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A NEW CLASS OF EXPONENTIAL RATIO TYPE ESTIMATORS IN RANKED SET SAMPLING

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Abstract

Ranked set sampling (RSS) gives an advantage in deriving an unbiased estimator for population parameters with some noticeable increase in efficiency. This study presents a new class of exponential ratio type estimators in ranked set sampling (RSS) and compared with an existing class of modified exponential ratio estimators in simple random sampling (SRS). The data set used in this paper is the data on enrolment of students (variable of interest) and staff strength (auxiliary variable) in secondary schools in Egba zone of Ogun State in 2015. The zone had 89 schools and a 3cycle ranked set sample of size 27 was selected. The descriptive statistics of the data set were obtained for the estimation of population ratio. The mean square errors (MSEs) for both the proposed estimators and Singh estimators were determined to obtain the efficiencies of the proposed estimators. The population mean for student enrolment and staff strength were 1581.1 and 66.44 respectively which gave a population ratio of 23.80. The MSEs of Singh estimators were 276,587.7; 269,512.8; 253,601.6; 271,224.3; 274,429.9; 267,024.4; 253,020.3; 246,060.8; 271,080.4 and 274,957.9 while those of proposed estimators were 241,662; 234,759.1; 219,234.8; 236,429; 239,556.7; 232331.2; 218,667.5; 211,887.3; 236,288.6 and 240,071.9 respectively. The MSEs for the members proposed class of estimators were found to be smaller than those of Singh class of estimators, hence they are more efficient estimators.

INTRODUCTION

Ranked set sampling, exponential ratio estimator, bias, mean square error (MSE), efficiency.

Ranked set sampling is a technique of gathering data that positively increases accuracy utilizing the sampler's judgement or available information about the relative sizes of the sampling units. In a bid to increase the precision of crop yield estimates without increasing the number of observations that need to be quantified, the ranked set sampling technique was proposed in [1].For comparable sample sizes, [2] demonstrated that the RSS procedure results in more accurate parameter estimators than simple random sampling (SRS). Equivalently, RSS requires fewer measured observations than SRS to attain the same level of precision. The improvement in precision is because RSS adds structure to the data in the form of the sampler's ranking that is absent in SRS.

It is well known that suitable use of auxiliary information in probability sampling results in considerable reduction in the variance of the estimators of population parameters viz. population mean (or total), median, variance, regression coefficient, and population correlation coefficient, etc.[3]

Several estimators of population mean are available in the literature when auxiliary variable is positively correlated with the variable of interest, the ratio estimator is considered,

The conventional ratio estimator and its mean square error are given as;

$$\bar{y}_{r(rss)} = \bar{y}_{rss}\frac{\bar{X}}{\bar{x}_{rss}} \quad (1)$$

$$MSE(\bar{y}_{r(rss)}) = \frac{1}{mr}\left[S_y^2 - 2RS_{xy} + R^2S_x^2\right] - \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - D_{x[i]}\right)^2\right] \quad (2)$$

$$where \quad \bar{x}_{rss} = \sum_{i=1}^{m}\sum_{h=1}^{r} x_{i[h]} \quad and \quad \bar{y}_{rss} = \sum_{i=1}^{m}\sum_{h=1}^{r} y_{i[h]}$$

Many authors have used the idea of RSS to improve the precision of many estimators in SRS which has spanned many years. In the recent time, a class of ratio estimators based on the linear combination of Co-efficient of Skewness and Quartile Deviation under ranked set sampling was proposed see [4].

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This study proposes a new class of exponential ratio type estimator under ranked set sampling, which is useful when the data is skewed. It should be noted that that N is assumed to be infinitely large so that;

$$\frac{(1-f)S^2}{n} = \frac{S^2}{n}$$

SOME EXISTING EXPONENTIAL ESTIMATORS IN SRS

Based on the modified versions of ratio estimators from [5], [6], [7], [8], [9], [10], [11], [12] and [13], aclass of ratio type exponential estimators for population mean $\bar{Y}$ was developed in [14] as;

$$t_{je} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{a(\bar{X} - \bar{x}_{srs})}{a(\bar{X} + \bar{x}_{srs}) + 2b}\right\} \quad (3)$$

where $\hat{\beta}$ **is the sample estimate of the population regression coefficient of y on x, a≠0 and b are real numbers or functions of population parameters.**

The bias and mean square error of the class of estimator can be obtained by letting

$\bar{y} = \bar{Y}(1+\delta_0)$, $\bar{x} = \bar{X}(1+\delta_1)s_x^2 = S_x^2(1+\delta_3)s_{xy} = S_{xy}(1+\delta_2)$so that

