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AN EMPIRICAL STUDY OF PROBABILISTIC DEMAND FOR MULTI – ITEM INVENTORY SITUATION IN AN INDUSTRIAL ENVIRONMENT IN NIGERIA

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Abstract

This paper empirically studied probabilistic demand inventory problem using iterative procedures. The answer to this inventory problem bothers on the quantity of the item to order and when to place order; the results revealed that a new order of 2306 litres is to be placed whenever inventory level drops to 1362 litres.

Keywords: Probabilistic demand, Inventory level, order, Multi – item

1. INTRODUCTION

Inventories are raw-materials, work-in-progress goods and completely finished goods ready for sale. Every manufacturing or industrial organization holds inventory. It is therefore, necessary to pay more attention to managing inventory effectively for too much inventory tied up working capital and incurs holding costs, reducing profit. Good inventory management has, therefore, become crucial to business as they seek to continually improve their customer service and profit margin in the heat of competition and demand variability.

Inventory replenishment is fundamental in production environment most especially in multistage production system under probabilistic demand situation. Manufacturers desire to satisfy customers demand always and at the same time avoid keeping unnecessary inventory that ties down money may be used for other productive purposes, In addition, manufacturers wish to maintain minimal inventory to avoid stock out situation so that it will not result in lost sales and loss of goodwill. Consequent upon these, it is necessary to maintain a balance between minimal inventory and customer demand satisfaction with the help of Economic Order Quantity, provided by Haris in 1915 [1].

It should be noted that the EOQ model assumes inventory situation with constant demand and delivery lead time which does not conform with reality because in most cases, demand are uncertain and therefore require model that can handle probabilistic demand situation [2].

Researchers in the area of probabilistic demand inventory have proposed different model approaches. Some of which are cited, for example,[3-7].

In the quest to control an inventory problem involving probabilistic demand, it requires employing a suitable inventory model that will minimize total inventory costs by determining the optimum amount of inventory that should be held and when inventory should be ordered for. In other words, the basic inventory problem is to balance the need for having enough inventories to meet expected demand but keeping the associated inventory costs (fixed, holding and shortage costs) at a minimum.

In any production inventory system, it is very important to determine an optimum production rate, because if the manufacturer produces a huge quantum of goods, it may result in a loss due to deterioration of the products, holding cost of the excess items and huge investment in the production. Also, the products may get obsolete and hence remained unsold. On the other hand, an insufficient amount of stock may result in a shortage.

Most industrial settings demand is uncertain; therefore it requires probabilistic model. It is on this basis that, in this paper we address an inventory problem using inventory model with probabilistic demand, assuming uniform distribution. The model make use of an iterative procedure that provides an approximation of the optimal solution which yields good solution to placing a new order, y^* whenever inventory dwindles to a certain level called re-order level, R^*.

MODEL SPECIFICATION

Description of the Probabilistic EOQ Model

Probabilistic EOQ model is a more accurate model in which the probabilistic nature of the demand is included directly in the formulation of the model. The model allows shortage of demand. The basic assumptions underlying the use of the model are listed as follows:

(a) Unfilled demand during lead time is backlogged.

(b) No more than one outstanding order is allowed.

(c) The distribution of demand during lead time remains stationary with time.

The model employed in this study using an iterative algorithm also known as a numeric algorithm to determine the best order quantity, (y^*) and the best order period, (R^*) that will minimize the expected total cost. The algorithm is developed in [8]. The following are the steps in the numeric algorithm.

(i) Check for existence of feasible solution to the problem using $\hat{y}$ and $\tilde{y}$ (see the proof in the Appendix). If $\tilde{y} \geq \hat{y}$, a unique solution to the optimum order quantity and optimum re-order level (i.e y^* and R^*) exist.

