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EFFECT OF FAULT CONFIGURATION ON REGIONAL GROUNDWATER FLOW

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Abstract

A model of the hydraulic head drawdown as a mathematical boundary-value problem is developed. This paper presents an analytic solution for calculating the drawdown in a pumping aquifer that has been intersected by a finite transmissivity fault or other similar permeable structures. The method enables one to locate such linear features and estimate their hydraulic properties. The governing equation can be used for the evaluation of drawdowns in pumping and observation wells. The transient solution presented allows one to find the drawdown at the pumping and observation wells in an unconfined aquifer of seemingly infinite areal extent.

***Keywords*:** Groundwater, Drawdown, Fault, Aquifer, Borehole

1. Introduction

Groundwater is a valuable and vital resource. Water covers over 70% of the earth's surface and although less than 5% of this water is defined as groundwater. It is the largest source of freshwater which is vital to all life. The natural superior quality of groundwater as well as the reliability of its supply makes it a resource to be valued. Groundwater must be studied and analyzed to efficiently use the resource without exploiting the source [1]. The determination of the aquifer hydraulic properties is a basic component of most ground water supply and contaminant transport investigations. A frequent method used for estimating
hydraulic properties is graphical type curve analysis of aquifer tests in which dimensionless type curves derived from an assumed analytical model of groundwater flow to a pumped well are used to analyze time drawdown measurements of hydraulic head in observation and piezometers [2]

Drawdown is the difference between the elevation of the piezometric surface prior to pumping $z\ (r, t = 0)$ and the elevation of the piezometric surface during pumping $z(\ r, t = 0)$ Thus $s(r, t) = z\ (r, t = 0) - z\ (\ r, t > 0)$ [3]. Or drawdown is the reduction in hydraulic head observed at a well in an aquifer, typically due to pumping a well as part of an aquifer test or well test [4].

One of the most important reasons for measuring drawdown is to make the source is adequate and not being depleted. The data collected to calculate drawdown can tell if the supply is slowly declining. Early detection of this can give off time to explore alternative sources, establish conservative measures or obtain special funding to get a new water source. Drawdown measurements give important information about the performance and efficiency of the wells.

One can combine drawdown data with well yield to evaluate the efficiency and performance of a well [5]. These analyses are done to estimate the transmissivity and storativity of confined or the hydraulic conductivity and specific yield of water table (unconfined) aquifers. An alternative approach to dimensionless type-curve analysis is to generate dimensional time drawdown curves from the analytical model that are compared directly to the measured values. In this approach the hydraulic properties of the model are adjusted in a series of model simulations until the model calculated drawdown closely match the measured values. This procedure is called model calibration and can be done graphically as in the dimensionless type-curve approach or automatically by the use of a parameter-estimation technique [6].

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The main source of water in the basement complex of Nigeria is generally the weathered / fractured top of the crystalline basement besides buried river channel and major fault zones. In the case of a fault zone, the fault may cut through various geologic formations in both the saturated and unsaturated zones. Depending on the relative displacement of the layers and the state of stress, a fault can provide a conductive pathway or be practically sealed. In terms of their hydrologic response on field pumping tests, faults may be classified as (1) tight faults, (2) constant-head faults and (3) non-constant head leaky faults.[7]

Tight faults are those that cut through an aquifer and for all practical purposes hydraulically disconnect one part of the aquifer from the rest of it and from other water bearing formations above and below.

Constant head faults are those that connect the aquifer to a large constant head water body, the path along the fault between the aquifer and the water body has practically infinite permeability. The method of images was used by [8] to calculate values of drawdown in a pumping aquifer that is intersected by tight or constant faults. Furthermore, Ferris [9] presented a method for finding the location of such boundaries by monitoring in at least two observation wells the drawdown due to a well with constant discharge.

This paper presents an analytic solution for calculating drawdown in a pumping aquifer that has been intersected by a finite transmissivity fault or other similar permeable geologic features.

2. Theory

Consider a two dimensional homogeneous isotropic aquifer that has been intersected by a fault of finite conductivity. A well located at a distance a from the fault is being pumped at a constant rate Q. The fault divides the aquifer into two regions. The drawdown at the point (x, y) in the aquifer at time t is represented by U (x, y, t). Then the problem can be formulated by the following governing partial differential equations and boundary conditions.

$$\frac{\partial U}{\partial t} = \alpha \left\{ \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right\} \qquad (1)$$

$$U(x, y, 0) = 0 \qquad (2)$$

$$U(\infty, y, t) = 0 \qquad (3)$$

$$U(0, y, t) = 0 \qquad (4)$$

$$U(x, \infty, t) = 0 \qquad (5)$$

$$U(x, 0, t) = \frac{\alpha Q}{k H} \delta(x - a) \qquad (6)$$

where $\delta(x)$ is the Dirac delta function, α and K are diffusivity and hydraulic conductivity of the aquifer, respectively. The storativity of the fault is negligible and the fault is connected to a constant source above or below the aquifer (Fig 1).