$E(\delta_1) = E(\delta_0) = E(\delta_2) = E(\delta_3) = 0$ and

$E(\delta_0^2) = \frac{1}{n}\frac{S_Y^2}{\bar{Y}^2}$, $E(\delta_1^2) = \frac{1}{n}\frac{S_x^2}{\bar{X}^2}$, $E(\delta_0\delta_1) = \frac{1}{n}\frac{S_{xy}}{\bar{X}\bar{Y}}$,

$E(\delta_1\delta_2) = \frac{1}{n}\frac{\mu_{21}}{\bar{X}S_{xy}}$, $E(\delta_1\delta_3) = \frac{1}{n}\frac{\mu_{30}}{\bar{X}S_x^2}$

where $\mu_{rs} = E[(x_i - \bar{X})^r(y_i - \bar{Y})^s]$, r and s are non-negative integers.

Taking $E(t_e - \bar{Y})$ and $E(t_e - \bar{Y})^2$ respectively, the Bias and MSE to the first degree of approximation are;

$$B(t_{je}) = \frac{1}{n}\left[\frac{3}{8}R_j^2\left(\frac{S_x^2}{\bar{Y}}\right) - \frac{N}{(N-2)}\beta\left(\frac{\mu_{21}}{S_{xy}} - \frac{\mu_{30}}{S_x^2}\right)\right] \quad (4)$$

and

$$MSE(t_{je}) = \frac{1}{n}\left[R_j^2\left(\frac{S_x^2}{4}\right) + S_y^2(1-\rho^2)\right] \quad (5)$$

where$R_j = \frac{a\bar{Y}}{(a\bar{X}+b)}$

The bias and MSE of members of the class of estimators were obtained by substituting for a and b appropriately.

The usual estimator when a=1 and b =0 is shown below. Estimators of population mean with known population correlation co-efficient(ρ), population coefficient of skewness(β_1) and population coefficient of kutosis(β_2) are also shown below:

$$t_{0e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\bar{X} - \bar{x}_{srs}}{\bar{X} + \bar{x}_{srs}}\right\} \quad for\ a = 1, b = 0 \quad (6)$$

$$t_{1e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\bar{X} - \bar{x}_{srs}}{(\bar{X} + \bar{x}_{srs}) + 2\beta_1}\right\} \quad for\ a = 1, b = \beta_1 \quad (7)$$

$$t_{2e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\bar{X} - \bar{x}_{srs}}{(\bar{X} + \bar{x}_{srs}) + 2\beta_2}\right\} \quad for\ a = 1, b = \beta_2 \quad (8)$$

$$t_{3e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\bar{X} - \bar{x}_{srs}}{(\bar{X} + \bar{x}_{srs}) + 2\rho}\right\} \quad for\ a = 1, b = \rho \quad (9)$$

$$t_{4e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\beta_2(\bar{X} - \bar{x}_{srs})}{\beta_2(\bar{X} + \bar{x}_{srs}) + 2\beta_1}\right\} \quad for\ a = \beta_2, b = \beta_1 \quad (10)$$

$$t_{5e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\rho(\bar{X} - \bar{x}_{srs})}{\rho(\bar{X} + \bar{x}_{srs}) + 2\beta_1}\right\} \quad for\ a = \rho, b = \beta_1 \quad (11)$$

$$t_{6e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\beta_1(\bar{X} - \bar{x}_{srs})}{\beta_1(\bar{X} + \bar{x}_{srs}) + 2\beta_2}\right\} \quad for\ a = \beta_1, b = \beta_2 \quad (12)$$

$$t_{7e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\rho(\bar{X} - \bar{x}_{srs})}{\rho(\bar{X} + \bar{x}_{srs}) + 2\beta_2}\right\} \quad for\ a = \rho, b = \beta_2 \quad (13)$$

$$t_{8e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\beta_1(\bar{X} - \bar{x}_{srs})}{\beta_1(\bar{X} + \bar{x}_{srs}) + 2\rho}\right\} \quad for\ a = \beta_1, b = \rho \quad (14)$$

$$t_{9e} = \bar{y}_{srs} + \hat{\beta}(\bar{X} + \bar{x}_{srs})exp\left\{\frac{\beta_2(\bar{X} - \bar{x}_{srs})}{\beta_2(\bar{X} + \bar{x}_{srs}) + 2\rho}\right\} \quad for\ a = \beta_2, b = \rho \quad (15)$$

