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MINIMUM TIME FOR MAXIMUM ALLOWABLE SEDIMENT ACCUMULATION IN A RESERVOIR FOR FLOOD CONTROL

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Abstract

Mathematical modelling has remained a cost-effective way to diagnose and analyse systems with varying degree of complexities. This approach to reservoir network modelling will provide a sustainable avenue in reservoir design, location, management and flood control. A set of a nonlinear system of ODE was developed to study the inflow and outflow of water from the reservoir in the network where runoff in each sub-catchment remain the major input into the model. Further analysis of the system of ODEgoverning this system gave more insight into reservoir operation for multipurpose use. An optimal control problem formulated to determine the minimum time for maximum allowable sediment accumulation. The failure of LQR to be a reasonable estimate of the optimal height of water in the reservoir led to the use of the tracked-LQR. The height obtained is used to estimate the current sediment height which determines the current useful life of the reservoir. The current information about the state of the reservoir is a tool that allows managers to control inflow into the reservoir to maintain the design useful life of the reservoir.

***Keywords**: ODE; Sediment, Useful Life; LQR; Tracked LQR; NOH*

1.1 Introduction

Reservoir network modelling is an age-long adventure of researcher for effective water management. The hydrological and hydraulic parameters of the catchment play a dominant role in the design and operation of the network. The runoff is affected by the perviousness of the catchment as a result of urbanization to determines the magnitude of flood expected. The interplay between environmental management in terms of drainage, vegetation, and soil type will determine the magnitude of the devastation of the flood. Closely related to this, landslide, mudflow and flood remain environmental problem which affects every part of the world. The first world countries are suffering from aging infrastructures, the second world countries are suffering from capacity to manage the flood while the third world countries lack infrastructure and proper enlightenment on the flood and how to get prepare for it. It is evident now that in the three categories above, weather forecasting, as well as flood forecasting in both time and magnitude, is not the problem. The problem however is the ability to utilize all resources to manage the excess water during the raining season for utilization during the dry season.

Several factors can affect the stability of the network, the most important factors are extreme rain rate, long rain event, short inter-event time and sediment transport. A mathematical model described in [1] and developed through [2]where the stability of the network was determined though the hydraulic and hydrologic parameters. To remedy these likely sources of instability, an alternative reservoir is considered in the model as well as sediment transport into the retention pond[2]. To guarantee effective network operation, the rate of sediment accumulation, criteria for pump constant and pump voltage as well as the useful life of retention pond is determined. The advent of Geographic Information System (GIS) provides an avenue to estimate the surface area of a small reservoir in the network[3]. The relationship between area and volume of these reservoirs studied in[3-5] was used to determine the relationship between the area of sub-catchment and the surface area of the reservoirs. This gives an optimal estimation since the surface area is derived from the catchment area, rain rate and time of concentration[1]. The range of validity of this estimate is in respect of land use regulation and catchment perviousness which affect the estimation of time of concentration. Relying on sediment estimate in catchment without data, the parameters $K \; and \; \rho$ which establish the relationship between water flow and sediment flow reported in [5-8] and found to be in the range $\rho \in (2,3)$ for $K = Q_s$ .Specifically for $\rho = 2$ in [8], $K = 1/\ln Q$ which makes both parameters to be catchment dependent.

The arbitrary selection of useful life, provision of equipment to measure sediment accumulated, and relying on the initial estimate have all been proven to have shortcoming since there are lots of other factors that may affect the initial estimate. The study of the reservoir during the flood event has also not done much to reduce the flood effect. The probable and convenient way is for the managers to routinely estimate sediment accumulation and dynamically study the useful life for effective flood control. For this reason, optimal

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estimation of height of water in the retention pond at the end of each rain event will on one hand provide timely information on the current state of sediment inflow. At the same time dynamically estimate the current useful life of the reservoir. Since useful life relied on the optimal height of water in the retention pond, the controller design of this problem failed. This is as a result failure of controllability and observability test. This failure necessitates the use of an optimal control approach to seeking an optimal solution[9]. Being a local method, we expect the local optimum value to strike a balance between stability and performance. The optimal height obtained in the retention pond and sediment accumulated in the reservoir is used to evaluate the useful life. High inflow from the connecting reservoir(s) may not make the useful life estimate reasonable, a benchmark is determined to get the maximum inflow that ensures the desired useful life. Either each flow is reduced by a factor determined by the benchmark of every flow above the benchmark will be replaced by maximum allowable inflow so that the target useful life can be achieved. Hence, the need to find maximum allowable inflow that guarantees minimum time from the current time to the closest dredging time as well as assist the managers in the information that may be utilized to control the inflow to prolong the time of dredging if any.