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$\tilde{y} = \frac{PD}{h}, \quad \hat{y} = \sqrt{\frac{2D[K + P \cdot E(x)]}{h}},$

where $E(x) = \int_0^\infty xf(x)dx.$

P represents penalty cost
D represents demand
h represents holding cost per item per period
K represents ordering or fixed cost

(ii) Compute R_i using $\int_{R_i}^{\infty} f(x)dx = \frac{hy_i}{PD}$

and $S = \int_{R_i}^{\infty} (x - R_i) f(x)dx$

(iii) Compute y_i

y_i is partitioned into $y_i = \sqrt{\frac{2DK}{h}}$ *and* $y_{i>1} = \sqrt{\frac{2D[K + P \cdot S]}{h}}$

The optimal values of y^* *and* R^* is reached when $y_{i+r} \approx y_{i+r-1}$ *and* $R_{i+r} \approx R_{i+r-1}$.

The algorithm converges in a finite number of iterations.

ANALYSIS

ILLUSTRATIVE EXAMPLE

A reputable manufacturing company in Nigeria uses diesel to power its generating sets in their production process and demanded for at the rate of 10,000 litres. This is so, because of incessant power failure. It costs the company ₦5000 to place an order. The holding cost per litre per month is ₦20 and the shortage cost per liter is ₦50. Historical data shows that the demand during lead time is uniform in the range $(0, 1500)$ litres.

D = 10,000, h = ₦20, P = ₦50, K = ₦5,000; $\cup \sim (0, 1500)$

$$f(x) = \frac{1}{1500}, \quad 0 \le x \le 1500.$$

$$E(x) = \int_0^{1500} x \cdot \frac{1}{1500} dx = \frac{x^2}{3000}\Big|_0^{1500} = \frac{(1500)^2}{3000} = 750 \ litres.$$

$$\hat{y} = \sqrt{\frac{2 \times 10{,}000\,[5000 + 50(750)]}{20}} = 6519.20 \ litres.$$

$$\tilde{y} = \frac{50 \times 10000}{20} = 25000 \ litres.$$

A unique solution exists for y^* and R^* since $\tilde{y} > \hat{y}$.

$$\int_{R_i}^{1500} \tfrac{1}{500} dx = \frac{20 \times y_i}{50 \times 10000}$$

$$\Rightarrow R_i = 1500 - \frac{3y_i}{50}$$

$$S = \frac{1}{500}\int_{R_i}^{1500} x\,dx - \frac{R_i}{1500}\int_{R_i}^{1500} dx$$

$$S = \frac{R_i^2 - 3000R_i + 2250000}{3000}$$

RESULTS AND INTERPRETATION

Table 1: Results of the Analysis

	1	2	3	4	5
y_i	2236.07	2302.17	2306.08	2306.31	2306.31
R_i	1365.84	1361.87	1361.64	1361.62	1361.62
S_i	5.9996	6.3599	6.3812	6.3830	6.3830

The Table 1 shows that $y_4 \approx y_5$ *and* $R_4 \approx R_5$. Therefore, optimal solution for y and R are approximately 2306 and 1362 litres respectively.

CONCLUSION

It was discovered that most manufacturing companies in Nigeria took no cognizance of the inventory systems in their production management, hence the need to showcase the usefulness of the systems to determine how much inventory to order, when necessary.

Therefore, it is concluded from the analysis of this study that a new order of 2306 litres is to be placed whenever inventory level drops to 1362 litres.

APPENDIX

$$TC(y,R)=\frac{DK}{y}+h\left(\frac{y}{2}+R-E[x]\right)+\frac{PD}{y}\int_R^{\infty}(x-R_i)f(x)dx \qquad \text{(i)}$$

Differentiate (i) with respect to y and set $\frac{\partial}{\partial y}(TC)=0,$ then make y the subject.

$$\frac{\partial}{\partial y}(TC)=-\frac{DK}{y^2}+\frac{h}{2}-\frac{PD}{y^2}\int_R^{\infty}(x-R_i)f(x)dx$$

$$0=-\frac{DK}{y^2}+\frac{h}{2}-\frac{PD}{y^2}\int_R^{\infty}(x-R_i)f(x)dx$$

$$\Rightarrow \frac{D}{y^2}\left[K+P\int_R^{\infty}(x-R_i)f(x)dx\right]=\frac{h}{2}$$

$$\Rightarrow y^2=\frac{2D\left[K+P\int_{R_i}^{\infty}(x-R_i)f(x)dx\right]}{h}$$

set $\int_{R_i}^{\infty}(x-R_i)f(x)dx=S$

$$\therefore y^2=\frac{2D[K+P\cdot S]}{h}$$

$$y^*=\sqrt{\frac{2D[K+P\cdot S]}{h}} \qquad \text{(ii)}$$

Differentiate (i) with respect to R_i and set $\frac{\partial}{\partial R_i}(TC)=0,$ then make $\int_{R_i}^{\infty}f(x)dx$, the subject.

