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A STOCHASTIC MODEL FOR THE EVALUATION AND MONITORING OF STUDENTS' ACADEMIC PROGRESS IN TERTIARY INSTITUTION

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Abstract

Evaluation and monitoring of students' progress is an essential part of any educational system. This paper focuses on the use of stochastic model to evaluate and monitor the progress of students in an higher institution. The academic programme is modelled by a discrete Markov chain with five transient and two absorbing states. The probability transition matrix is constructed. The quantitative characteristics of the absorbing Markov chain, like the expected time until absorption and the probabilities of absorption, are used to determine chosen indicators of the programme. Probabilities of graduation and withdrawal were evaluated. Besides, a prediction on the students' enrolment for the next four academic years was made.

Keywords: Markov chain, probability transition matrix, absorption probability, educational system

Introduction

Evaluation of students' progress is an essential part of any educational system. Every higher education institution can be considered as a hierarchical organization in which a student stays in a given study stage for one academic year, and then moves to the next stage or leaves the system as a graduate or dropped out. Due to continuous changing and the increasing amount of data, the problem of understanding and assessing the students' progress through the educational system is very important [1]. It can help the managers of the education institution to establish an optimal educational policy, which ensures better position in the educational market.

Markov chains are an important family of stochastic processes, defined as a sequence in which the dependency of the successive events goes back only one unit of time. In other words, as defined by Tijms [2], the future probability behavior of the process depends only on the present state of the process and is not influenced by its past history. This is called the Markovian property. Despite a very simple structure, Markov chains are extremely useful in a wide variety of practical probability problems [2]. The application of Markov chains can be found in various branches of natural sciences, engineering, and medical sciences [3]. In the literature, there are many attempts to apply the Markov chain to analyse the higher education study process. For instance, Moody [4]applied the Markov chain to analyse and predict the mathematical achievement gap between African American and white American students. Furthermore, Hlavatý [5] presented the Markov chain model of students' progress throughout a particular course. To finish the course successfully, each student has to go throughout various stages of the course requirements where his success depends on the completion of the previous duties. Another approach is proposed in[6] where the theory of Non-Homogeneous Markov Systems (NHMS) with fuzzy states for describing students' educational progress in Greek Universities is used. Very interesting and useful are the studies which modeled the students' progression and their performance during higher education study using an absorbing Markov chain (see e.g., [7-14]). Such application provides a means for projecting the number of students' graduation and withdrawing by age, gender, and by study programme, and provides estimates of the average time a student stays in the system, the probability of completion as well as the average time to complete the study. Since the theory of the absorbing Markov chain is relatively simple, such applications indicate high practical value and therefore offer great opportunities for implementation in practice.

2.0 Discrete-time Markov chains

Discrete-time Markov chains have been used over some years to date in several disciplines including health in predicting the disease progression, education in predicting enrollment, and other forecast projects based on present events. For instance, a discrete-time Markov chain model has been used to forecast daily admission scheduling and resource planning in a cost or capacity constrained healthcare system. Another area where discrete-time Markov chain has been used was in investigating the effects of treatment programs and healthcare protocols for chronic diseases. One major characterization of discrete-time Markov chains is that the next event to occur depends only on the present state of the system and does not depend on the history of the system. Hence, this Markov chain process is said to be "memory-less". A discrete-time Markov chain is said to be a stochastic process which satisfies the Markov property given by

P(X_{n+1}=j/$X_{n=i\ \ n}$,X_{n-1}=i_{n-1},…,X_0=i_0)= P(X_n=j/$X_{n=i}$) where X_n is a sequence of random variables with discrete time steps t_n, t_1,….,t_n. A discrete-time Markov chain can be homogeneous or non-homogeneous. A homogeneous Markov chain is the one in which the probability transition matrices do not change overtime from one state to the other (i. e cycles) and across subjects. In a 10 non-homogeneous Markov chain, the probability transition matrices change over time from state to state and/or across subjects.

In this work, we use a discrete-time Markov chain model to estimate and monitor of students' academic progress in Ekiti State University (EKSU), Nigeria. We assume that the discrete-time Markov chain is non-homogeneous so that the probability transition matrix changes over the observation time. A discrete-time Markov chain model is applicable to this study, because the state to which a student transitions depends on only

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the current state (or academic year) and not on the past. Therefore, student enrollment from semester to semester satisfies the Markov property with the time increments being semesters. This makes it possible for us to model student enrollment and then use the model to predict graduation rates, dropout rates and total student enrollment using data obtained on student progression. Enrollment retention rates are an accepted indicator of a university's success in providing quality degree programs and learning environments which will lead to a student's continued enrollment and timely graduation from the university campus.