The mean square errors of the estimators were given as;

$$MSE(t_{0e}) = \frac{1}{n}\left[R^2\left(\frac{S_x^2}{4}\right) + S_y^2(1-\rho^2)\right] \quad (16)$$

$$MSE(t_{1e}) = \frac{1}{n}\left[R_1^2\left(\frac{S_x^2}{4}\right) + S_y^2(1-\rho^2)\right] \quad (17)$$

$$MSE(t_{2e}) = \frac{1}{n}\left[R_2^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (18)$$

$$MSE(t_{3e}) = \frac{1}{n}\left[R_3^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (19)$$

$$MSE(t_{4e}) = \frac{1}{n}\left[R_4^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (20)$$

$$MSE(t_{5e}) = \frac{1}{n}\left[R_5^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (21)$$

$$MSE(t_{6e}) = \frac{1}{n}\left[R_6^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (22)$$

$$MSE(t_{7e}) = \frac{1}{n}\left[R_7^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (23)$$

$$MSE(t_{8e}) = \frac{1}{n}\left[R_8^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (24)$$

$$MSE(t_{9e}) = \frac{1}{n}\left[R_9^2\left(\frac{\boldsymbol{S_x^2}}{4}\right) + \boldsymbol{S_y^2}(\boldsymbol{1}-\boldsymbol{\rho^2})\right] \quad (25)$$

where

$R_0 = \frac{\bar{Y}}{\bar{X}}, R_1 = \frac{\bar{Y}}{\bar{X}+\beta_1(x)}, R_2 = \frac{\bar{Y}}{\bar{X}+\beta_2(x)}, R_3 = \frac{\bar{Y}}{\bar{X}+\rho}, R_4 = \frac{\beta_2(x)\bar{Y}}{\beta_2(x)\bar{X}+\beta_1(x)}, R_5 = \frac{\rho\bar{Y}}{\rho\bar{X}+\beta_1(x)} R_6 = \frac{\beta_1(x)\bar{Y}}{\beta_1(x)\bar{X}+\beta_2(x)}$

$, R_7 = \frac{\rho\bar{Y}}{\rho\bar{X}+\beta_2(x)}\ , R_8 = \frac{\beta_1(x)\bar{Y}}{\beta_1(x)\bar{X}+\rho}\ ,\ R_9 = \frac{\beta_2(x)\bar{Y}}{\beta_2(x)\bar{X}+\rho}$

MATERIALS AND METHODS

Motivated by the class of estimators proposed in [14] which shows that the incorporation of more and more parameters on auxiliary variable improves the efficiency of ratio estimators using simple random sampling, this work proposes anew class of exponential ratio type estimators for population mean $\bar{Y}$ **using ranked set sampling as;**

$$\boldsymbol{t_{rje}} = \bar{\boldsymbol{y}}_{\boldsymbol{rss}} + \widehat{\boldsymbol{\beta}}(\bar{\boldsymbol{X}} + \bar{\boldsymbol{x}}_{\boldsymbol{rss}})\boldsymbol{exp}\left\{\frac{\boldsymbol{a}(\bar{\boldsymbol{X}} - \bar{\boldsymbol{x}}_{\boldsymbol{rss}})}{\boldsymbol{a}(\bar{\boldsymbol{X}} + \bar{\boldsymbol{x}}_{\boldsymbol{rss}}) + \boldsymbol{2b}}\right\} \quad (\boldsymbol{26})$$

where $\widehat{\boldsymbol{\beta}}$, **a≠0 and b are same as defined for the class of estimators 't' above. This suggested class of estimators 't_{rje}' can be used in obtaining numerous estimators for conformed values of *a*and *b*.**

Expressing t_{rie} in terms of δ, we have

$$\boldsymbol{t_{rje}} = \bar{Y}\left[(1+\delta_0) + \frac{\boldsymbol{\beta}}{\bar{Y}}(1+\delta_2)(1+\delta_3)^{-1}\bar{\boldsymbol{X}}\delta_1\right]\boldsymbol{exp}\left\{-\frac{\boldsymbol{\alpha}_j\delta_1}{\boldsymbol{2}}\left(\boldsymbol{1}+\frac{\boldsymbol{\alpha}_j\delta_1}{\boldsymbol{2}}\right)^{-1}\right\}$$

Expanding the right-hand side of the equation and neglecting error terms with degree greater than 2, then;

$$(\boldsymbol{t_{rje}} - \bar{Y}) = \bar{Y}\left(\delta_0 - \frac{\boldsymbol{\alpha}_j\delta_1}{2} - \mathbf{K}\delta_1 - \frac{\boldsymbol{\alpha}_j\delta_0\delta_1}{2} - \mathbf{K}(\delta_1\delta_2 - \delta_1\delta_3) + \frac{3}{8}\boldsymbol{\alpha}_j^2\delta_1^2 + 4K\delta_1^2\right)$$

$$(\boldsymbol{t_{rje}} - \bar{Y})^2 = \bar{Y}^2\left(\delta_0^2 + \frac{1}{4}\boldsymbol{\alpha}_j^2\delta_1^2 + K^2\delta_1^2 - \boldsymbol{\alpha}_j\delta_0\delta_1 - 2K\delta_0\delta_1 + \boldsymbol{\alpha}_j\boldsymbol{K}\delta_1^2\right)$$

where, $\boldsymbol{K} = \frac{\boldsymbol{\beta}\bar{X}}{\bar{Y}}$**. Identifying the bias and mean square error of the class of estimators,** let

