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ON THE SEMI-ANALYTICAL APPROACH TO NONLINEAR FIRST ORDER FRACTIONAL FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract

In this paper, we enlarge a reliable modification of the Adomian decomposition approach offered in literature for solving first order fractional Fredholm Integro-differential equations. In an effort to verify the applicability and the advantages of our method, we do not forget some illustrative examples.

Keywords: Fractional derivative, nonlinear fractional-order Volterra Integro-differential equation, modified decomposition Method, Adomian polynomials, reduced Adomian polynomials, fractional-order Fredholm integro-differential equations.

1. INTRODUCTION

An application of fractional derivatives was first given in 1823 through [1] who applied the fractional calculus in solving an integral equation that arises inside the method of the Tautochrone problem. The fractional integro-differential equations have attracted much greater interest of mathematicians and physicists, which presents an efficiency for the description of many realistic dynamical arising in engineering and scientific disciplines including, physics, electrochemistry, economy, biology, chemistry, electromagnetic and viscoelasticity e.t.c.

The idea of ADM emerged in a pioneering paper by Adomian [2]. Researchers who made significant contributions within the developments of ADM are [3-5], amongst others. The modification of the Adomian approach was first delivered by [4]. In current years, many authors paid attention on the development of numerical, analytical techniques as well as semi-analytical approach for fractional differential and integro-differential equations. As an example, we can keep in mind the following works. In [6], Yang and Hou implemented the Laplace decomposition technique to remedy the fractional integro-differential equations. Also in [7] they implemented some iterative methods for solving fuzzy Volterra-Fredholm integral equations, Ma and Huang in [8] used the hybrid collocation technique to take a look at integro-differential equations of fractional order, Mittal and Nigam [9] applied the Adomian decomposition method to find approximate solutions for fractional integro-differential equations and Zurigat [10] carried out HAM for system of fractional integro-differential equations. Furthermore, the properties of fractional differential equations were studied by some of the above authors.

The principle objective of the existing paper is to have a look at the behavior of the solutions that can be formally obtained from nonlinear first order fractional integro-differential equations of Fredholm type via semi-analytical approximated approach called the improved decomposition method by using a reduced form of the Adomian polynomial in the decomposition of the nonlinear part.

Considering the above equation owing to the basic principles of ADM

$$Lu + Ru + Nu = g \qquad \dots (1)$$

Where in L is an invertible operator that is taken as the highest order differential operator,

R is the rest of the linear operator, N represents the nonlinear terms and g is the specified analytic function. Applying the inverse operator L^{-1} on both sides of equation (1) yields

$$u = \varphi + L^{-1}[g] - L^{-1}[Ru] - L^{-1}[Nu] \qquad \dots (2)$$

where φ is determined by the usage of the given initial values. This approach decomposes the results $u(x)$ right into a hastily convergent series of solution components, after which decomposes the analytic nonlinearity Nu into the series of the Adomian polynomials [5].

$$u(x) = \sum_{n=0}^{\infty} u_n \qquad \dots (3)$$

$$Nu(x) = \sum_{n=0}^{\infty} A_n \qquad \dots (4)$$

Where $A_n = A_n(u_0, u_1, u_2, \dots u_n)$ are the Adomian polynomials, define as follows:

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$$A_n(t) = \frac{1}{k!}\frac{d^k}{d\theta^k}\left[N\left(\sum_{i=0}^{k}\theta^i v_i\right)\right]_{\theta=0} . \qquad \ldots(5)$$

Then the same old Adomian recursion scheme is given in [5] as follows

$u_0(x) = \varphi + L^{-1}[g]$,

$u_{n+1}(x) = -L^{-1}[Ru_n + A_n]$ ... (6)

Rach in [11] proposed a new modification of the Adomian decomposition method for resolution of higher-order inhomogeneous nonlinear differential equations. This new modified decomposition approach gives a huge benefit for computing the Taylor's expansion systematically.