1.2 Background of the study

Modelling reservoir network connection for flood control is born out of studies on flood events at the expense of the main causes of flood. The reservoirs that handle runoff within a catchment range from small to medium depending on the rainfall pattern, relief, vegetation and land use regulations[10]. Sediment transport and accumulation, on the other hand, receive attention but concentrates on the large body of water like lake and ocean with continuous water inflow. Design and management of construct like reservoirs in terms of retention and detention pond receive little attention. These forms of reservoirs are easy to construct, manage, and range from small to big in capacity and effective for flood control, irrigation, recreation and other water use. Efforts to get a simple and concise tool for pond design and management for flood prevention is worth encouraging. It is a fact that the network of reservoirs in a catchment is most effective for water management[11, 12]. Therefore, there is a need for a mathematical approach to network modelling which will aid design, operation and analysis of reservoir network for effective and efficient operation at reducing cost[13].

A system of a nonlinear system of ODE was presented in [2] which study the interaction between *(m+2)* reservoir network. There are *(m-1)* detention ponds, one retention pond, a connecting stream, and an alternative reservoir. Sediment consideration was added to the model and benchmarks set by determination of Normal operating height (NOH) of all the reservoirs as well as criteria for stability of the network. Sediment accumulation and useful life of retention pond in the network were also considered to give a complete outlook and equation capable of providing basic information about the network. Through this approach to reservoir network modelling, an opportunity to study all aspects of water management where flood control is of particular interest is presented. This formulation utilizes systems of nonlinear ordinary differential equation with hydraulic and hydrologic inputs as well as other parameters which have been estimated through catchment parameters. This approach allows a comprehensive outlook in the design, analysis and operation of the network. However, the model is used to study the stability of the network [14, 15] on which NOH is used to determine water and sediment inflow. The mathematical description of the network of reservoirs for flood control is given by

$$\left.\begin{array}{l}
A_j\dot{H}_j = \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le H_j^0 \end{cases} \qquad j = 1,2,\ldots,m-1 \\
A_m\dot{H}_m = \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} + B_{A_r}\sqrt{H_{A_r} - H_m} & \qquad j = m \\ -B_m(H_m - H_s)^{3/2} - A_m f - V_pK_pB_n\sqrt{\left|H_m - H_{Ar}\right|} - K(1-T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} & ; \quad H_m < H_m^{temp} \end{cases} \\
A_{A_r}\dot{H}_{A_r} = \begin{cases} 0 & ; \quad H_{A_r} > H_{A_r}^0 \qquad j = A_r \\ V_pK_pB_n\sqrt{\left|H_m - H_{Ar}\right|} - B_{A_r}\sqrt{H_{A_r} - H_m} & ; \quad H_{A_r} < H_{A_r}^0 \end{cases} \\
A_s\dot{H}_s = B_m\left(H_m - H_s\right)^{3/2} - L_s \qquad ; \quad H_S < H_S^0 \; ; \qquad j = s
\end{array}\right\} [2] \qquad (1)$$

Where $B_n = c_{d,m}a_m\sqrt{2g}$, $B_i = c_{d,i}a_i\sqrt{2g}$, $B_m = c_{d,m}L$, $B_{A_r} = c_{d,m}a_{A_r}$ and $c_{d,m} = \left(\frac{2}{3}\right)^{3/2}\sqrt{g}$ [10]

The equation (1) is a full description of the network with m reservoirs, an alternative reservoir and the connecting stream. The inputs into the model range from hydraulic and hydrologic parameters while other parameters were derived and evaluated from the catchment parameters thereby reducing dependency on long term data.