$\bar{y}_{(rss)} = \bar{Y}(1+\delta_0),\ \bar{x}_{(rss)} = \bar{X}(1+\delta_1) s_{x(rss)}^2 = S_x^2(1+\delta_3) s_{xy(rss)} = S_{xy}(1+\delta_2)$so that

$E(\delta_1) = E(\delta_0) = E(\delta_2) = E(\delta_3) = 0$ and

$$V(\delta_0) = E(\delta_0^2) = \frac{V(\bar{y}_{[rss]})}{\bar{Y}^2} = \frac{1}{mr\bar{Y}^2}\left[S_y^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right] = \frac{1}{mr}\left[\frac{S_y^2}{\bar{Y}^2} - \frac{1}{m}\sum_{i=1}^{m}D_{y[i]}^2\right]$$

Similarly,

$$V(\delta_1) = E(\delta_1^2) = \frac{V(\bar{x}_{[rss]})}{\bar{X}^2} = \frac{1}{mr}\left[\frac{S_x^2}{\bar{X}^2} - \frac{1}{m}\sum_{i=1}^{m}D_{x[i]}^2\right]$$

$$Cov(\delta_0\delta_1) = E(\delta_0\delta_1) = \frac{Cov(\bar{y}_{[rss]}, \bar{x}_{[rss]})}{\bar{Y}\bar{X}} = \frac{1}{mr}\left[\frac{S_x^2}{\bar{Y}\bar{X}} - \frac{1}{m}\sum_{i=1}^{m}D_{y(i)x[i]}\right]$$

$$Cov(\delta_1\delta_2) = E(\delta_1\delta_2) = \frac{Cov(S_{xy_{[rss]}}, \bar{x}_{[rss]})}{S_{xy}\bar{X}} = \frac{1}{mr}\left[\frac{\mu_{21}}{S_{xy}\bar{X}} - \frac{1}{m}\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right]$$

$$Cov(\delta_1\delta_3) = E(\delta_1\delta_3) = \frac{Cov(S_{x[rss]}^2, \bar{x}_{[rss]})}{S_x^2\bar{X}} = \frac{1}{mr}\left[\frac{\mu_{30}}{S_x^2\bar{X}} - \frac{1}{m}\sum_{i=1}^{m}D_{x[i]}^3\right]$$

The Bias and MSE of the estimator t_{r1} to the first degree of approximation are respectively, given by

$$E(\boldsymbol{t_{rje}}-\bar{Y})=\bar{Y}E\left(\delta_0-\frac{\boldsymbol{\alpha}_j\delta_1}{2}-\mathbf{K}\delta_1-\frac{\boldsymbol{\alpha}_j\delta_0\delta_1}{2}-\mathbf{K}(\delta_1\delta_2-\delta_1\delta_3)+\frac{3}{8}\boldsymbol{\alpha}_j^2\delta_1^2+4K\delta_1^2\right)$$

$$B(\boldsymbol{t_{rje}})=\bar{Y}\left\{\frac{3}{8}\boldsymbol{\alpha}_j^2\left(\frac{1}{mr}\left[\frac{S_x^2}{\bar{X}^2}-\frac{1}{m}\sum_{i=1}^{m}D_{x[i]}^2\right]\right)+K\frac{1}{mr}\left[\frac{\mu_{30}}{S_x^2\bar{X}}-\frac{1}{m}\sum_{i=1}^{m}D_{x[i]}^3\right]-K\frac{1}{mr}\left[\frac{\mu_{21}}{S_{xy}\bar{X}}-\frac{1}{m}\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right]\right\}$$

or

$$B(\boldsymbol{t_{rje}})=\frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_j^2}{8}\frac{R^2S_x^2}{\bar{Y}}-\boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}}-\frac{\mu_{30}}{S_x^2\bar{X}}\right)\right]-\frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_j^2}{8}\sum_{i=1}^{m}D_{x[i]}^2+K\left(\sum_{i=1}^{m}D_{x[i]}^3-\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right)\right] \quad (27)$$

