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A NEW STATISTICAL MODEL FOR ANALYZING THE NUMBER OF SUCCESSIVE FAILURES OF AIR CONDITIONING SYSTEM

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Abstract

This paper presents a new Statistical model for analyzing time-to-event data set. Some Statistical properties of the distribution such as the density function, cumulative distribution function, survival function, hazard rate function, mean residual life function, quantile function, median, moments and Renyi entropy were derived. The maximum likelihood estimation method was employed to estimate the unknown parameters of the distribution and a Monte Carlo simulation study was carried out to investigate the asymptotic behaviour of the maximum likelihood estimators of the distribution. The proposed distribution was applied to a data set consisting of the number of successive failures of air conditioning system and its fit were compared with the fits attained by some existing lifetime distributions.

Keywords: Akash distribution, Hazard function, Moments, Simulation Study

1. Introduction

In reliability, some parametric models have been successfully studied and served as a statistical tool for modeling lifetime data. The length of life of any device or organism is expected to exhibit a decreasing failure rate (DFR), if its behaviour is characterized by "work-hardening" (in Engineering terms) and "Immunity" (in Biological terms). The shape of the failure rate of these models can be monotone (increasing or decreasing) or non-monotone (bathtub, upside down bathtub or unimodal). Distributions whose failure rate exhibits monotonic shape are studied in the works of [1-8] among others. While distributions whose failure rate exhibits non-monotonic shape are studied in the works of [9-14].

Here we consider the Akash distribution with increasing failure rate (IFR) proposed by [15] and its probability density function (pdf) is of the form

$$f(x,\theta)=\frac{\theta^3}{\theta^2+2}\left(1+x^2\right)e^{-\theta x} \quad ; x>0,\theta>0 \tag{1}$$

The density function in Equation (1) is obtained from the mixture method of generalization given by

$$f(x) = pf_1(x) + (1-p)f_2(x). \tag{2}$$

Where $f_1(x)$ and $f_2(x)$ are the density functions of a known probability distribution and p is called the mixing proportion.

The Akash distribution is a two-component mixtures of an exponential (θ) and gamma ($3,\theta$) distributions with their mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$ respectively.

The corresponding cumulative distribution function (cdf) of (1) is given by

$$F(x,\theta)=1-\left[1+\frac{\theta x(\theta x+2)}{\theta^2+2}\right]e^{-\theta x} \quad ; x>0,\theta>0 \tag{3}$$

The work of [15]suggests that the Statistical properties of the one-parameter Akash distribution tends to be more flexible than the Lindley and Exponential distributions. In spite of the flexibility of this distribution over Lindley and Exponential distributions in modeling lifetime data set, there are situations where the Akash distribution may not provide a better fit when analyzing a lifetime data. Thus, to increase the flexibility of this distribution, we wish to extend the classical one-parameter Akash distribution to a two-parameter lifetime distribution by introducing another scale parameter to the existing parameter. We shall call the proposed distribution, the Two-Parameter Generalized Akash Distribution (TPGAD). The rest Sections of this paper are organized as follows: Section 2 presents the statistical properties of the TPGA distribution. The estimation of the unknown parameters and a Monte Carlo simulation study to examine the asymptotic behaviour of the maximum likelihood estimators of the TPGA distribution are discussed in Section 3. Finally, the application of the TPGA distribution to a data set consisting of the number of successive failures of air conditioning system is presented in Section 4.

2. Statistical Properties of the TPGA Distribution

2.1 The Density Function and Cumulative Distribution Function of the TPGAD

Let X be a continuous random variable, then using the method of generalization in Equation (2), the probability density function of the Two-Parameter Generalized Akash distribution is defined by

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$$f(x,\theta,\beta) = \frac{\theta}{\beta+2}\left(\beta+(\theta x)^2\right)e^{-\theta x}, \qquad x>0,\ \theta>0,\ \beta>-2. \tag{4}$$

Where $f_1(x)=\theta e^{-\theta x}$ is the density function of the exponential distribution, $f_2(x)=\dfrac{\theta^3 x^2 e^{-\theta x}}{2}$ is the density function of a special case of gamma distribution and $p=\dfrac{\beta}{\beta+2}$ is the mixing proportion.

It is readily observed that for $\beta = \theta^2$, the density function of the TPGAD reduces to the density function of the one parameter Akash distribution and for $\beta = 0$, it reduces to the density function of a special case of gamma $(3,\theta)$.