The linearized (1) provides sufficient evidence for the stability of the network which was studied in [15, 16] and used to determine the NOH of each reservoir in the network. The unreliability of stability analysis in the face of the decision matrix not satisfying the observability and controllability of the system led to seeking a solution through optimal control LQR formulation with a performance index

$$\left.\begin{aligned}
&Min\quad J = X^T(t_f)SX(t_f) + \int_{t_0}^{t_f}\left\{XQ^TX + UR^TU\right\}dt\\
&s.t.\quad \dot{X} = AX + BU\\
&h_j^{\min} \le x_j \le h_j^{\max} \qquad ;\ j = number\ of\ detention\ ponds\\
&h_{perm} \le x_j \le h_{flood} \qquad ;\ j = number\ of\ retention\ ponds\\
&h_{perm} \ne 0\\
&h_s^0 \le x_s \le h_s^{\max} \qquad ;\ s = the\ connecting\ stream\\
&h_s^0 \ne 0\\
&h_{A_r}^{\min} \le x_{A_r} \le h_{A_r}^{\max} \qquad ;\ A_r = alternative\ reservoir
\end{aligned}\right\} \tag{2}$$

The robustness of LQR formulation is exploited by the analytic approach for small scale problem and numerical solution for large scale problem. Matrices *A* and *B* have been obtained from the formulation while effective control of the system relied on proper choice of matrices *Q* and *R*. For the proper selection of *Q* and *R* the concept of controllability and observability of the system is considered. In the case of matrix *S*, the solution to the Riccati differential equation

$$-\dot{P}(t) = P(t)A + A^TP(t) - P(t)BR^{-1}B^TP(t) + Q \tag{3}$$

$P(t_f) = S(t)$ will ensure positive semi-definiteness of *S*. Equation (3) is a Riccati differential equation which is solved to get *P(t)*and makes the minimization of LQR robust with the analytical solution. The analytical solution to (2) rely on the Jacobian matrix associated with linearization of the nonlinear system which is in itself an approximation. The constraints (2)are exploited bytracked LQR which is more suitable for this formulation with great control effort. The solution to this problem presented in[9]which gives the optimal height of water level in retention pond with associated sediment inflow.

1.3 Normal Operating Height

The NOHs is the expected height for proper operation of the reservoir because it is used to determine the stability of the network. In [2] the NOH of all reservoirs in the network is determined by setting(1) to zero. This gives

$$H_m = \left(\frac{L_s}{B_m}\right)^{2/3} + H_s \ ; \qquad j = m \tag{4}$$

This is contrary to the NOH obtained in [2] since $K\ and\ \rho$ have been determined in[8] and average stream discharge can be used for design purpose. The NOH of other reservoirs becomes

$$H_j = \left[\frac{I}{B_j}\left(A_{scj} + \sum_{i=\sigma_i}^{\sigma_r} A_{sci}\right)\right]^2 + H_k \ ; \quad j = 1, 2, \ldots\ldots, m-1 \tag{5}$$

and

$$H_{Ar} = H_m + \frac{(AI - L_s - A_m f)}{(V_pK_pB_n - B_{Ar})} + \frac{\left[\sum_{i=\sigma_1}^{\sigma_r} A_{sci}I\right]^2 - (1-T_e)L_s^2}{\ln Q\,(V_pK_pB_n - B_{Ar})}; \quad j = A_r \tag{6}$$

where H_s is given by the height of the connected stream and at equilibrium for $\rho = 2$, and $K = 1/\ln Q$ .The three parts in (6) represent the height of reservoir *m* (retention pond), the water inflow, and the sediment inflow into the alternative reservoir H_{Ar} respectively. Since $V_pK_pB_n - B_{Ar} > 0$ [2], the choice of $V_p\ and\ K_p$ as determined by the ratio of B_{Ar}/B_n will determine the value to be added to H_m. The choice of varying the parameter at the denominator provides the opportunity to consider several locations for the citing of the alternative reservoir. Equation (4), (5) and (6) are for design, the inflow *Q* will remain the initial design parameter since it has been replaced by the height of connected reservoirs during operation.