Also,

$$E(\boldsymbol{t_{rje}}-\bar{Y})^2=\bar{Y}^2E\left(\delta_0^2+\frac{1}{4}\boldsymbol{\alpha}_j^2\delta_1^2+K^2\delta_1^2-\boldsymbol{\alpha}_j\delta_0\delta_1-2K\delta_0\delta_1+\boldsymbol{\alpha}_j\boldsymbol{K}\delta_1^2\right)$$

$$MSE(\boldsymbol{t_{rje}})=\bar{Y}^2\left\{\frac{1}{mr}\left[\frac{S_y^2}{\bar{Y}^2}-\frac{1}{m}\sum_{i=1}^{m}D_{y[i]}^2\right]+\frac{\boldsymbol{\alpha}_j^2}{4}\left(\frac{1}{mr}\left[\frac{S_x^2}{\bar{X}^2}-\frac{1}{m}\sum_{i=1}^{m}D_{x[i]}^2\right]\right)+K^2\left(\frac{1}{mr}\left[\frac{S_x^2}{\bar{X}^2}-\frac{1}{m}\sum_{i=1}^{m}D_{x[i]}^2\right]\right)-2K\frac{1}{mr}\left[\frac{S_x^2}{\bar{Y}\bar{X}}-\frac{1}{m}\sum_{i=1}^{m}D_{y(i)x[i]}\right]\right\}$$

Or

$$MSE(\boldsymbol{t_{rje}})=\frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_j^2R^2S_x^2}{4}+S_y^2(1-\rho^2)\right]-\frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]}-KD_{x[i]}\right)^2+\frac{\boldsymbol{\alpha}_j^2}{4}\sum_{i=1}^{m}D_{x[i]}^2\right] \quad (28)$$

where n=mr, $D_{y\{i\}}^2=\frac{\tau_{y[i]}^2}{\bar{Y}^2}$, $D_{x\{i\}}^2=\frac{\tau_{x(i)}^2}{\bar{X}^2}$and $D_{x(i)y\{i\}}=\frac{\tau_{x(i)y[i]}}{\bar{X}\bar{Y}}$, it should also be noted that $\tau_{y[i]}=\mu_{y[i]}-\bar{Y}$,$\tau_{x[i]}=\mu_{x(i)}-\bar{\boldsymbol{X}}$ and $\tau_{x(i)y[i]}=(\mu_{x(i)}-\bar{\boldsymbol{X}})(\mu_{y[i]}-\bar{Y})$ where $\mu_{x(i)}=E[x_{(i)}]$, $\mu_{y[i]}=E[y_{[i]}]$,R= $\frac{\bar{Y}}{\bar{X}}$, $\boldsymbol{\alpha}_j=\frac{a\bar{X}}{(a\bar{X}+b)}$

Adapting estimator $\boldsymbol{t_{1e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r1e}}=\bar{\boldsymbol{y}}_{rss}+\hat{\boldsymbol{\beta}}(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})\boldsymbol{exp}\left\{\frac{\bar{\boldsymbol{X}}-\bar{\boldsymbol{x}}_{rss}}{(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})+2\beta_1}\right\} \quad (\mathbf{29})$$

$$B(\boldsymbol{t_{r1e}})=\frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_1^2}{8}\frac{R^2S_x^2}{\bar{Y}}-\boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}}-\frac{\mu_{30}}{S_x^2\bar{X}}\right)\right]-\frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_1^2}{8}\sum_{i=1}^{m}D_{x[i]}^2+K\left(\sum_{i=1}^{m}D_{x[i]}^3-\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{30})$$

$$MSE(\boldsymbol{t_{r1e}})=\frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_1^2R^2S_x^2}{4}+S_y^2(1-\rho^2)\right]-\frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]}-KD_{x[i]}\right)^2+\frac{\boldsymbol{\alpha}_1^2}{4}\sum_{i=1}^{m}D_{x[i]}^2\right] \quad (31)$$

Adapting estimator $\boldsymbol{t_{2e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r2e}}=\bar{\boldsymbol{y}}_{rss}+\hat{\boldsymbol{\beta}}(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})\boldsymbol{exp}\left\{\frac{\bar{\boldsymbol{X}}-\bar{\boldsymbol{x}}_{rss}}{(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})+2\beta_2}\right\} \quad (\mathbf{32})$$

$$B(\boldsymbol{t_{r2e}})=\frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_2^2}{8}\frac{R^2S_x^2}{\bar{Y}}-\boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}}-\frac{\mu_{30}}{S_x^2\bar{X}}\right)\right]-\frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_2^2}{8}\sum_{i=1}^{m}D_{x[i]}^2+K\left(\sum_{i=1}^{m}D_{x[i]}^3-\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{33})$$

$$MSE(\boldsymbol{t_{r2e}})=\frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_2^2R^2S_x^2}{4}+S_y^2(1-\rho^2)\right]-\frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]}-KD_{x[i]}\right)^2+\frac{\boldsymbol{\alpha}_2^2}{4}\sum_{i=1}^{m}D_{x[i]}^2\right] \quad (34)$$