2. PREMINARIES

The mathematical definitions of fractional integrals and fractional derivatives are the problem of several different processes. The maximum frequently used definition of the fractional calculus involves the Riemann-Liouville fractional derivative and the Caputo derivative ([12 13]).

Definition 1

Fraction Calculus involves differentiation and integration of arbitrary order (all real numbers and complex values). e.g. $D^{2.5}, D^{\pi}, D^{i+1}, D^{\frac{1}{2}}$ $, J^{1.5}, J^{\pi}, J^{i+2}, J^{\frac{1}{2}}$ e.t.c

Definition 2

Gamma function is defined as

$$\Gamma(z) = \int_0^\infty t^{z-1}\, e^{-t} dt$$

This integral converges when real part of z is positive $(Re(z) \leq 0)$.

$\Gamma(z+1) = z\Gamma(z)$

When z is a positive integer

$\Gamma(z) = (z-1)!$

Definition 3

Beta function is defined as

$$\mathrm{B}(v,m) = \int_0^1 (1-u)^{v-1}u^{m-1}\, du = \frac{\Gamma(v)\Gamma(m)}{\Gamma(v+m)} = \mathrm{B}(\mathrm{v},\mathrm{m}), \text{where } \mathrm{v},\mathrm{m} \in \mathrm{R}_+$$

Definition 4

Riemann – Liouville fractional integral is defined as

$$J^{\propto}f(x) = \frac{1}{\Gamma(\propto)}\int_0^x \frac{f(x)}{(x-t)^{1-\propto}} dt\,, \quad \propto > 0, x > 0$$

$J^{\propto}$ denotes the fractional integral of order $\propto$

Definition 5

Riemann – Liouville fractional derivative denoted $D^{\propto}$ is defined as

$D^{\propto}J^{\propto}f(x) = f(x)$

Definition 6

Riemann-Liouville fractional derivative defined as

$$D^{\propto}f(x) = \frac{1}{\Gamma(m-\propto)}\int_0^x (x-s)^{m-\propto-1}f^m(s)ds\,.$$

m is positive integer with the property that $m-1 < \propto m$.

Definition 7

The Caputo Factional Derivative is defined as

$$D^{\propto}f(x) = \frac{1}{\Gamma(m-\propto)}\int_0^x (x-s)^{m-\propto-1}f^m(s)ds\,.$$

Where m is a positive integer with the property that $m-1 < \propto < m$.

For example if $0 < \propto < 1$ the caputo fractional derivative is

$$D^{\propto}f(x) = \frac{1}{\Gamma(m-\propto)}\int_0^x (x-s)^{-\propto}f^1(s)ds\,.$$

Hence, we have the following properties:

1. $J^{\propto}J^{v}f = J^{\propto+v}f, \propto, v > 0, f \in C_\mu, \mu > 0$
2. $J^{\propto}x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\propto+\gamma+1)}x^{\propto+\gamma}\,,\ \propto > 0\,, \gamma > -1\,,\ x > 0$
3. $J^{\propto}D^{\propto}f(x) = f(x) - \sum_{k=0}^{m-1} f^k(0)\frac{x^k}{k!}\ ,\ x > 0\,,\ m-1 < \propto \leq m$
4. $D^{\propto}J^{\propto}\, f(x) = f(x),\ x > 0\,,\ m-1 < \propto \leq m$
5. $D^{\propto}C = 0,\ where\ C\ is\ a\ constant.$
6. $D^{\propto}x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\propto+1)}x^{\beta-\propto}\ ,\ \beta \epsilon N_0\ ,\ \ \beta \geq \propto$

Where $\propto$ is an integer and N_0 are natural numbers

7. The n-fold integral formula

$$\int_a^{x_1}\int_a^{x_2}\dots\int_a^{x_n} u(x_n)dx_n\dots dx_2dx_1 = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1}u(t)\,dt$$

Definition 8

In this work, we will define the absolute error as:

Absolute Error= $|u(x)-u_n(x)|: 0\le x\le 1$.