1.4 Dynamic Estimation of Useful Life of Reservoir

The basic thrust of this work is to determine the real and dynamic time of dredging the reservoir which has been storing the accumulated sediment. Since retention pond serves the purpose of storing water, the performance index (2) is to generate optimal height h^{opt} of water in each reservoir between times t_0 and t_f, where t_0 is the time rain begins and $t_f = t_{currenttime}$. At the end of every rain cycle, all detention ponds are expected to be emptied and the height of water in the retention pond at t_f is used to evaluate the useful life at that instant. The useful life of a reservoir is the loss of half of the initial capacity [8]. In [17], the useful life is found to be

$$t_f = \frac{C_0 - C}{K\left(\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right)^{\rho}} + t_0 \tag{7}$$

Where H_i and H_m are NOHs of the detention ponds and $\sigma_1,\ldots.\sigma_r$ count the number of detention ponds connected to the retention pond.

The dynamic nature of sediment inflow also gave rise to the constants K, ρ as determined in[2] based on the stability of the network. But the equation (7) will only be used to provide a benchmark for the inflow that will guarantee the design or the expected useful life. The choice of height in the retention pond and the connected detention ponds will go beyond NOH rather than the optimal height obtained through tracked LQR. If the benchmark is for maximum allowable sediment accumulation is $0.5C_0$ [5, 18]with current sediment capacity C_{csc} , the equation (7) becomes

$$t_f = \frac{1/2\, C_0 - C_{csc}}{KT_e\left(\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{h_i - h_m^{opt}}\right)\right)^{\rho}} + t_0 \quad (8).$$

This is the minimum time for maximum allowable sediment to accumulate in the reservoir provided the projected parameters for the design remain the same. When the design parameters change especially the sediment inflow, the current sediment capacity is $C_{scs} = C_{osc} + q_{csi}t_i$ where t_i is the total rain cycle time from beginning to the current time, C_{osc} is the old sediment capacity and the current sediment capacity q_{csi} is given by

$$q_{csi} = K\left(B_i\sqrt{h_i - h_m^{opt}}\right)^{\rho} \quad (9)$$

Using (9) in (8)have we

$$t_f = \frac{1/2\, C_0 - C_{osc} - K\left(B_i\sqrt{h_i - h_m^{opt}}\right)^{\rho}}{KT_e\left(B_i\sqrt{h_i - h_m^{opt}}\right)^{\rho}} + t_0$$

and

$$t_f = \frac{1/2\, C_0 - C_{osc}}{KT_e\left(B_i\sqrt{h_i - h_m^{opt}}\right)^{\rho}} - \frac{1}{T_e} + t_0 \quad (10)$$

At $t=0$, $t_0 = 0$ and $C_{osc} = 0$ hence,

$$t_f = \frac{1}{T_e}\left[\frac{1/2\, C_0}{K\left(B_i\sqrt{h_i - h_m^{opt}}\right)^{\rho}} - 1\right] \quad (11)$$

Some factors can affect the inflow into the reservoir at any given time, these factors may adversely affect the initial computed useful life. The management of the reservoir now relies on the monitoring of inflow into the reservoir so that after every rain event the current state of the reservoir can be recorded and an appropriate decision taken. With a 50% reduction throughout 50years, the inflow characteristics should be

$$\left[\frac{0.5C_0}{K\left(t_f T_e + 1\right)}\right]^{1/\rho} = B_i\sqrt{h_i - h_m^{opt}} \quad (12)$$

In [8], $K = 1/\ln Q$ at equilibrium, with $Q = B_i\sqrt{h_i - h_m^{opt}}$ and $\rho = 2$ the equation (12) becomes

$$t_f = \frac{1}{T_e}\left\{\frac{C_o \ln Q}{2Q^2} - 1\right\} \quad (13)$$

The equation (11) will be used to determine the state of the reservoir after every rain event, (12) will be used to benchmark the inflow that will sustain the usefulness of the reservoir for the time the reservoir is designed to last and (13) will be used to obtain the current useful life from current sediment inflow.