Theorem 1:

The shape of the density function of the Two-Parameter Generalized Akash distribution is decreasing for $\beta \geq 1$ and unimodal for $\beta < 1$ with mode at $x_0 = \dfrac{1+\sqrt{(1-\beta)}}{\theta}$.

Proof:

Taking the natural logarithm of Equation (4), the first derivative is given by

$$\frac{d}{dx} ln\ f(x) = \frac{2\theta^2 x}{\beta+(\theta x)^2} - \theta,$$

it follows that for,

(i) $\beta \geq 1,\ \dfrac{d}{dx} ln\ f(x) < 0,$ that is, $f(x)$ is decreasing in x,

(ii) $\beta < 1,\ \dfrac{d}{dx} ln\ f(x) = 0,$ which implies that $f(x)$ has a unique mode at x_0,

where $x_0 = \dfrac{1+\sqrt{(1-\beta)}}{\theta}$.

The corresponding cumulative distribution function of the TPGAD is given by

$$F(x,\theta,\beta) = 1 - \left[1 + \frac{\left((\theta x)^2 + 2\theta x\right)}{\beta+2}\right] e^{-\theta x} \qquad x>0, \theta>0, \beta>-2 \tag{5}$$

The graphical plots of the density function and the cumulative distribution of TPGA distribution for different values of θ and β are shown in Figure 1 below

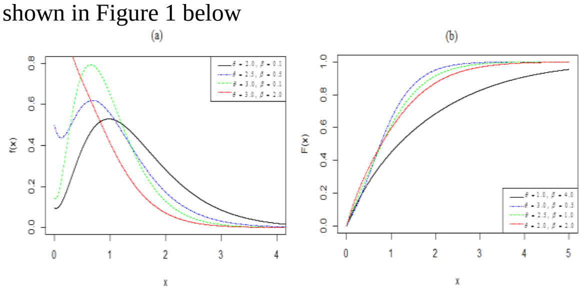

Figure 1: Density Function and Cumulative Distribution Function of TPGAD

The plots in Figure 1(a) suggest that the density function of the TPGA distribution can be right skewed (uni-modal) or decreasing (reversed J-shaped) and Figure 1(b) indicates that cumulative distribution function of the TPGA distribution tends to 1 as x tends to infinity, for different values of the parameters.

2.2 The Survival, Hazard Rate and Mean Residual Life Functions of the TPGAD

Let X be a continuous random variable with density function $f(x)$ and cumulative distribution function $F(x)$. The survival (reliability), hazard rate (failure rate)and Mean residual life functions of the two-parameter generalized Akash distribution are defined as

$$s(x) = 1-F(x) = \left(1+\frac{\left((\theta x)^2+2\theta x\right)}{\beta+2}\right)e^{-\theta x}, \tag{6}$$

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{\theta\left(\beta+(\theta x)^2\right)}{((\beta+2)+(\theta x)(\theta x+2))}, \tag{7}$$

and

$$m(x) = \frac{1}{1-F(x)}\int_x^\infty (1-F(t))\,dt = \frac{\left((\beta+6)+(\theta x)(\theta x+4)\right)}{\theta\left((\beta+2)+(\theta x)(\theta x+2)\right)}. \tag{8}$$

Figure 2 displays the graphical plots of the two-parameter generalized Akash distribution for selected values of the parameters.

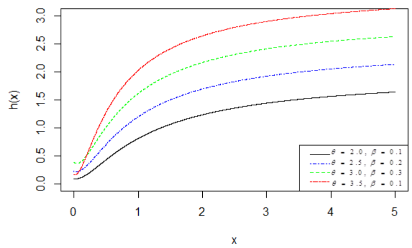

Figure 2: Hazard rate function of the Two-Parameter Generalized Akash Distribution

The plots in Figure 2 suggest that the two-parameter generalized Akash distribution exhibits an increasing failure rate property for selected values of the parameters

2.3 Quantile Function of the TPGAD

The quantile function of the two-parameter generalized Akash distribution can be obtained from the expression $F(x) = u$. Where *F(x)* is the distribution function given by Equation (5) and $0 < u < 1$. Thus, using numerical methods, the quantile function of the TPGA distribution is obtained by solving the system of nonlinear equation

$$\theta x + \log[(1-u)(\beta+2)] - \log[(\theta x)^2 + 2\theta x + \beta + 2] = 0 \tag{9}$$

Subsequently, random samples from the two-parameter generalized Akash distribution can be generated using equation (9) for some selected values of the parameters as shown in Table 1.

