total average inventory cost per unit time $TC(T_1)$*, increases due to higher holding cost.

iv. With increase in the value of the parameter ρ (holding cost), the values of T_1*, t_1* and Z_1*decrease while $TC(T_1)$* increases. This is so because increase in the holding cost of the items will also increase the stocking cost and so increases the total average inventory cost per unit time. To reduce the stocking holding cost, the model now lowers the value of Z_1* thereby reducing both t_1* and the cycle time T_1*.

v. With increase in the value of the parameter μ (deterioration rate), the values of t_1*, T_1^* and Z_1^* decreases. While the value of $TC(T_1)$* increases. The total average inventory cost per unit time increases which is expected because the deterioration cost is high. The model now forces a decrease in the value of T_1* thereby resulting to a decrease of the optimal production quantity Z_1* and time for maximum inventory also reduces.

vi. With increase in the value of the parameter a (constant part of the demand during production), the values of t_1*and $TC(T_1)$* increase while the values of T_1*and Z_1* decrease. This is so because since a higher the demand rate is high, stock will take less time to finish and so T_1* and Z_1* reduce. Increasing the demand will also in turn increase the optimal time for maximum inventory and the total average inventory cost per unit time.

vii. With an increase in the value of the parameter b (stock dependent part of the demand before production), the values of t_1* and $TC(T_1)$* increase, while the values of T_1*and Z_1* decrease. This is expected since if the stock dependent demand rate is higher, the inventory will finish earlier and so T_1* and Z_1* will decrease. Increasing the demand will also in turn increase the optimal time for maximal inventory t_1*, and the total average inventory cost per unit time $TC(T_1)$*.

viii. With increase in the value of the parameter γ (constant part of the demand part after production). The values of t_1* and Z_1* are unstable, while the values of T_1* decreases and $TC(T_1)$* increases. This is so because increase in the value of the parameter γ will result to higher demand. The stock will finish earlier and this will lower the value of T_1^*.This will in turn to increase the value of total average inventory cost per unit time.

ix. With increase in the value of the parameter d (stock dependent part of the demand after production), the values of t_1* and Z_1* are on unstable, while T_1* decreases and $TC(T_1)$* increase. This is so because if the stock dependent part of the demand rate is higher, the overall demand will also be higher and so stock will finish in time thereby resulting to lower value of T_1^*.This will also in turn to increase the value of the total average cost per unit time $TC(T_1)$*.

Conclusion

In this paper, we present a production inventory model for items with little decay starting with buffer stock. The demand during production is assumed to be different from the demand after production even though they are both linear level dependent. The production rate is constant. The cost function has shown to be convex and a numerical example is given is given to show the application of the model. A sensitivity analysis is then carried out to see the effect of parameter changes of the model.

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