$$\frac{\partial}{\partial R_i}(TC)=h-\frac{PD}{y^*}\int_{R_i}^{\infty}f(x)dx$$

$$0=h-\frac{PD}{y^*}\int_{R_i}^{\infty}f(x)dx$$

$$\Rightarrow \frac{PD}{y^*}\int_{R_i}^{\infty}f(x)dx=h$$

$$\Rightarrow \int_{R_i}^{\infty}f(x)dx=\frac{hy^*}{PD} \qquad \text{(iii)}$$

Substituting (iii) into (i) gives:

$$TC(y,R)=\frac{DK}{y^*}+h\left(\frac{y^*}{2}+R-E[x]\right)+\frac{PD}{y^*}\left(E[x]-\frac{hy^*}{PD}\right) \qquad \text{(iv)}$$

Differentiate (iv) with respect to y^*

$$\frac{\partial}{\partial y^*}(TC)=-\frac{DK}{y^{*2}}+\frac{h}{2}-\frac{PD}{y^{*2}}E[x]$$

$$\frac{\partial}{\partial y^*}(TC)=-\frac{D}{y^{*2}}[K+PE(x)]+\frac{h}{2}$$

set $\frac{\partial}{\partial y^*}(TC)=0$

$$\Rightarrow \frac{D}{y^{*2}}[K+P\cdot E(x)]=\frac{h}{2}$$

$$\therefore y^{*2}=\frac{2D[K+P\cdot E(x)]}{h}$$

Then, $\hat{y}^*=\sqrt{\frac{2D[K+P\cdot E(x)]}{h}}$ (v)

Differentiate (iv) with respect to $E(x)$, and equate to zero.

$$\frac{\partial}{\partial E(x)}(TC)=-h+\frac{PD}{y^*}$$

$$\Rightarrow h=\frac{PD}{y^*}$$

$$hy^*=PD$$

$$\tilde{y}^*=\frac{PD}{h} \qquad \text{(vi)}$$

REFERENCES

[1] Muchstadt J.A and Roundy R.O. (1993). Analysis of Multistage Production System. Handbooks in OR & MS, 4.

[2] Cetinkaya S. and Lee C-Y. (2000). Stock Replenishment and Shipment Scheduling for Vendor-Managed Inventory System. Management Science, 46(2), 217-232.

[3] Chen L.H, Ouyang L.Y, Teng J.T(2006). On an EOQ Model with Ramp Type Demand Rate and Time Dependent Deterioration Rate. Int Journal of Inf Manag Sci , 17: 51- 66.
[4] Patra S.K (2010). An Order Level Inventory Model For Deteriorating Items with Partial Backlog and Partial Lost Sales. Int Journal of Adv Oper Manag ; 2: 185.
[5] Prasad K, Mukherjee B (2016). Optimal Inventory Model Under Stock and Time Dependent Demand for Time Varying Deterioration Rate with Shortage. Ann Oper Res 2016; 243-334.
[6] Palanivel M, Uthayakumar R.(2017). A Production-Inventory Model with Promotional Effort, Variable Production Cost and Probabilistic Deterioration. Int J Syst Assur Eng Manag., 8: 290-300.
[7] Saha S, Chakrabarti T. Fuzzy (2017). Inventory Model for Deteriorating Items in a Supply Chain System with Price Dependent Demand and Without Backorder. Am J Eng Res; 6: 183-187.
[8] Hadley, G. and T. Whitin (1963). Analysis of Inventory Systems, Prentice Hall, Upper saddle River, NK.