Table 1: Random Samples fromTwo-Parameter Generalized Akash distribution

U	$(\theta = 1, \beta = 0.1)$	$(\theta = 2, \beta = 0.1)$	$(\theta = 3, \beta = 0.1)$	$(\theta = 2, \beta = 2)$
0.1	0.965365	0.482682	0.321788	0.110616
0.2	1.422851	0.711426	0.474284	0.244261
0.3	1.814070	0.907035	0.604690	0.401608
0.4	2.193622	1.096811	0.731207	0.581459
0.5	2.588802	1.294401	0.862934	0.784060
0.6	3.025180	1.512590	1.008393	1.014866
0.7	3.539797	1.769899	1.179933	1.288836
0.8	4.207441	2.103720	1.402480	1.642267
0.9	5.255199	2.627600	1.751733	2.190217

2.4 The r^{th} Raw and k^{th} Central Moments of the TPGAD

Let *X*be a continuous random variable with probability density function $f(x)$, then the r^{th} moment about the origin of X is defined by,

$$\mu_r' = E(X^r) = \int_{-\infty}^{\infty} x^r f(x)dx. \tag{10}$$

the r^{th} moment of the TPGA distribution is obtained as

$$= \int_0^{\infty} x^r \frac{\theta(\beta + (\theta x)^2)}{\beta + 2} e^{-\theta x} dx$$

$$= \frac{\theta}{\beta + 2} \int_0^{\infty} x^r (\beta + (\theta x)^2) e^{-\theta x} dx$$

$$= \frac{1}{\beta + 2}\left[\frac{\beta}{\theta^r}\Gamma(r+1) + \frac{1}{\theta^r}\Gamma(r+3)\right]$$

$$= \frac{\beta\Gamma(r+1) + \Gamma(r+3)}{\theta^r(\beta+2)} \cdot \tag{11}$$

Using Equation (11), the first four raw moments of the two-parameter generalized Akash distribution are obtained as follows

$$\mu_1' = \frac{\beta + 6}{\theta(\beta+2)} = \mu \qquad \mu_2' = \frac{2\beta + 24}{\theta^2(\beta+2)}$$

$$\mu_3' = \frac{6\beta + 120}{\theta^3(\beta+2)} \qquad \mu_4' = \frac{24\beta + 720}{\theta^4(\beta+2)}$$

The k^{th} central moments of the two-parameter generalized Akash distribution are obtained as

$$\mu_k = E\{(X-\mu)^k\} = \sum_{r=0}^{k}\binom{k}{r}\mu_r'(-\mu)^{k-r},$$

in particular,

$$\mu_2 = \left(\mu_2' - \mu^2\right) = \frac{\beta^2 + 16\beta + 12}{\theta^2(\beta+2)^2},$$

$$\mu_3 = \left(\mu_3' - \mu_2'\mu + 2\mu^3\right) = \frac{2\left(\beta^3 + 30\beta^2 + 36\beta + 24\right)}{\theta^3(\beta+2)^3},$$

$$\mu_4 = \left(\mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^4\right) = \frac{3\left(3\beta^4 + 128\beta^3 + 408\beta^2 + 576\beta + 240\right)}{\theta^4(\beta+2)^4}.$$

Using the k^{th} central moments above, the measures of skewness (S_k) and kurtosis (K_s) of the two-parameter generalized Akash distribution are obtained as

$$S_k = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2\left(\beta^3+30\beta^2+36\beta+24\right)}{\left(\beta^2+16\beta+12\right)^{3/2}},$$

$$K_s = \frac{\mu_4}{(\mu_2)^2} = \frac{3\left(3\beta^4+128\beta^3+408\beta^2+576\beta+240\right)}{\left(\beta^2+16\beta+12\right)^2}.$$

Table 2 gives a summary statistic of the first four moments, variance (μ_2), measure of skewness (S_k) and measure of kurtosis (K_s) of the two-parameter generalized Akash distribution for selected values of the parameters.