Adapting estimator $\boldsymbol{t_{3e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r3e}}=\bar{\boldsymbol{y}}_{rss}+\hat{\boldsymbol{\beta}}(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})\boldsymbol{exp}\left\{\frac{\bar{\boldsymbol{X}}-\bar{\boldsymbol{x}}_{rss}}{(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})+\mathbf{2}\boldsymbol{\rho}}\right\} \quad (\mathbf{35})$$

$$B(\boldsymbol{t_{r3e}})=\frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_3^2}{8}\frac{R^2S_x^2}{\bar{Y}}-\boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}}-\frac{\mu_{30}}{S_x^2\bar{X}}\right)\right]-\frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_3^2}{8}\sum_{i=1}^{m}D_{x[i]}^2+K\left(\sum_{i=1}^{m}D_{x[i]}^3-\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{36})$$

$$MSE(\boldsymbol{t_{r3e}})=\frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_3^2R^2S_x^2}{4}+S_y^2(1-\rho^2)\right]-\frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]}-KD_{x[i]}\right)^2+\frac{\boldsymbol{\alpha}_3^2}{4}\sum_{i=1}^{m}D_{x[i]}^2\right] \quad (37)$$

Adapting estimator $\boldsymbol{t_{4e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r4e}}=\bar{\boldsymbol{y}}_{rss}+\hat{\boldsymbol{\beta}}(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})\boldsymbol{exp}\left\{\frac{\beta_2(\bar{\boldsymbol{X}}-\bar{\boldsymbol{x}}_{rss})}{\beta_2(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})+2\beta_1}\right\} \quad (\mathbf{38})$$

$$B(\boldsymbol{t_{r4e}})=\frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_4^2}{8}\frac{R^2S_x^2}{\bar{Y}}-\boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}}-\frac{\mu_{30}}{S_x^2\bar{X}}\right)\right]-\frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_4^2}{8}\sum_{i=1}^{m}D_{x[i]}^2+K\left(\sum_{i=1}^{m}D_{x[i]}^3-\sum_{i=1}^{m}D_{x[i]y[i]}^{21}\right)\right] \quad (39)$$

$$MSE(\boldsymbol{t_{r4e}})=\frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_4^2R^2S_x^2}{4}+S_y^2(1-\rho^2)\right]-\frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]}-KD_{x[i]}\right)^2+\frac{\boldsymbol{\alpha}_4^2}{4}\sum_{i=1}^{m}D_{x[i]}^2\right] \quad (40)$$

Adapting estimator $\boldsymbol{t_{5e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r5e}}=\bar{\boldsymbol{y}}_{rss}+\hat{\boldsymbol{\beta}}(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})\boldsymbol{exp}\left\{\frac{\boldsymbol{\rho}(\bar{\boldsymbol{X}}-\bar{\boldsymbol{x}}_{rss})}{\boldsymbol{\rho}(\bar{\boldsymbol{X}}+\bar{\boldsymbol{x}}_{rss})+2\beta_1}\right\} \quad (\mathbf{41})$$

$$B(\boldsymbol{t_{r5e}}) = \frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_5^2}{8}\frac{R^2S_x^2}{\bar{Y}} - \boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}} - \frac{\mu_{30}}{S_x^2\bar{X}}\right)\right] - \frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_5^2}{8}\sum_{i=1}^{m} D_{x[i]}^2 + K\left(\sum_{i=1}^{m} D_{x[i]}^3 - \sum_{i=1}^{m} D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{42})$$

$$MSE(\boldsymbol{t_{r5e}}) = \frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_5^2R^2S_x^2}{4} + S_y^2(1-\rho^2)\right] - \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - KD_{x[i]}\right)^2 + \frac{\boldsymbol{\alpha}_5^2}{4}\sum_{i=1}^{m} D_{x[i]}^2\right] \quad (43)$$

