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GRAVITATIONAL FIELDS STRENGTH AT THE SURFACE OF A STATIC HOMOGENEOUS OBLATE SPHEROIDAL EARTH ALONG THE POLE AND THE EQUATOR

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Abstract

Gravitational field strength in spherical coordinates and their applications are well-known. It is however, known that almost all rotating astronomical bodies are more precisely spheroidal in geometry. Consequently, in this paper we formulated gravitational field strength in oblate spheroidal earth along the pole and the equator.

***Keywords*:** Gravitational field strength, Oblate spheroidal, Pole and Equator

Introduction

Before 1950, almost all theoretical study of astronomical massive bodies was restricted to a perfect spherical geometry [1]. The only reason for these restrictions are because of the mathematical convenience and simplicity. Most planetary bodies have been assumed to be spherical and consequently, many treatments of motion involving these bodies have been taken into consideration of the spherical approximation of these bodies [2]. However, despite the spherical assumption of planetary bodies, since 1950, studies have shown that the real fact of nature is that the equilibrium shape of a rotating star or planet, for that matter is not a sphere but rather a flattened oblate spheroid [3]and almost all major astronomical bodies in the universe are spheroidal in geometry which is a more approximate description of these bodies. And it is obvious that their spheroidal geometry will have corresponding consequences and effects in the motion of all particles in their gravitational field strength [4]. These effects will exist in both Newtonian theories of gravitation and Einsteinian theory of gravitation. consequently, we hereby prepare the way for the study of motion of all particles in the gravitational field strength of oblate spheroidal earth along the pole and the equator.

Methodology

In this paper we applied the well-known gravitational field strength equation in oblate spheroidal [4] given by

$$g = -\nabla \Phi_g(\eta, \xi) \quad (1)$$

where $\Phi_g(\eta, \xi)$ is the gravitational scalar potential [3] given by

$$\Phi_g(\eta, \xi) = B_0 Q_0(-i\xi) P_0(\eta) + B_2 Q_2(-i\xi) P_2(\eta) \quad (2)$$

where Q_0 and Q_2 are the Legendre functions of the second kind linearly independent to the Legendre polynomials P_0 and P_2 respectively given by.

$$Q_0(-i\xi) = \frac{(1+3\xi^2)}{3\xi^3} i \quad (3)$$

$$Q_2(-i\xi) = -\frac{(7+15\xi^2)}{15\xi^3} i \quad (4)$$

$$P_0(\eta) = 1 \quad (5)$$

$$P_2(\eta) = \frac{1}{2}(3\eta^2 - 1) \quad (6)$$

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B_0 and B_2 are constant given by

$$B_0 = \frac{4\pi G\rho_0 a^2 \xi_0^5}{3\left(1+\xi_0^2\right)} i \tag{7}$$

$$B_2 = \frac{40\pi G\rho_0 a^2 \xi_0^5}{3\left\{44\xi_0^2 + \left(1+3\xi_0^2\right)\left(7+10\xi_0^2\right)\right\}} i \tag{8}$$

Theoretical Analysis

Consider the earth to be static homogeneous massive oblate spheroid as shown in Figure 1. It is obviously seen from the figure that the x-coordinate point on the surface of the static oblate spheroidal earth corresponds to its equatorial radius while, the z-coordinate point on the surface correspond the polar radius [3].