Where $u(x)$ is the exact solution and $u_n(x)$ will be the approximate solution from the method.

3. METHODOLOGY

We consider the following nonlinear fractional first order Fredholm integro-differential equation:

$$D^{\propto}u(x) = g(x) + \lambda\int_a^1 k(x,t)F\big(u(t)\big)\,dt \qquad \dots(7)$$

With initial conditions

$u(0)=C_0$

The main idea of the new method is replacing the forcing terms $g(x)$ which is either an exponential or trigonometric function by a series of infinite components. According to [11], the forcing terms was expressed in series, that is $g(x)=g_1(x)+g_2(x)+g_3(x)+\cdots$ owing to this expression and the introduction of the reduced Adomian polynomials in place of the original Adomian polynomials and from the properties of fractional integral and derivative we know that

$$J^{\propto}D^{\propto}u(x) = u(x) - \sum_{k=0}^{m-1}u^k(0)\frac{x^k}{k!},$$

Where $D^{\propto}$ is the operator that defines fractional derivative and

$\sum_{k=0}^{m-1}u^k(0)\frac{x^k}{k!}=c_0$, this implies

$$J^{\propto}D^{\propto}u(x) = u(x)-c_0$$

This was obtained from the given initial conditions

And also we define the nonlinear terms as:

$$Nu(x)=\sum_{n=0}^{\infty}X_n \qquad \dots(8)$$

$X_n = X_n(u_0,u_1,\dots u_n)$ Are called the reduced Adomian polynomials defined as follows, which we tend to substitute for the A_n .

$$X_n(t) = \begin{cases} N(X_0), n=0 \\ u_nN'[u_0], n\ge 1\end{cases} \qquad \dots(9)$$

Where the first few terms of $X_n(t)$ are:

$X_0 = F(u_0)$

$X_1 = u_1F'(u_0)$,

$X_2 = u_2F'(u_0)$,

$X_3 = u_3F'(u_0)$

…

$X_n = u_nN'[u_0]$ $\qquad \dots(10)$

Now, applying the integral operator $J^{\propto}$ to both sides of Eq. (7) we obtain

$$u(x) = \sum_{k=0}^{r-1}u^k(0)\frac{x^k}{k!} + J^{\propto}[\,g_1(x) + g_2 + g_3(x),\dots] + J^{\propto}\left(\lambda\int_a^1 k(x,t)F\big(u(t)\big)\,dt\right) \qquad \dots(11)$$

The method defines the solution $u(x)$ by the series (3), and the nonlinear function $F\big(u(t)\big)$ is decomposed using the proposed reduced form of Adomian polynomials (9).

$$\sum_{n=0}^{\infty}u_n(x) = \sum_{k=0}^{m-1}u^k(0)\frac{x^k}{k!} + J^{\propto}[g_1(x) + g_2+,\dots] + J^{\propto}\left(\lambda\int_0^1 k(x,t)\sum_{n=0}^{\infty}X_n\,dt\right) \qquad \dots(12)$$

Where $\sum_{k=0}^{m-1}u^k(0)\frac{x^k}{k!}=c_0$

The components $u_0,\ u_1,\ u_2,\dots$ are determined recursively by the following scheme:

$u_0(x)=C_0$

This will be obtained from the given initial condition(s)