1.5 Design Sediment Capacity

Based on the assumption in [19], the cumulative capacity of all detention pond in the network should not exceed the permanent mark of the retention pond, the design capacity

$$C_o = \sum_{j=1}^{m-1} V_j + v \le V_m(h_m^{perm}) \quad (14)$$

where v is the capacity of retention pond based on the characteristics of the sub-catchment, V_m is the volume of the permanent mark of the retention pond.In(14), the volumes of the reservoirs can be calculated by the rational method [20-22] and the peak runoff can be estimated by

$$Q_p = AcI \quad (15)$$

where Q_p is the peak runoff, A is the drainage area, C is the runoff coefficient and I is the rainfall intensity. These parameters are valid on the impervious drainage basin. The runoff coefficient determines the level of perviousness of the catchment and its value is ranged between 0 and1 [22-24]. Notice that in(15) $Q_p \approx AI$ for c close to 1 in an impervious catchment (c < 1 is accounted for the losses through evaporation and seepage). In [25], the water losses due to infiltration and evaporation is removed from the rainfall rather than from the stored water on the surface of the catchment. The total runoff has been considered with the losses, which is equal to rainfall volume in a completely impervious basin. The complete impervious basin is used to get the maximum runoff so that the losses are considered negligible. Hence,

$$Q_p t = V = V_{stored} + V_{flowin} + V_{precipitation} \tag{16}$$

since the peak runoff over time give the volume of runoff. For catchment that is divided into sub-catchments (16) can be expressed as

$$V = \sum_{j=1}^{m} A_{sc_j} c_j I_j t_{c_j} \tag{17}$$

where all the parameters in (17) are define for all the sub-catchments that make up the catchment. The volume of the basin for stormwater purposes is expected to be 3-4 times the volume of runoff [26].

$$V_j := 4kA_j^{1.5} \; ; \; j = 1, 2, \ldots, m-1 \tag{18}$$

$$\sum_{j=1}^{m-1} 4kA_j^{1.5} + v = V_m(h_m^{perm}) \tag{19}$$

The volume with regular cross-section is used in [27] to determine the average depth of the permanent pool but where information about the surface area of the bottom shelf is not available[28].

$$h_{j_{ave}} = \frac{V_j}{A_j} \tag{20}$$

This is in contrast with reality where the pool is considered and estimated by half pyramid in [29] and used to estimate the surface areas. The volume of a regular cross-section is six times volume of half pyramid i.e.

$$V = A_j h_j = 6kA_j^{1.5} \tag{21}$$

which shows that the use of regular cross-section has taking care of 3-4 times volume of runoff that gave rise to (18). The equation (19) can be replaced by

$$\sum_{j=1}^{m-1} 6kA_j^{1.5} + v = V_m(h_m^{perm}) \tag{22}$$

The height of the reservoir can be estimated from the characteristic length of the surface of the reservoir in the formulation of area-volume relationship in [3, 29-31] i.e.

$$h = \frac{l}{20} \tag{23}$$

where l is the characteristic length of the reservoir, but $l = \sqrt{A_j}$ and

$$h = \frac{1}{20}\sqrt{A_j} \tag{24}$$

where

$$A_j = \left(\frac{A_{sc_j} I_{sc_j} t_{c_j}}{k_j} \right)^{2/3} \quad [1] \tag{25}$$

The volume in (21) and the height in (24) for *j=1,2,…,m-1* will be used to estimate the volume and height of the permanent part of the retention pond. With the assumption of regular cross-section, this surface area will be used to estimate the sediment that will ensure the stability of the network i.e.

$$Sediment\ Ratio\ to\ Reservoir\ Capacity = \frac{vol\ of\ sediment\ accumulated\ with\ NOH}{design\ vol\ of\ retention\ pond} \times 100\% \tag{26}$$

since the NOH is used to determine the stability of the network. Sediment accumulation beyond this does not necessarily imply useful life but capacity that may trigger flood when other parameters exceeded their limit point.