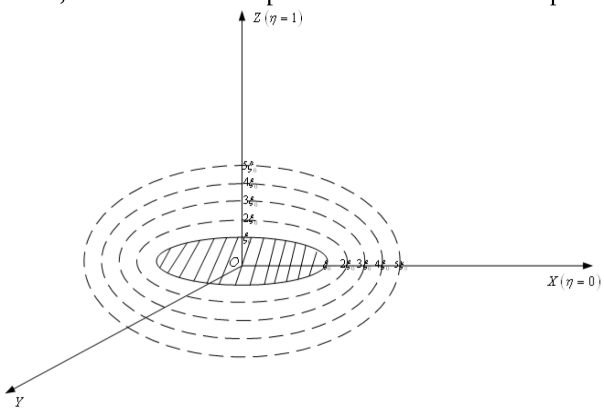

Figure 1: Static Homogeneous Oblate Spheroidal Massive Body

Now, we recall equation (1) given as

$$g = -\nabla \Phi_g(\eta,\xi) \tag{9}$$

$$g = -\frac{1}{h_\xi}\frac{\partial}{\partial \xi}\Phi_g(\eta,\xi) \tag{10}$$

where h_ξ is the scale factor given by

$$h_\xi = \frac{1}{a}\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}} \tag{11}$$

Putting equations (2) and (11) into equation (9) we get

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}}\frac{\partial}{\partial \xi}\left(B_0 Q_0(-i\xi)P_0(\eta) + B_2 Q_2(-i\xi)P_2(\eta)\right) \tag{12}$$

Putting equations (3), (4), (5) and (6) into equation (12) we get

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}}\frac{\partial}{\partial \xi}\left(B_0\frac{\left(1+3\xi^2\right)}{3\xi^3}i - B_2\frac{\left(7+15\xi^2\right)}{15\xi^3}i\frac{1}{2}\left(3\eta^2-1\right)\right)$$

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}}\left(B_0\frac{\partial}{\partial \xi}\frac{\left(1+3\xi^2\right)}{3\xi^3}i - B_2\frac{\partial}{\partial \xi}\frac{\left(7+15\xi^2\right)}{15\xi^3}i\frac{1}{2}\left(3\eta^2-1\right)\right)$$

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}}\left(B_0\frac{\partial}{\partial \xi}\frac{\left(1+3\xi^2\right)}{3\xi^3}i - B_2\frac{\partial}{\partial \xi}\frac{\left(7+15\xi^2\right)}{30\xi^3}i\left(3\eta^2-1\right)\right) \tag{13}$$

From the figure 1. We can see that at the equator $\eta = 0$ then equation (13) become

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\xi^2}\right)^{\frac{1}{2}}\left(B_0\frac{\partial}{\partial \xi}\frac{\left(1+3\xi^2\right)}{3\xi^3}i + B_2\frac{\partial}{\partial \xi}\frac{\left(7+15\xi^2\right)}{30\xi^3}i\right) \tag{14}$$

Similarly, at the pole $\eta = 1$ then equation (13) become

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{1+\xi^2}\right)^{\frac{1}{2}}\left(B_0 \frac{\partial}{\partial\xi}\frac{\left(1+3\xi^2\right)}{3\xi^3}i - 2B_2\frac{\partial}{\partial\xi}\frac{\left(7+15\xi^2\right)}{30\xi^3}i\right)$$

$$g = -\frac{1}{a}\left(B_0 \frac{\partial}{\partial\xi}\frac{\left(1+3\xi^2\right)}{3\xi^3}i - B_2\frac{\partial}{\partial\xi}\frac{\left(7+15\xi^2\right)}{15\xi^3}i\right) \quad (15)$$

Now we differentiate both equations (14) and (15) with respect to ξ we get

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\xi^2}\right)^{\frac{1}{2}}\left(B_0 \frac{\partial}{\partial\xi}\left(\frac{1}{3}\xi^{-3}+\xi^{-1}\right)i + B_2\frac{\partial}{\partial\xi}\left(\frac{7}{30}\xi^{-3}+\frac{1}{2}\xi^{-1}\right)i\right)$$

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\xi^2}\right)^{\frac{1}{2}}\left(B_0\left(-\xi^{-4}-\xi^{-2}\right)i + B_2\left(-\frac{7}{10}\xi^{-4}-\frac{1}{2}\xi^{-2}\right)i\right)$$

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\xi^2}\right)^{\frac{1}{2}}\left(-B_0\left(\frac{1}{\xi^4}+\frac{1}{\xi^2}\right)i - B_2\left(\frac{7}{10\xi^4}+\frac{1}{2\xi^2}\right)i\right)$$