And the other terms are obtained from the decomposition of the forcing terms and the reduced polynomials as shown below:

$$u_{n+1}(x) = J^{\propto}(g_1(x) + g_2(x) + g_3(x) + \cdots) + J^{\propto}\left(\lambda \int_a^1 k(x,t) \sum_{n=0}^{\infty} X_n \, dt\right), n \geq 0 \quad \ldots (13)$$

$$u_1(x) = J^{\propto}[\ g_1(x)] + J^{\propto}\lambda \int_a^1 k(x,t) X_0(t)\, dt$$

$$u_2(x) = J^{\propto}[\ g_2(x)]\ + J^{\propto}\lambda \int_a^1 k(x,t) X_1(t)\, dt$$

$$u_3(x) = J^{\propto}[\ g_3(x)] + J^{\propto}\lambda \int_a^1 k(x,t) X_2(t)\, dt$$

$$u_4(x) = J^{\propto}\lambda \int_a^1 k(x,t) X_3(t)\, dt \quad \ldots (14)$$

4 Numerical Examples:

EXAMPLE 1: Consider the following equation

$$D^{\propto}u(x) = 1 - \frac{x}{4} + \int_0^1 xtu^2(t)\, dt, \qquad u(0) = 0\,, \quad 0 < \propto \leq 1 \quad \ldots (15)$$

Applying $J^{\propto}$ to both sides of Eq. (15) gives

$$u(x) = 0 + J^{\propto}\left(1 - \frac{x}{4}\right) + J^{\propto}\left(\int_0^1 xtX_n(t)\, dt\right), n \geq 0$$

For $\propto = 1$

$$u_0(x) = J^{\propto}(1) = x$$

$$u_1(x) = -J^{\propto}\left(\frac{x}{4}\right) + J^{\propto}\left(\int_0^1 xtX_0(t)\, dt\right) = -\frac{x^2}{8} + \frac{x^2}{8} = 0$$

$$u_2(x) = J^{\propto}\left(\int_0^1 xtX_1\, dt\right) = 0$$

$$u_3(x) = 0$$

$$u_4(x) = 0$$

$$u_5(x) = 0$$

…

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x) + u_5(x) + \cdots = x$$

For $\propto = 0.9$

$$u_0(x) = J^{0.9}(1) = 1.039754134 x^{9/10}$$

$$u_1(x) = -J^{0.9}\left(\frac{x}{4}\right) + J^{0.9}\left(\int_0^1 xtX_0(t)\, dt\right) = 0.0188781129 x^{19/10}$$

$$u_2(x) = J^{0.9}\left(\int_0^1 xtX_1\, dt\right) = 0.004475638986 x^{19/10}$$

$$u_3(x) = 0.001061088280 x^{19/10}$$

…

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots = 1.039754134 x^{9/10} + 0.02441484017 x^{19/10}$$

For $\propto = 0.8$

$$u_0(x) = J^{0.8}(1) = 1.073671274 x^{4/5}$$

$$u_1(x) = -J^{0.8}\left(\frac{x}{4}\right) + J^{0.8}\left(\int_0^1 xtX_0(t)\, dt\right) = 0.0418814651 x^{9/5}$$

$$u_2(x) = J^{0.8}\left(\int_0^1 xtX_1\, dt\right) = 0.01166176249 x^{9/5}$$

$$u_3(x) = 0.003247181159 x^{9/5}$$

…

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots = 1.073671274 x^{4/5} + 0.05679040875 x^{9/5}$$

For $\propto = 0.7$

$$u_0(x) = J^{0.7}(1) = 1.100547406 x^{7/10}$$

$$u_1(x) = -J^{0.7}\left(\frac{x}{4}\right) + J^{0.7}\left(\int_0^1 xtX_0(t)\, dt\right) = 0.0687755671 x^{17/10}$$

$$u_2(x) = J^{0.7}\left(\int_0^1 xtX_1\, dt\right) = 0.02227307024 x^{17/10}$$

$$u_3(x) = 0.007213167102 x^{17/10}$$

…

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots = 1.100547406 x^{7/10} + 0.09826180444 x^{17/10}$$

For $\propto = 0.5$

$u_0(x) = J^{0.5}(1) = 1.128379167\sqrt{x}$

$u_1(x) = -J^{0.5}\left(\frac{x}{4}\right) + J^{\propto}\left(\int_0^1 xtX_0(t)\,dt\right) = 0.1312028006\,x^{3/2}$