1.6 Results and Discussion

Before the determination of the useful life of a reservoir, we need to know the design capacities of all reservoirs in the network. The catchment and sub-catchment parameters as well as the computed surface area of the reservoir using(25), the design height using (24), the volume of the reservoir using (22)are presented in Table1.The volumes and heights of reservoirs (1-10) are from the original design from the catchment parameters, while, reservoir 10(design) is based on Assumption (c) in [19] which give rise to (22). Using equations (4), (5) and(6), the computed NOH due to water and sediments are also reported in Tables 1. From this table, the network configuration does not affect the estimated sediment generated in the catchment. The surface area as well as the rate of inflow of sediment will over time determine the time of dredging. The solution of all systems described in this section must, therefore, be constrained between the NOH H_j^0 and the height of the reservoir $H_j^0 + h_{j_{min}}$ or $H_j^0 + h_{j_{ave}}$ $H_j^0 + h_{j_{max}}$ for minimum, average and maximum rain rate respectively. The difference between the NOH in all reservoirs is $H^0_{\max(I)s} - H^0_{\max(I)w} = 0.8661$ for both the series and hypothetical reservoir network while from equations (24) and(25), the design height, the surface area of the reservoir, and design volume are presented in Table2.The design surface area of reservoir *j=10* is 2744.98 with sediment height of 0.8661 gives the design sediment capacity which ensures the stability of this network.

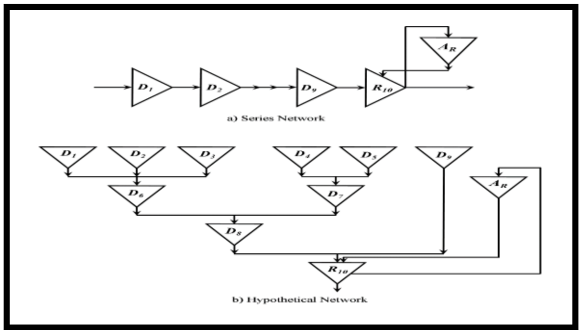

Figure1: Reservoir Network Connection

In this catchment, 8% of the design capacity of the retention pond will guarantee the stability of the network based on (26). The time at which the reservoir loses its purpose of design is useful life and it is caused by sediment accumulated in the retention pond only. At $t = 0$, the design capacity is the useful capacity which keeps reducing as sediment flows in and trap efficiency of the reservoir. It worth estimating at $t=0$ what the useful life will be based on the catchment information. Principally this will involve the network topology since the inflow varies from one configuration to another. However, on the assumption of a retention pond in the network, whatever types of configuration all sediments eventually end in the last reservoir which is retention pond. The rate of sedimentation which depends on inflow pattern (Q_s), percentage of trapped sediment (T_e), and density of the deposited sediment factors responsible for sediment deposition [30]. Moreover, the useful life of the reservoir depends heavily on the ratio of densities of deposited sediment which has been replaced by the inflow pattern in[17]. Using (7) for 100%, 50%, and 25% capacity loss in permanent, temporary and flood capacities of the reservoir, This result is consistent with the two network configurations. However, all values obtained are unreasonable for the estimation of useful life. For example, 10.1165 years for 100% capacity loss of permanent pond is too small of a construct that will be used to manage retention pond. To correct this anomaly, a benchmark is required to obtain the expected useful life. This will become the criteria for dynamic estimation of this parameter during operation. Suppose the expected useful life is t_{Le}, from (11), a benchmark based on inflow will be used to design the required useful life. This is based on the regulation of inflow in(12), the manager of the network will develop a flow regime to satisfy the inequality based on (12) i.e.

$$B_i\sqrt{h_i - h_m^{opt}} \le \left[\frac{0.5C_0}{K\left(t_f T_e + 1\right)}\right]^{1/\rho} \tag{27}$$

The inequality (27) must be satisfied to have the useful life of proposed t_f for 50% capacity reduction. With the expected minimum useful life of 50 years, the inflow benchmark for a 50% reduction in reservoir capacity is $B_i\sqrt{H_i - H_j} = 13.42m^3s^{-1}$. In this catchment, however, the average inflow into the retention pond with maximum rain rates is estimated to be 154.37 m^3s^{-1}. Since this discharge is far above the benchmark, we defined a standard reduction factor which all discharges have to satisfy i.e.

$$\vartheta = \frac{Expected\ disch\arg e}{Computed\ disch\arg e} \tag{28}$$

In this case, $\vartheta = 13.4147/154.3699 = 0.0869$. This value will be used to reduce all inflows above the benchmark to achieve the required useful life. The normalized useful life is presented with the reduction factor makes (7) to become

$$t_f = \frac{C_0 - C}{KT_e\left(B_i\vartheta_{\max}\sqrt{H_i - H_j}\right)^{\rho}} + t_0 \tag{29}$$

The maximum reduction factor $\vartheta_{\max}$ will bring all inflow to a level that will ensure the expected minimum useful life is achieved. The required useful life is obtained through (29) with 50% capacity reduction for maximum rain rate and the benchmark produced 50years. While the volume of flood height will take 100years. For subsequent estimation of useful life, the height obtained through tracked LQR is utilized since the result of LQR produced unreliable results as can be seen in Figure1. The *x1* through *x12* are the state variables corresponding to the height of water in each reservoir, *x1* to *x9* are heights in the detention pond, *x10* is the height in the retention pond, *x11* is the height in the connecting stream and *x12* is the height in the alternative reservoir.