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\xi^2}\right)^{\frac{1}{2}}\left(-B_0\left(\frac{1+\xi^2}{\xi^4}\right)i - B_2\left(\frac{7+5\xi^2}{10\xi^4}\right)i\right) \quad (16)$$

Equation (16) is the gravitational field strength at the equator

$$g = -\frac{1}{a}\left(B_0 \frac{\partial}{\partial\xi}\left(\frac{1}{3}\xi^{-3}+\xi^{-1}\right)i - B_2\frac{\partial}{\partial\xi}\left(\frac{7}{15}\xi^{-3}+\xi^{-1}\right)i\right)$$

$$g = -\frac{1}{a}\left(B_0\left(-\xi^{-4}-\xi^{-2}\right)i - B_2\left(-\frac{7}{5}\xi^{-4}-\xi^{-2}\right)i\right)$$

$$g = -\frac{1}{a}\left(-B_0\left(\frac{1}{\xi^4}+\frac{1}{\xi^2}\right)i + B_2\left(\frac{7}{5\xi^4}+\frac{1}{\xi^2}\right)i\right)$$

$$g = -\frac{1}{a}\left(-B_0\left(\frac{1+\xi^2}{\xi^4}\right)i + B_2\left(\frac{7+5\xi^2}{5\xi^4}\right)i\right) \quad (17)$$

Equation (17) is the gravitational field strength at the pole

Results and Discussion

In this paper we have successfully formulated the gravitational field strength of a static homogeneous oblate spheroidal massive body along the equator and the pole of the earth (16) and (17) respectively.

These results (16) and (17) extend the Newton's theory of classical mechanics from the well-known spherical bodies to those of spheroidal bodies. This sets of equations pave the way for the equations of motion for a particle of non-zero rest mass using spheroidal coordinates. It is most interesting and instructive to note that by the definition of the Legendre functions of the second kind Q_0 and Q_2 the gravitational field strength due to oblate spheroidal body are all even orders in the inverse coordinate ξ.

Conclusion

In this paper, gravitational field strength for a static homogeneous oblate spheroidal massive body along the equator and the pole of the earth is formulated as given by equations (16) and (17). Consequently, the physical interpretation of these results which we hope it would come up in the next edition of this paper and hence this paper opens the door for the experimental investigation in the motion of all bodies in the earth's atmosphere and solar system as well as all other gravitating systems in the universe.

$$g = -\frac{1}{a}\left(\frac{1+\xi^2}{\xi^2}\right)^{\frac{1}{2}}\left(-B_0\left(\frac{1+\xi^2}{\xi^4}\right)i - B_2\left(\frac{7+5\xi^2}{10\xi^4}\right)i\right) \quad (18)$$

and

$$g = -\frac{1}{a}\left(-B_0\left(\frac{1+\xi^2}{\xi^4}\right)i + B_2\left(\frac{7+5\xi^2}{5\xi^4}\right)i \right) \tag{19}$$

References

[1] Chifu Ebenezer Ndikilar, Adam Usman and Osita Meludu (2009). Gravitational Scalar Potential Values Exterior to the Sun and Planets. *The Pacific Journal of Science and Technology*. Vol. 10, Number 1. Pp. 663-673

[2] N. E. J Omaghali and S. X. K. Howusu (2016). Riemannian Acceleration in Oblate Spheroidal Coordinate System. *Journal of Applied Mathematics and Physics*. Vol. 4. Pp. 279-285

[3] Peter Sutherland (1999, January 4). When a massive object: such as a star spins, how is its gravitational field affected? *The New York*. Retrieved July 10, 2020, from https://www.scientificamaerica.com/article/when-a-massive-object-su/

[4] Nura Yakubu and S.X.K Howusu (2011). Gravitational Field of a Stationary Homogeneous Spheroidal Massive Body. *Proceedings of Annual Conference of IRDI Research and Development Network*. Vol. 6, Number 1. Pp. 68-77.