$u_2(x) = J^{0.5}\left(\int_0^1 xtX_1\,dt\right) = 0.05568419801\,x^{3/2}$

$u_3(x) = 0.02363310764\,x^{3/2}$

…

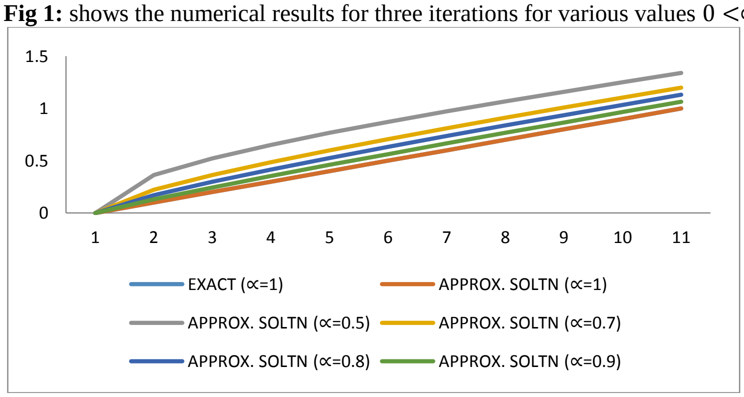
$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots = 1.128379167\sqrt{x} + 0.2105201062\,x^{3/2}$

TABLE 1: The numerical results for three iterations for various values $0 < \propto \leq 1$

X	EXACT ($\propto$=1)	APPROX. SOLTN ($\propto$=1)	APPROX. SOLTN ($\propto$=0.5)	APPROX. SOLTN ($\propto$=0.7)	APPROX. SOLTN ($\propto$=0.8)	APPROX. SOLTN ($\propto$=0.9)
0	0	0	0	0	0	0
0.1	0.1	0.1	0.363482053	0.221548657	0.171065497	0.131204655
0.2	0.2	0.2	0.523455995	0.363091828	0.299409437	0.245410106
0.3	0.3	0.3	0.652630706	0.486489302	0.41629928	0.354314077
0.4	0.4	0.4	0.766907489	0.600192423	0.526759096	0.460092062
0.5	0.5	0.5	0.872314658	0.707710013	0.632970975	0.563732239
0.6	0.6	0.6	0.971879648	0.810920556	0.736140992	0.665798441
0.7	0.7	0.7	1.067363374	0.910973972	0.83702654	0.766653878
0.8	0.8	0.8	1.159888935	1.008636242	0.936143191	0.866551191
0.9	0.9	0.9	1.250219687	1.104448355	1.033862095	0.965675739
1	1	1	1.338899273	1.19880921	1.130461683	1.064168974

These charts reveal that the depicted numerical results are in good agreement with the exact solution as $\propto$ gets close to 1 and our approximate result for $\propto$=1 coincides with the exact solution .

Fig 1: shows the numerical results for three iterations for various values $0 < \propto \leq 1$.



The comparison shows that as $\propto \to 1$, the approximate solution tends to x , which is the exact solution of the equation in the case $\propto$=1.

EXAMPLE 2: Consider the following equation

$$D^{\propto}u(x) = 2e^x - \frac{1}{24}e^x + \frac{1}{24}\int_0^1 e^{x-4t}u^2(t)\,dt, \qquad u(0) = 1\,, 0 < \propto \leq 1 \qquad \ldots(16)$$

Applying $J^{\propto}$ to both sides of Eq. (16) gives

$$u(x) = 1 + J^{\propto}\left(2e^x - \frac{1}{24}e^x\right) + J^{\propto}\left(\frac{1}{24}\int_0^1 e^{x-4t}u^2(t)\,dt\right)$$

Taking $\propto = 1$, we can rewrite the above Eq. as

$$u(x) = 1 + 1.9583\,x + 1.9792\,x^2 + 1.3264\,x^3 + 0.66493\,x^4 + 0.26632\,x^5 + 0.088831\,x^6 + J^{\propto}\left(\tfrac{1}{24}\int_0^1 e^{x-4t}u^2(t)\,dt\right)$$