Adapting estimator $\boldsymbol{t_{6e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r6e}} = \boldsymbol{\bar{y}_{rss}} + \boldsymbol{\hat{\beta}}(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}})\boldsymbol{exp}\left\{\frac{\beta_1(\boldsymbol{\bar{X}} - \boldsymbol{\bar{x}_{rss}})}{\beta_1(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}}) + 2\beta_2}\right\} \quad (\mathbf{44})$$

$$B(\boldsymbol{t_{r6e}}) = \frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_6^2}{8}\frac{R^2S_x^2}{\bar{Y}} - \boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}} - \frac{\mu_{30}}{S_x^2\bar{X}}\right)\right] - \frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_6^2}{8}\sum_{i=1}^{m} D_{x[i]}^2 + K\left(\sum_{i=1}^{m} D_{x[i]}^3 - \sum_{i=1}^{m} D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{45})$$

$$MSE(\boldsymbol{t_{r6e}}) = \frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_6^2R^2S_x^2}{4} + S_y^2(1-\rho^2)\right] - \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - KD_{x[i]}\right)^2 + \frac{\boldsymbol{\alpha}_6^2}{4}\sum_{i=1}^{m} D_{x[i]}^2\right] \quad (46)$$

Adapting estimator $\boldsymbol{t_{7e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r7e}} = \boldsymbol{\bar{y}_{rss}} + \boldsymbol{\hat{\beta}}(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}})\boldsymbol{exp}\left\{\frac{\boldsymbol{\rho}(\boldsymbol{\bar{X}} - \boldsymbol{\bar{x}_{rss}})}{\boldsymbol{\rho}(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}}) + 2\beta_2}\right\} \quad (\mathbf{47})$$

$$B(\boldsymbol{t_{r7e}}) = \frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_7^2}{8}\frac{R^2S_x^2}{\bar{Y}} - \boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}} - \frac{\mu_{30}}{S_x^2\bar{X}}\right)\right] - \frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_7^2}{8}\sum_{i=1}^{m} D_{x[i]}^2 + K\left(\sum_{i=1}^{m} D_{x[i]}^3 - \sum_{i=1}^{m} D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{48})$$

$$MSE(\boldsymbol{t_{r7e}}) = \frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_7^2R^2S_x^2}{4} + S_y^2(1-\rho^2)\right] - \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - KD_{x[i]}\right)^2 + \frac{\boldsymbol{\alpha}_7^2}{4}\sum_{i=1}^{m} D_{x[i]}^2\right] \quad (49)$$

Adapting estimator $\boldsymbol{t_{8e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r8e}} = \boldsymbol{\bar{y}_{rss}} + \boldsymbol{\hat{\beta}}(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}})\boldsymbol{exp}\left\{\frac{\beta_1(\boldsymbol{\bar{X}} - \boldsymbol{\bar{x}_{rss}})}{\beta_1(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}}) + \boldsymbol{2\rho}}\right\} \quad (\mathbf{50})$$

$$B(\boldsymbol{t_{r8e}}) = \frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_8^2}{8}\frac{R^2S_x^2}{\bar{Y}} - \boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}} - \frac{\mu_{30}}{S_x^2\bar{X}}\right)\right] - \frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_8^2}{8}\sum_{i=1}^{m} D_{x[i]}^2 + K\left(\sum_{i=1}^{m} D_{x[i]}^3 - \sum_{i=1}^{m} D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{51})$$

$$MSE(\boldsymbol{t_{r8e}}) = \frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_8^2R^2S_x^2}{4} + S_y^2(1-\rho^2)\right] - \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - KD_{x[i]}\right)^2 + \frac{\boldsymbol{\alpha}_8^2}{4}\sum_{i=1}^{m} D_{x[i]}^2\right] \quad (52)$$

Adapting estimator $\boldsymbol{t_{9e}}$ **a new exponential ratio type estimator in ranked set sampling is being proposed as;**

$$\boldsymbol{t_{r9e}} = \boldsymbol{\bar{y}_{rss}} + \boldsymbol{\hat{\beta}}(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}})\boldsymbol{exp}\left\{\frac{\beta_2(\boldsymbol{\bar{X}} - \boldsymbol{\bar{x}_{rss}})}{\beta_2(\boldsymbol{\bar{X}} + \boldsymbol{\bar{x}_{rss}}) + \boldsymbol{2\rho}}\right\} \quad (\mathbf{53})$$

$$B(\boldsymbol{t_{r9e}}) = \frac{1}{mr}\left[\frac{3\boldsymbol{\alpha}_9^2}{8}\frac{R^2S_x^2}{\bar{Y}} - \boldsymbol{\beta}\left(\frac{\mu_{21}}{S_{xy}\bar{X}} - \frac{\mu_{30}}{S_x^2\bar{X}}\right)\right] - \frac{\bar{Y}}{m^2r}\left[\frac{3\boldsymbol{\alpha}_9^2}{8}\sum_{i=1}^{m} D_{x[i]}^2 + K\left(\sum_{i=1}^{m} D_{x[i]}^3 - \sum_{i=1}^{m} D_{x[i]y[i]}^{21}\right)\right] \quad (\mathbf{54})$$

$$MSE(\boldsymbol{t_{r9e}}) = \frac{1}{mr}\left[\frac{\boldsymbol{\alpha}_9^2R^2S_x^2}{4} + S_y^2(1-\rho^2)\right] - \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - KD_{x[i]}\right)^2 + \frac{\boldsymbol{\alpha}_9^2}{4}\sum_{i=1}^{m} D_{x[i]}^2\right] \quad (55)$$