The data used for this research work was Students enrolment into Ekiti State University, Ado – Ekitifrom2011/2012 to 2017/2018 academic sessions for a five-year academic programme, collected from the records Department of the University. The Figure 1below depicts the transition state diagram for our Markov chain model for the progression of undergraduate students at Ekiti State University Ado-Ekiti Nigeria (EKSU). The transition probability diagram consists of Five (5) states, namely freshman (100L), 200L, 300L, 400L and 500L. In the model, we assume there are only forward transitions and no backward transitions. For example, we assume, freshman can retain their freshman class level or progress to either a 200l, 300l, 400l, graduate, drop out or take a break from school. 200l either retains their status of 200l or transition to 300l, 400l, graduate, drop out or vacate, but 200l cannot move back to 100l freshman. 300l and 400l follow a similar progression as 100l freshmen and 200l. However, some of the transitions are uncommon (such as freshman to 400l), but they are included in the schematic because the data indicates these potential transitions (most likely due to erroneous classification of students within the data). Also, there are some backward transitions also indicated in the data which are believed to be errors in the incorrect categorization of transfer students initially. However, in the later years of data collection, these backward transitions are infrequent. We disregard all backward transitions in the calculation of probability measurements.

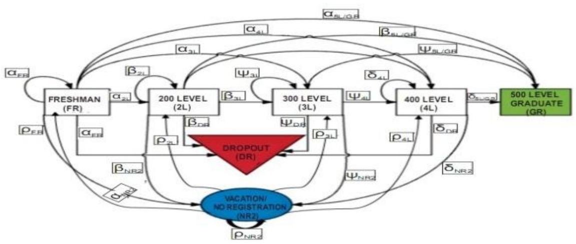

Fig1:Discrete-Time Markov Chain Probability Transition Diagram

In the diagram α, β, γ, δ and ρ are the probabilities of transitions by freshmen, 200L, 300L, 400L, 500L and vacating state respectively to other states. Graduation and dropout states are said to be absorbing states. An absorbing state is a state that once entered, cannot be left; i.e. once a student enters this state, they are assumed to no longer transition to any other states within the system. The vacating state is the only state that allows a student to enter, leave and re-enter or return. This can be described as a 'pendulum state'. It allows for free entry and exit of other states. However, we assume that there is no direct transition from the vacating state to any of the two absorbing states.

3.0Discrete-Time Markov Chain Transition Probability State Matrices

Now, we build a general probability transition matrix which is constructed of four smaller matrices, $P_1, P_{2,} P_{3,} and\ P_4$

$$\boldsymbol{P} = \begin{bmatrix} P_1 & P_2 \\ \\ P_3 & P_4 \end{bmatrix}$$

A discrete-time Markov chain is represented by a probability transition matrix P

Where $P_{ij}(n) = \Pr(X_{n+1} = j / X_n = i)$ is the probability that a student is in state j

A total of 3279 students enrolled in first year, 2305 in second year, 4060 in third year, and 4618 in fourth year, and 2764 in fifth year, for the five academic sessions with various categories of graduated, repeated and withdrawn,

Table 1Summary of the students' enrolment and their performances over years

	1st year	2nd year	3rd year	4th year	5th year
P	1000	800	1500	1900	2100
R	2000	1200	2200	2500	500
W	279	305	360	218	164
G	0	0	0	0	0
N_i	3279	2305	4060	4618	2764

Wherepromoted, repeated, withdrawn and graduating are represented by P, R, W and G respectively

4.0 The Transition Probability Matrix

From the table above, we can develop our one- step transition probability matrix as

$\boldsymbol{n_{ij}}$ =

		W	G	1L	2L	3L	4L	5L
	W	1326	0	0	0	0	0	0
	G	0	2100	0	0	0	0	0
1L	279	0	2000	1000	0	0	0	0
	2L	305	0	0	1200	800	0	0
	3L	360	0	0	0	1500	2200	0
	4L	218	0	0	0	0	2500	1900
	5L	164	2100	0	0	0	0	500

Now, to deduce the transition probability matrix (one step matrix), we divide each element of the row by its row total.

$$p_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.085 & 0 & 0.609 & 0.304 & 0 & 0 & 0 \\ 0.132 & 0 & 0 & 0.521 & 0.347 & 0 & 0 \\ 0.088 & 0 & 0 & 0 & 0.369 & 0.542 & 0 \\ 0.047 & 0 & 0 & 0 & 0 & 0.541 & 0.411 \\ 0.059 & 0.759 & 0 & 0 & 0 & 0 & 0.181 \end{pmatrix}$$

The one step transition probability matrix $\boldsymbol{p_{ij}}$ above can be decomposed into $\boldsymbol{Q_i R}$ and $\boldsymbol{Q_j}$which are defined as

$$\mathbf{Q=} \begin{pmatrix} 0.609 & 0.304 & 0 & 0 & 0 \\ 0 & 0.521 & 0.347 & 0 & 0 \\ 0 & 0 & 0.369 & 0.542 & 0 \\ 0 & 0 & 0 & 0.541 & 0.411 \\ 0 & 0 & 0 & 0 & 0.181 \end{pmatrix}$$

$$\mathbf{R=} \begin{pmatrix} 0.085 & 0 \\ 0.132 & 0 \\ 0.088 & 0 \\ 0.047 & 0 \\ 0.059 & 0.759 \end{pmatrix} \qquad \mathbf{0} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