Using the recursive relation, we take

$u_0(x) = 1$

$u_1(x) = 1.9583x + J^{\propto}\left(\frac{1}{24}\int_0^1 e^{x-4t}({u_0}^2)\,dt\right) = 1.9685\,x + 0.0051129\,x^2 + 0.0017043\,x^3 + 0.00042608\,x^4 + 0.000085216\,x^5$

$u_2(x) = 1.9792x^2 + J^{\propto}\left(\frac{1}{24}\int_0^1 e^{x-4t}(2u_0u_1)\,dt\right) = 0.0093261\,x + 1.9839\,x^2 + 0.0015543\,x^3 + 0.00038859\,x^4 + 0.000077717\,x^5$

$u_3(x) = 1.3264\,x^3 - 0.003982435322 + 0.003982435322\,e^x$

$u_4(x) = 0.66493\,x^4 - 0.001491334267 + 0.001491334267\,e^x$

$u_5(x) = 0.26632\,x^5 - 0.0004908893509 + 0.0004908893509\,e^x$

…

$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x) + u_5(x)\ldots = 1.0000 + 1.9838\,x + 1.9919\,x^2 + 1.3306\,x^3 + 0.66599\,x^4 + 0.26653\,x^5$

For $\propto = 0.9$, we obtain the following results

$$u(x) = 1 + 0.10704\,x^{59/10} + 0.31557\,x^{49/10} + 0.77215\,x^{39/10} + 1.5018\,x^{29/10} + 2.1662\,x^{19/10} + 2.0362\,x^{9/10}$$

$$+J^{0.9}\left(\frac{1}{24}\int_0^1 e^{x-4t}u^2(t)\,dt\right)$$

$$u_0(x) = 1$$

$$u_1(x) = 2.0468\,x^{9/10} + 0.000017115\,x^{59/10} + 0.00010098\,x^{49/10} + 0.00049478\,x^{39/10} + 0.0019296\,x^{29/10} + 0.0055961\,x^{19/10}$$

$$u_2(x) = 2.1721\,x^{19/10} + 0.000018140\,x^{59/10} + 0.00010703\,x^{49/10} + 0.00052444\,x^{39/10} + 0.0020453\,x^{29/10} + 0.011270\,x^{9/10}$$

$$u_3(x) = 1.5026\,x^{29/10} + 0.0000078492\,x^{59/10} + 0.000046311\,x^{49/10} + 0.00022692\,x^{39/10} + 0.0025665\,x^{19/10} + 0.0048763\,x^{9/10}$$

$$u_4(x) = 0.77224\,x^{39/10} + 0.0000029773\,x^{59/10} + 0.000017566\,x^{49/10} + 0.00033569\,x^{29/10} + 0.00097350\,x^{19/10} + 0.0018496\,x^{9/10}$$

$$u_5(x) = 0.31558\,x^{49/10} + 9.9285\,10^{-7}\,x^{59/10} + 0.000028703\,x^{39/10} + 0.00011194\,x^{29/10} + 0.00032463\,x^{19/10} + 0.00061681\,x^{9/10}$$

…

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x) + u_5(x) \ldots =$$

$$1. + 1.5070\,x^{29/10} + 2.1816\,x^{19/10} + 2.0654\,x^{9/10} + 0.000047074\,x^{59/10} + 0.31585\,x^{49/10} + 0.77351\,x^{39/10}$$