Table 2: Theoretical Moments of the TPGA Distribution for selected value of the Parameters

θ	β	μ_1'	μ_2'	μ_3'	μ_4'	μ_2	S_k	K_s
1	0.1	2.9048	11.5238	57.4286	3440	3.0859	1.1117	4.8883
	0.5	2.6000	10.0000	49.2000	292.80	3.2400	1.0892	4.7275
	0.9	2.3793	8.8966	43.2414	255.72	3.2355	1.1473	4.7982
2	0.1	1.4524	2.8810	7.1786	21.500	0.7715	1.1113	4.8888
	0.5	1.3000	2.5000	6.1500	18.300	0.8100	1.0892	4.7275
	0.9	1.1897	2.2241	5.4052	15.983	0.8087	1.1481	4.7987
3	0.1	0.9683	1.2804	2.1270	4.2469	0.3428	1.1127	4.8876
	0.5	0.8667	1.1111	1.8222	3.6148	0.3599	1.0898	4.7293
	0.9	0.7931	0.9885	1.6015	3.1571	0.3595	1.1473	4.7988
4	0.1	0.7262	0.7202	0.8973	1.3438	0.1928	1.1130	4.8917
	0.5	0.6500	0.6250	0.7687	1.1438	0.2025	1.0886	4.7319
	0.9	0.5948	0.5560	0.6756	0.9989	0.2022	1.1476	4.8001

From Table2, we observed that the TPGA distribution is strictly right-skewed ($S_k > 0$) and strictly leptokurtic $(K_s > 3)$ in terms of the peakness of the distribution. Also, Table 2 further supports the claim in the graphical plots shown in Figure 1(a).

2.5 The Renyi Entropy of the TPGAD

An entropy of a random variable *X* is a measure of variation of uncertainty associated with the random variable *X*. [16] defined the Renyi entropy of *X* with density function $f(x)$, as

$$\tau_R(\gamma) = \frac{1}{1-\gamma} ln\left(\int f^{\gamma}(x)dx\right), \qquad \gamma>0,\ \gamma\neq 1. \tag{12}$$

By substituting the density function of the TPGA distribution defined in equation (4) into (12), we obtain

$$\tau_R(\gamma) = \frac{1}{1-\gamma} ln\left(\int_0^{\infty}\frac{\theta^{\gamma}\left(\beta+(\theta x)^2\right)^{\gamma}}{(\beta+2)^{\gamma}}e^{-\theta\gamma x}dx\right),$$

$$= \frac{1}{1-\gamma} ln\left(\frac{\theta^{\gamma}}{(\beta+2)^{\gamma}}\int_0^{\infty}\left(\beta+(\theta x)^2\right)^{\gamma}e^{-\theta\gamma x}dx\right),$$

$$= \frac{1}{1-\gamma} ln\left(\sum_{r=0}^{\infty}\binom{\gamma}{r}\beta^{\gamma-r}\frac{\theta^{\gamma}}{(\beta+2)^{\gamma}}\int_0^{\infty}(\theta x)^{2r}e^{-\theta\gamma x}dx\right),$$

but

$$\int_0^{\infty}(\theta x)^{2r}e^{-\theta\gamma x}dx = \frac{\Gamma(2r+1)}{\theta\left(\gamma^{2r+1}\right)},$$

thus,

$$\tau_R(\gamma) = \frac{1}{1-\gamma} ln\left(\sum_{r=0}^{\infty}\binom{\gamma}{r}\beta^{\gamma-r}\frac{\theta^{\gamma-1}\Gamma(2r+1)}{(\beta+2)^{\gamma}\left(\gamma^{2r+1}\right)}\right). \tag{13}$$

3. Estimation of Unknown Parameters of the TPGA Distribution

3.1 Estimation by maximum likelihood

Let $(x_1, x_2, ..., x_n)$ be random samples from the two-parameter generalized Akash distribution with density function defined in Equation (4), then the likelihood function is of the form

$$L(x,\eta) = \prod_{i=1}^{n}\left[\frac{\theta(\beta+(\theta x)^2)e^{-\theta x}}{\beta+2}\right], \tag{14}$$

The log-likelihood function based on the random sample of size *n* from the two-parameter generalized Akash distribution is given by

$$\ell(x,\eta) = n\log\theta + \sum_{i=1}^{n}\log\left(\beta+(\theta x)^2\right) - n\log(\beta+2) - n\theta\overline{X} \tag{15}$$

and the corresponding gradients are obtained as

$$\frac{\partial\ell}{\partial\beta} = \sum_{i=1}^{n}\frac{1}{(\beta+(\theta x)^2)} - \frac{n}{(\beta+2)}, \tag{16}$$

$$\frac{\partial \ell}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \frac{2\theta x^2}{(\beta + (\theta x)^2)} - n\overline{X} \qquad (17)$$

Where $\overline{X}$ is the sample mean.