EFFICIENCY COMPARISM

On comparing equation5 with equation28 , we have;

$$MSE(\boldsymbol{t_{je}}) - MSE(\boldsymbol{t_{rje}}) = \boldsymbol{G_j} \geq \boldsymbol{0} \quad \boldsymbol{where}\ \boldsymbol{G_j} = \frac{\bar{Y}^2}{m^2r}\left[\sum_{i=1}^{m}\left(D_{y[i]} - KD_{x[i]}\right)^2 + \frac{\boldsymbol{\alpha}_j^2}{4}\sum_{i=1}^{m} D_{x[i]}^2\right]$$

Therefore, $MSE(\boldsymbol{t_{je}}) \geq MSE(\boldsymbol{t_{rje}})$

This is true for all $\boldsymbol{G_j}$ **for j =0, 1, 2,……….,9**

EMPIRICAL STUDY

For empirical study, a set of secondary data on the enrolment of students in 2015 in secondary schools in Egba zone of Ogun State, Nigeria was used. Y is the number of enrolled students(variable of interest) and X is the number of staff(auxiliary variable). In this study, the sample size is 27, the total population size is 89, population mean for Y is 1581.10 and that of X is 66.44 which give the population ratio as 23.7974.

Exponential Ranked Set Sampling Analysis

The tables below show the population characteristics of the data set and the estimated MSE and the relative efficiency for estimators t_{je} and t_{rje} for j= 0, 1,2,3,4,5,6,7,8,9

Table 1: Data Statistics

Parameters	Statistics	Parameters	Statistics
$\boldsymbol{N}$	89	R	3
$\boldsymbol{n}$	27	$\boldsymbol{S_{yx}}$	40496.18
$\boldsymbol{m}$	9	$\boldsymbol{S_y^2}$	1746369
$\boldsymbol{\bar{X}}$	66.44	$\boldsymbol{S_x^2}$	1742.79
$\boldsymbol{\bar{x}}$	64.85	$\boldsymbol{S_y}$	1321.50
$\boldsymbol{\bar{Y}}$	1581.10	$\boldsymbol{S_x}$	41.75
$\boldsymbol{\bar{y}}$	1706.89	β_1	0.974
$\boldsymbol{C_y}$	0.8358	β_2	3.329
Cx	0.6285	$\boldsymbol{P}$	0.7340

Table 2: The estimates, MSE of the estimators and their relative efficiencies

Estimators	Estimates	MSEsrs	MSErss	MSE diff	R.E
tr0e	1727.724	276587.7	241662	34925.68	1.1445
tr1e	1727.419	269512.8	234759.1	34753.68	1.1480
tr2e	1726.715	253601.6	219234.8	34366.86	1.1568
tr3e	1727.493	271224.3	236429	34795.29	1.1472
tr4e	1727.632	274429.9	239556.7	34873.22	1.1456
tr5e	1727.31	267024.4	232331.2	34693.19	1.1493
tr6e	1726.689	253020.3	218667.5	34352.72	1.1571
tr7e	1726.373	246060.8	211877.3	34183.53	1.1613
tr8e	1727.487	271080.4	236288.6	34791.79	1.1472
tr9e	1727.654	274957.9	240071.9	34886.06	1.1453

From table 2 above, we inferred that introduction of population parameters improved the estimators when compared with the usual exponential estimator $\mathbf{t_{r0e}}$. Estimator $\mathbf{t_{r7e}}$with a population mean estimate of 1726.373 which hasa = ρ and b = β_2 has the lowest MSE for both SRS and RSS sampling methods of 246060.8 and 211877.3 respectively. It was also clear that the **MSE** difference between the existing estimators and proposed estimators is greater than 3400 for all estimators and the relative efficiencies for all estimators are greater than 1. Therefore, the proposed class of estimators is better than existing class of estimators.

CONCLUSION

We have developed new class exponential ratio type estimators in ranked set sampling with bias and mean square errors of the proposed estimators obtained theoretically. In the application of the estimators to the data set, it was seen that the proposed class of estimatorsgave a more efficient result based on the comparison of the MSEs of the proposed estimators and that of the exponential ratio type estimators in simple random sampling proposed by Singh and Yadav (2020). We then submit that the proposed class of estimators are more precise as shown theoretically and empirically.

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