For $\propto = 0.5$, we obtain the following results

$$u_0(x) = 1$$

$$u_1(x) = 2.2212\sqrt{x} + 0.000035521\,x^{11/2} + 0.00019536\,x^{9/2} + 0.00087913\,x^{7/2} + 0.0030770\,x^{5/2} + 0.0076925\,x^{3/2}$$

$$u_2(x) = 2.9924\,x^{3/2} + 0.000068046\,x^{11/2} + 0.00037425\,x^{9/2} + 0.0016841\,x^{7/2} + 0.0058945\,x^{5/2} + 0.022104\sqrt{x}$$

$$u_3(x) = 2.3974\,x^{5/2} + 0.000031074\,x^{11/2} + 0.00017090\,x^{9/2} + 0.00076907\,x^{7/2} + 0.0067294\,x^{3/2} + 0.010094\sqrt{x}$$

$$u_4(x) = 1.3723\,x^{7/2} + 0.000012405\,x^{11/2} + 0.000068228\,x^{9/2} + 0.0010746\,x^{5/2} + 0.0026865\,x^{3/2} + 0.0040297\sqrt{x}$$

$$u_5(x) = 0.61058\,x^{9/2} + 0.0000043602\,x^{11/2} + 0.00010791\,x^{7/2} + 0.00037770\,x^{5/2} + 0.00094425\,x^{3/2} + 0.0014164\sqrt{x}$$

…

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x) + u_5(x) \ldots =$$

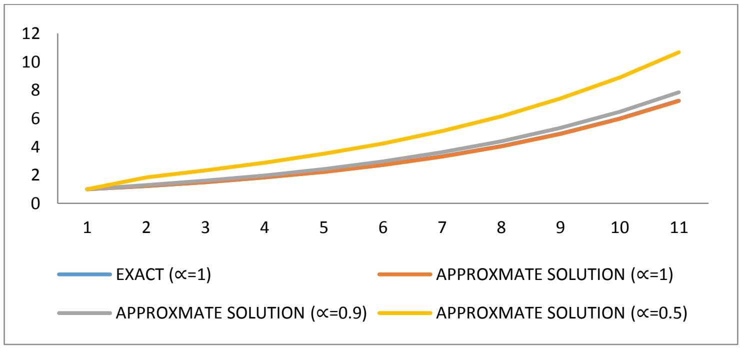
$$1 + 2.2588441\sqrt{x} + 0.0001514062\,x^{11/2} + 0.611388738\,x^{9/2} + 1.37574021\,x^{7/2} + 2.40782380\,x^{5/2} + 3.01045265\,x^{3/2}$$

TABLE 2: the numerical results for three iterations for various values $0 < \propto \leq 1$

X	EXACT (∝=1)	APPROX. SOLTN (∝=1)	APPROX. SOLTN (∝=0.9)	APPROX. SOLTN (∝=0.5)
0	1	1	1	1
0.1	1.221402667	1.219698864	1.289481727	1.817576684
0.2	1.491818667	1.488231674	1.603447198	2.327880818
0.3	1.822048	1.816379387	1.974190174	2.873639268
0.4	2.225130667	2.217161011	2.418937502	3.499449997
0.5	2.716666667	2.706153438	2.955654773	4.235869426
0.6	3.315136	3.301811277	3.604648521	5.111822404
0.7	4.042218667	4.025786696	4.38923011	6.157752537
0.8	4.923114667	4.903249254	5.336154693	7.406841788
0.9	5.986864	5.963205739	6.475982554	8.895630136
1	7.266666667	7.23882	7.843407074	10.6644009

These charts reveal that the depicted numerical results are in good agreement with the exact solution as ∝ gets close to 1 and our approximate results for ∝=1 coincides with the exact solution

Fig 2: shows the numerical results for three iterations for various values 0 <∝≤ 1.



The comparison shows that as ∝→ 1, the approximate solution tends to e^{2x} , which is the exact solution of the equation in the case ∝=1.

5. Conclusion

In this paper, we have developed a reliable amendment of the Adomian decomposition approach provided in [11] for solving first order fractional Fredholm Integro-differential equation. The exceptional advantage of this new method is that, we used a reduced form of the Adomian polynomials which has proved to be very effective. The results obtained from numerical examples display that the existing method can supply a greater correct approximation.

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