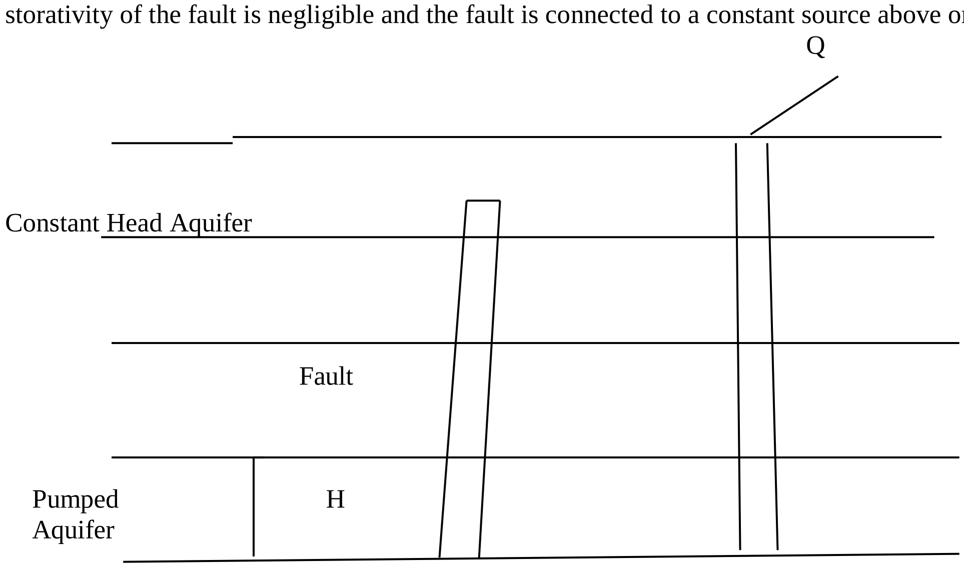

Figure 1. Aquifer intersected by a fault

Method

Let u(x, y, s) be the Laplace transform of £{ U(x, y, t) }, then taking the Laplace transform of Eq(1), we get

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = s\,u(s) - C_0 \qquad (7)$$

where C_0 is a constant. Using condition (2) in Eq (7) we get

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = s u \qquad (8)$$

Taking the Fourier sine transform of Eq (8) and integrating from 0 to ∞, we get

$$\int_0^\infty \frac{\partial^2 u}{\partial x^2} \sin nx \, dx + \int_0^\infty \frac{\partial^2 u}{\partial y^2} \sin nx \, dx = \int_0^\infty s u \sin nx \, dx \qquad (9)$$

$$\text{Let } \tilde{u} = \int_0^\infty u \sin nx \, dx \qquad (10)$$

where the tilde denotes a function which has been transformed.
Apply Eq (10) to Eq (9) while differentiating under the integral we get

$$-n^2 \tilde{u} + n\, u(\infty, y, s) \cos nx + n\, u(0, y, s) + \frac{d^2 \tilde{u}}{d y^2} = s \tilde{u} \qquad (11)$$

the Laplace transform of conditions (2) and (3) are

$$u(\infty, y, s) = 0 \text{ and } u(0, y, s) = 0 \qquad (12)$$

applying (12) to (11) yields

$$\frac{d^2 \tilde{u}}{d y^2} - (n^2 + s)\tilde{u} = 0 \qquad (13)$$

Eq (13) has the solution

$$\tilde{u} = A \exp\{ \overline{y \sqrt{(n^2 + s)}} \} + B \exp\{ \overline{-y \sqrt{(n^2 + s)}} \} \qquad (14)$$

from the boundedness of $\tilde{u}$ as $y \to \infty$, $A = 0$ so that

$$\tilde{u} = B \exp\{ -y\sqrt{(n^2 + s)} \} \qquad (15)$$

from condition (5)

$$u(n, 0, s) = \int_0^\infty \frac{\alpha Q}{KHs} \delta(x-a) \sin nx \, dx \qquad (16)$$

$$= -\frac{\alpha Q}{K H s}(x-a) \qquad (17)$$

from Eq (15) if y = 0 then

$$\tilde{u} = B e^0 = B \qquad (18)$$

comparing Eq (17) and Eq (18), we get

$$B = \frac{-\alpha Q}{K H s}(x-a) \qquad (19)$$

thus Eq (15) can be written as

$$\tilde{u} = -\frac{\alpha Q}{K H s}(x-a) \exp\{ -y \sqrt{(n^2 + s)} \} \qquad (20)$$

Applying inverse Fourier sine transform to Eq (20) we get

$$u = \frac{2}{\pi} \sum -\frac{\alpha Q}{K H s}(x-a) \exp\{ -y (\sqrt{n^2 + s}) \} \sin nx \qquad (21)$$

$$£^{-1} \exp\{ -- \frac{y(\sqrt{s + n^2})}{s} \} = \int_0^t \frac{y}{2\sqrt{\pi}\, v^3} \exp(--- y^2/4v) \exp(--- n^2 v)\, dv \ [10] \qquad (21a)$$

applying the inverse Laplace transform Eq (21a) to Eq (21) to obtain

$$U(x, y, t) = -\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\alpha Q}{KH}\sin nx\left\{\frac{2}{\sqrt{\pi}}\int_{t}^{\infty}\exp\left[-\left(p^2 + n^2 y^2/4p^2\right)\right]\right\}dp \qquad (22)$$

where $y^2 / 4v = p^2$ in Eq(22)

rewriting Eq (22) in the form

$$U(x, y, t) = -\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\alpha Q}{KH}\sin nx \exp\left(-n^2 y^2/4p^2\right) \operatorname{erf} p \qquad (23)$$

where $$\int_{t}^{\infty}\frac{2}{\sqrt{\pi}}\exp - p^2\, dp = \operatorname{erf} p \qquad (24)$$

and erf p is the error function.

Conclusion

In this study a model of the hydraulic head drawdown as a mathematical boundary-value problem was developed. Using the Laplace and Fourier transforms consecutively, a solution to the mathematical model was derived. The exponential integral stands for the drawdown due to a pumping well in an infinite aquifer. The shape of the exponential integral for this solution is in agreement with the S-shaped drawdown –time behavior of unconfined aquifers. The exponential function indicates that if the pumping continues for relatively large periods of time, most of the water extracted from the aquifer is essentially supplied by the fault.

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