In estimation of the parameters θ and β, the equation $\frac{\partial \ell}{\partial \theta} = 0$ and $\frac{\partial \ell}{\partial \beta} = 0$ cannot be solved directly, hence the need of an iterative scheme known as the Newton-Raphson procedure. The hessian matrix is given by

$$H(\eta) = \left(\frac{\partial^2 \ell}{\partial \eta^2}\right) = \begin{bmatrix} \frac{\partial^2 \ell}{\partial \theta' \partial \theta} & \frac{\partial^2 \ell}{\partial \theta \partial \beta} \\ \frac{\partial^2 \ell}{\partial \beta \partial \theta} & \frac{\partial^2 \ell}{\partial \beta' \partial \beta} \end{bmatrix}$$

where,

$$\frac{\partial^2 \ell}{\partial \theta' \partial \theta} = \sum_{i=1}^{n} \frac{2x^2[\beta - (\theta x)^2]}{[(\beta + (\theta x)^2)]^2} - \frac{n}{\theta^2}$$

$$\frac{\partial^2 \ell}{\partial \theta \partial \beta} = \frac{\partial^2 \ell}{\partial \beta \partial \theta} = -\sum_{i=1}^{n} \frac{2\theta x^2}{[(\beta + (\theta x)^2)]^2}$$

$$\frac{\partial^2 \ell}{\partial \beta' \partial \beta} = \frac{n}{(\beta + 2)^2} - \sum_{i=1}^{n} \frac{1}{[(\beta + (\theta x)^2)]^2}$$

3.2 Interval estimate

The asymptotic confidence intervals (CIs) for the parameters of TPGAD(β,θ) is obtained according to the asymptotic distribution of the maximum likelihood estimate of the parameters.

Let $\hat{\eta} = (\hat{\theta}, \hat{\beta})$ be MLE of η, the estimators are approximately bi-variate normal with mean (θ, β) andthe Fisher information matrix is given by:

$$I(\eta_k) = -E(H(\eta_k)) \qquad (18)$$

The approximate (1-δ)100 CIs for the parameters θ and β, respectively, are

$\hat{\beta} \pm Z_{\delta/2}\sqrt{\text{var}(\hat{\beta})}$ and $\hat{\theta} \pm Z_{\delta/2}\sqrt{\text{var}(\hat{\theta})}$

where $\text{var}(\hat{\beta})$ and $\text{var}(\hat{\theta})$ are the variance of β and θ which are given by the first and second diagonal element of the variance-covariance matrix $I^{-1}(\eta_k)$ and $Z_{\delta/2}$ is the upper ($\delta/2$) percentile of the standard normal distribution. It is important to note that for a given set of data, the matrix given by equation (18) is obtained after the convergence of the Newton-Raphson procedure.

3.3 Simulation Study

Here, we investigate the behaviour of the maximum likelihood estimates of the two-parameter generalized Akash distribution, through a simulation study for different parameter values as well as different sample sizes. Random data from the two-parameter generalized Akash distribution are generated using equation (9). The Monte Carlo simulation study is repeated 1000 times for different sample sizes n = 50,75,100,150, 200 and parameter values (θ= 0.2, β = 0.3), (θ= 2, β = 1) and (θ= 0.2, β = 1).

Table 3: Monte Carlo Simulation Results for Average Bias and Mean Square Error of the MLE

Parameter	*N*	Average Bias (θ)	Mean Square Error (θ)	Average Bias (β)	Mean Square Error (β)
θ = 0.2, β = 0.3	50	0.002548	0.000468	0.040295	0.106971
	75	0.001778	0.000354	0.013571	0.054527
	100	0.001232	0.000213	0.013320	0.034141
	150	0.000354	0.000132	0.012081	0.018789
	200	0.000337	0.000107	0.007674	0.015753
θ = 2, β = 1	50	0.033950	0.076063	0.104344	0.505837
	75	0.026366	0.050011	0.059496	0.278788
	100	0.025002	0.034048	0.026177	0.217560
	150	0.016622	0.023410	0.020208	0.114670
	200	0.010187	0.019016	0.015528	0.101886
θ = 0.3, β = 1	50	0.004199	0.001806	0.042610	0.509725
	75	0.003853	0.001076	0.042068	0.248743
	100	0.002723	0.000883	0.025215	0.234749
	150	0.001223	0.000554	0.015007	0.135750
	200	0.000906	0.000384	0.012564	0.076713

From Table 3, we clearly observe that the values of the average bias and the mean square error of the parameter estimates decreases as the sample size n increases which validates the consistency property of a good estimator.