Table 1: Catchment and Sub-Catchment Parameters with Design Parameters and Computed NOHs of Reservoirs

Reservoir	**Sub-Catchment Parameters**				**Series**		**Hypothetical**		**Estimated Surface Area, Height and Volume of Reservoirs**		
					Water + Sediment	**Water**	**Water + Sediment**	**Water**			
J	$c_{d,j}$	A_{sc_j}	a_j	t_c	$H^0_{\max(I)}$	$H^0_{\max(I)}$	$H^0_{\max(I)}$	$H^0_{\max(I)}$	A_j	$h_{j(\max)}$	$V_{j(\max)}$
1	0.33	2200	1.0	11.13	836.32	835.46	217.24	216.37	594.45	1.21907	724.68
2	0.32	3100	1.5	19.20	781.17	780.30	162.08	161.21	1074.51	1.638987	1761.11
3	0.62	600	2.0	17.96	779.76	778.89	161.60	160.73	343.88	0.927203	318.85
4	0.33	1850	2.5	20.67	751.66	750.80	161.31	160.44	799.99	1.414208	1131.36
5	0.32	4600	2.5	19.03	642.15	641.28	155.07	154.20	1389.86	1.864044	2590.77
6	0.62	1630	2.5	20.13	639.40	638.54	154.69	153.82	722.46	1.343928	970.93
7	0.33	1780	3.0	31.88	538.45	537.58	125.40	124.53	1040.79	1.61306	1678.85
8	0.32	7400	3.5	33.99	223.94	223.08	39.63	38.77	2808.98	2.64999	7443.77
9	0.62	8140	4.0	30.00	219.01	218.15	34.70	33.84	2753.98	2.623919	7226.22
10	0.33	9000	5.0	27.00	21.33	20.47	21.33	20.47	2744.98	2.619627	7190.82
									Design(10)		**31037.35**
$a_{A_r}/a_m = 1.2$ and V_pK_p is taking as 1.2x10^{-12}. $L_s = 7m$ and $H_s(S) = 7$											

The reliability of the results of tracked LQR allows each height to be constrained between the maximum and minimum point in the reservoirs. This is contrary to the results in regular LQR which brought all heights to zero.