THEOREM[8]

Given,

$$P = \begin{bmatrix} I & O \\ R & Q \end{bmatrix}$$

Then $\boldsymbol{M} = (\boldsymbol{M_{ij}})$ is given by

$$\boldsymbol{M_{ij}} = \begin{cases} \dfrac{1}{1-P_{ii}}, \ if\ i = j \\ \dfrac{\overset{i-1}{\underset{r=1}{\pi}} P_{rr+1}}{\overset{i}{\underset{r=1}{\pi}} 1 - P_{rr+1}}, \ if\ i \neq j \\ 0, elsewhere \end{cases}$$

4.1 Fundamental Matrix

To find the fundamental matrix

$$\boldsymbol{N = (I - Q)^{-1}}$$

$$\text{M=} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1.1111 & 1.0561 & 4.0806 & 0.7272 & 4.4999 \\ 0 & 1.1111 & 4.2929 & 0.6502 & 0.6314 \\ 0 & 0 & 4.5454 & 0.6869 & 0.5329 \\ 0 & 0 & 0 & 1.1363 & 0.9378 \\ 0 & 0 & 0 & 0 & 1.0663 \end{pmatrix}$$

4.2 Probability of Absorption and (Withdrawal And Graduation)

The probability that the process will enter the jth absorbing state if it starts in the ith transient state is called the probability of absorption. It is given as **B = MR** where **M** is the fundamental matrix. $\boldsymbol{R} = (\boldsymbol{N} - \boldsymbol{K}) \times \boldsymbol{K}$Matrix showing the probability of transition from a transient state to an absorbing state. Then b_{ij} is the (i, j)th entry of matrix B (Aulck, et al.,2016)..

Table 2 Estimation of probabilities from the student's enrolments

I	W	G	1	2	3	4	5	Total
n_i	1035	1840	450	500	2340	300	126	6591
p_j	0.1570	0.2791	0.0682	0.0758	0.3550	0.0455	0.0191	1.000

4.3 Predicting the future enrollment of students

Given an initial vector which contains the current enrolment of students in a five years academic programme, the future performance can be predicted by

$\boldsymbol{P^{(n)}} = \boldsymbol{P^{(0)}}\boldsymbol{P^{*(n)}}$

If we take the new students into consideration, then the total enrolment of students at the beginning of the nth academic year will be given as:

$\boldsymbol{P^{(n)}} = \boldsymbol{P^{(0)}}\boldsymbol{P^{*(n)}} +$ **r(n)** Where $\boldsymbol{P^{(n)}}$is the state of the cohort of students at the beginning of the nth year.

$\boldsymbol{P^{(0)}}$is the initial vector $\boldsymbol{P^{*(n)}}$ is the matrix of transition probabilities after removing the absorbing states at nth year.

Table 3.PREDICTING THE FUTURE ENROLMENT AND PERFORMANCE

Academic session	1st Year	2nd Year	3rd Year	4th Year	5thYear
2018/2019	3600	4100	3700	3900	4000
2019/2020	1500	3300	3300	3300	3300
2020/2021	900	2400	2800	2900	3000
2021/2022	500	1700	2200	2500	2900

5.0 Conclusion

It is discovered from the transition probability matrix that the rate of withdrawal decreases as the student's progress to highest levels. The movement of students in a particular level depends on the previous level occupied by individual. Again student's performances improve over time as they move from one level to another. This may be as a result of the fact that they understand the system better as they pass from one level to another. It is often very high in 1st year because most of the students are not stable. In essence, change of environment, inability to understand their new environment and tenets of academic work often contribute to their instability. Furthermore, the vacating state was introduced to reduce the error in our model due to misclassification of dropouts,

The simulation of the model shows it is fairly close to making accurate predictions about the total number of new students enrolled but over estimates the total number of students entirely. Hence, the reasons for the inaccurate prediction by the model is not only associated with misclassification of students but there are other factors which need to be considered in future work using the model.

We hypothesize that in the future it might be necessary to reexamine how the probabilities are generated for the matrices. Finally, present students enrolment help to give the insight of the minimum number of students that will enroll in each level in few years to come. These results can be extended to its cohort universities for prediction all things being equal. Markov Chain Model or input – Out model is very good in education planning. The models show movement of students in through out of tertiary institution. It is useful in projecting the number of students that graduates, a good academic programme should have the probabilities of withdrawal being non- decreasing function if iteming to zero while probabilities of graduation should be a non – decreasing function if it approaches unity. This means that prospects should increase as one approaches graduation.

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