4. Application of the two-parameter generalized Akash distribution to Lifetime Data

In this Section, we fit the two-parameter generalized Akash distribution to a real data set and compare its fit with some existing lifetime distribution which includes; Akash distribution with density function, $f(x,\theta) = \frac{\theta^3}{\theta^2 + 2}(1 + x^2)e^{-\theta x}$, Lindley distribution with density function,

$f(x,\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}$ and Quasi Lindley with density function, $f(x,\theta)=\frac{\theta}{\beta+2}(\beta+x\theta)e^{-\theta x}$. The estimates of the parameters of the distribution, *-2log(L)*, Akaike Information Criterion [AIC = *2k-2log(L)*], Akaike Information Criterion corrected [AICC = $AIC+\frac{2k(k+1)}{n-k-1}$], Bayesian Information Criterion [BIC = *klog(n)-2log(L)*] and Kolmogorov-Smirnov Statistic (*K-S*), were considered for the comparison. Where n is the number of observations, k is the number of estimated parameters and L is the value of the likelihood function evaluated at the parameter estimates.

Data Set: The data set consists of the number of successive failures for the air conditioning system of each member in a fleet of 13 Boeing 720 jet airplanes reported in [3]. The data set is presented in Table 4.

Table 4: Air Conditioning System Data

194	413	90	74	55	23	97	50	359	50	130	487	57 102	15
	14	10	57	320	261	51	44	9	254	493	33		
18	209	41	58	60	48	56	87	11	102	12	5	14	
14	29	37	186	29	104	7	4	72	270	283	7	61	
100	61	502	220	120	141	22	603	35	98	54	100	11	
181	65	49	12	239	14	18	39	3	12	5	32	9	
438	43	134	184	20	386	182	71	80	188	230	152	536	
79	59	33	246	1	79	3	27	201	84	27	156	21	
16	88	130	14	118	44	15	42	106	46	230	26	59	
153	104	20	206	5	66	34	29	26	35	5	82	31	
118	326	12	54	36	34	18	25	120	31	22	18	216 139	67
	310	3	46	210	57	76	14	111	97	62	39		
30	7	44	11	63	23	22	23	14	18	13	34	16	
18	130	90	163	208	1	24	70	16	101	52	208	95	
62	11	191	14	71	-	-	-	-	-	-	-	-	

Table 5: Estimates,*-2logL, AIC, AICC, BIC and K-S Statistic* for Air Conditioning System Data

Distributions	Parameter estimates	*-2log L*	*AIC*	*AICC*	*BIC*	*K-S* Statistic
TPGAD (θ,β)	θ = 0.01347 β = 14.4996	2072.91	2076.91	2076.9729	2083.381	0.0706
Quasi Lindley (θ,β)	θ = 0.01154 β = 14.9596	2076.76	2080.76	2080.8239	2087.232	0.0854
Akash (θ)	θ= 0.03256	2331.16	2331.16	2331.1825	2333.639	0.2894
Lindley (θ)	θ= 0.02149	2165.31	2167.31	2167.3305	2170.546	0.2149

The best fitted distribution is considered by investigating the distribution with the minimum *-2logL, AIC, AICC, BIC* and *K-S Statistic* value. Table 5, indicates that two-parameter generalized Akash distribution gives the best fit and thus demonstrate more flexibility over the examined lifetime distributions in modeling real lifetime data.

Concluding Remark

In this paper, a new two-parameter generalized Akash distribution is introduced and the Statistical properties such as the shape of the density function, cumulative distribution function, survival function, hazard rate function, mean residual life function, quantile function, median, and Renyi entropy were derived. The maximum likelihood method for estimating the unknown parameters of the proposed distribution have been as well discussed. A Monte Carlo simulation study was carried out to examine the performance and accuracy of the maximum likelihood estimators of the parameters of the proposed distribution. The application of the proposed distribution to a real data set alongside with Akash distribution, Lindley distribution and Quasi Lindley distribution reveals that the proposed distribution fits better in modeling lifetime data.

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