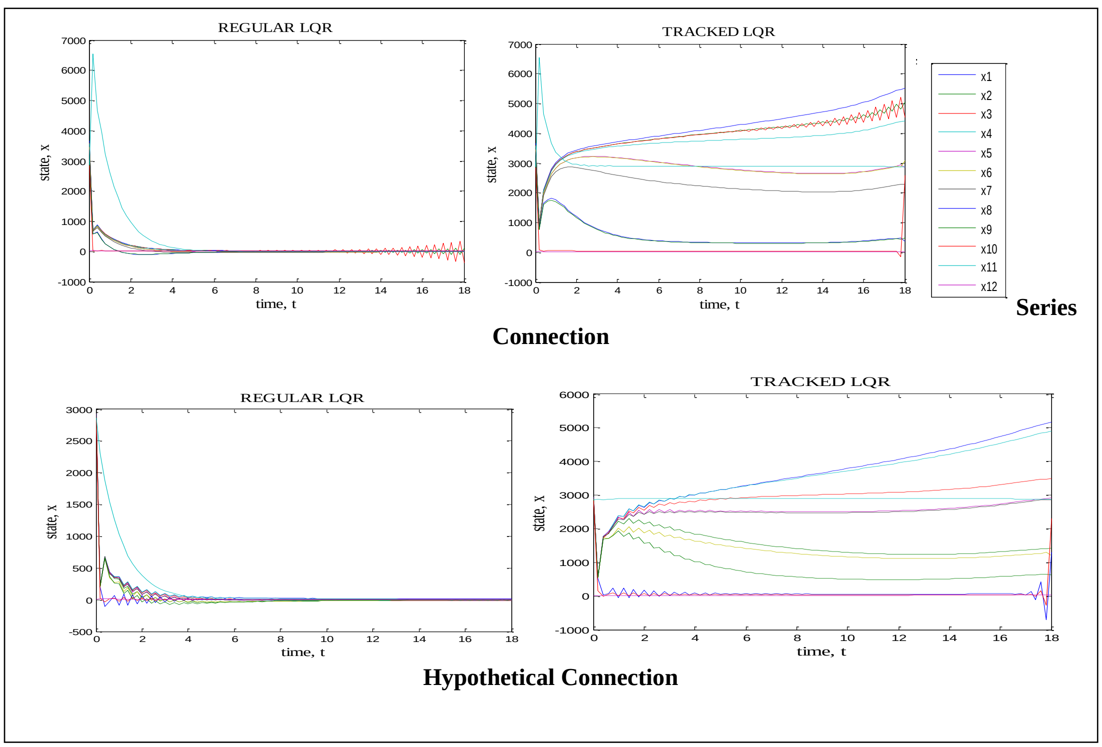

Figure 2: Result of LQR and Tracked LQR

Table2: Estimation of optimal Height $x_{10}(t_f)$ for the Determination of Dynamic Useful Life

Rain Rate	**Series**				**Hypothetical**					
	$H_9^0 - h_9$	**stdct**	**D9**	t_L	$H_8^0 - h_8$	$H_9^0 - h_9$	**htdct**	***D8***	***D9***	t_L
Min	25.970	31.587	-	-	24.709	24.775	31.125	-	-	-
Ave	567.791	295.938	181.03	2.04E-2	532.027	533.611	167.766	981.840	210.00	7.1E-5
Max	3036.62	2599.59	229.53	9.98E-3	2843.85	2852.31	2311.84	1186.57	225.25	4.E-5

Stdct **s**eries connection in time $\mathbf{t_d}$ for the **c**oupled system in a **t**racking LQR
Htdct hypothetical connection in time $\mathbf{t_d}$ for the **c**oupled system in a **t**racking LQR

The discharge D_9 in series connection and the discharges D_8 and D_9 in hypothetical connection as in Table2 are far bigger than the discharges used for benchmarking in this catchment. This is the reason why 10.1165 years for 100% capacity loss of retention pond is too small. The reduction factors due to (28)is design for managers of these constructs to reduce the flow to maintain and sustain the inflow that will guarantee the design useful life. These values are found to be about 31years and 15.2years for average and maximum rain rate in series connection respectively. For a hypothetical connection, the average and maximum rain rates produced about 87 years and 61 years of useful life respectively. From these figures, the hypothetical network seems to be better managed the network to reduce sediment accumulation.

1.7 Conclusion

Determination of useful life of the reservoir can be a rewarding endeavour considering the rate of flooding been experience around the globe. It is never out of place looking for solutions to a problem of this magnitude. The non-availability of data is a major source of concern when dealing with modelling of this type of event. Nevertheless, we have tried as much as possible to reduce dependency on long time data to get the description and behaviour of floodwater in a network.

In this work, however, we can dynamically determine the time of dredging of retention pond which is based on the information about the useful life of the reservoir. The rain has been a natural phenomenon have been predicted over the years but the effect of it and attendance human behaviour has made it a source of worry due to flooding. The dynamic estimation of useful life using the optimal height obtained through tracked LQR suggests that for a phenomenon like this constraining the performance index within the minimum and the maximum value is a best to avoid values outside the feasible region. The whole formulation makes no sense if the inflow benchmark is not included for the managers to manage the flow that will ensure maximum utilization of the constructs. The overall performance now shows that the hypothetical network will better manage water and sediment inflow to get the desired useful life of the reservoirs.

The modelling phase of this project is complete, the next phase is the implementation of the result on the catchment which houses Kangimi dam in Kaduna State Nigeria and Dadin Kowa dam in Bauchi with collaboration with the National Institute of Water Resources, Kaduna Nigeria.

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