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ESTIMATION OF MONTHLY MEAN HOURLY GLOBAL SOLAR RADIATION FROM DAILY GROUND MEASUREMENT DATA USING WHILLIER/ LIU AND JORDAN MODEL FOR KADUNA, NIGERIA.

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Abstract

Estimating global solar radiation incident on any surface of various orientations is essential for designing and analyzing PV systems and to evaluate their long-term average performance. However, the only available global solar radiation dataset are provided by meteorological stations and are often available for a limited number of locations. The global solar radiation dataset can be daily or hourly. In this study, Whillier/ Liu and Jordan (WLJ) model was employed to estimate monthly- mean hourly global solar radiation from the daily ground measured data of global solar radiation of Kaduna obtained from Nigerian Meteorological Agency (NIMET). The performance of the model was assessed with hourly global solar radiation measurements using Mean Absolute Bias Error (MABE), Root Mean Square Error (RMSE), Mean Bias Error (MBE) and t-statistical test. From the obtained results, it can be shown that the WLJ model gives good estimates, and therefore recommended for further study and applicability to different parts of Nigeria.

***Keywords*:** Hourly Global solar radiation, WLJ model, MABE, RMSE, MBE, t-statistical test.

INTRODUCTION

Energy from the sun is environmentally friendly, replenishable and is in great abundance; therefore its effective use for electricity generation in photovoltaic (PV) system is essential [1].

The design and sizing of these PV systems depends on the availability, accessibility, and accuracy of global solar radiation data for any location.

The three distinct techniques through which the global solar radiation can be made available are through ground measurement, satellite measurement and estimation. Unfortunately, global solar radiation data from ground measurement and satellite-derived measurements are not commonly obtainable for every location due to the complexity of the tools and apparatus, technologies, lack of expertise and cost implications [2].

This made researchers to place more emphasis on estimation technique in determining the magnitude of global solar radiation of a location.

To successfully estimate the global solar radiation for a known location using estimation technique, it is necessary to use a theoretical model with certain known input parameters.

The first theoretical model for estimating monthly average daily global solar radiation based on sunshine duration was first proposed in 1924 and is known as Angstrom model [3]. This model was revisited and modified in 1940 to resolve the ambiguity characterizing the definition of the clear sky global solar radiation [4]. The modified relation is known as Angstrom- Prescott model. The Angstrom model again was modified in 1964 to base it on extraterrestrial radiation on a horizontal surface rather than on clear-day radiation [5].

These set of models are known as the temperature- based models and sunshine duration-based models which uses sunshine duration and other input parameters to output daily estimate of global solar radiation of a chosen location. These daily estimation models of global solar radiation have been successfully applied to quantify the magnitude of global solar radiation in different locations in Nigeria [6] – [8].

While daily global solar radiation information is important in many energy applications in renewable energy and research areas, hourly global solar radiation information provides a better, clearer and detailed description of the magnitude of the global solar radiation

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received in a particular day for better accuracy in analyzing, modeling, sizing and designing of PV systems.
For this reason, there is need to provide hourly solar radiation information for every location.
The set of models that compute hourly estimates of global solar radiation are known as the radiation ratio models. Hourly Estimation of global solar radiation H_h using these types of models is not easy or straightforward because one has to first calculate the radiation ratio r, to estimate the hourly solar radiation. These types of models also require measured daily global solar radiation H_d to calculate the hourly global solar radiation as shown in equation1:

$$H_h = rH_d \qquad (1)$$

Where H_d is the measured monthly daily radiation to be collected from any meteorological station and r radiation ratio.
The first radiation ratio model for estimating hourly global solar radiation from daily measured global solar radiation was introduced in 1953 known as the Whillier model [9] – [10]. This radiation ratio model was later modified in 1960 by providing a generalization [11]. This generalization reduces the Whillier radiation ratio model by assuming $\left(\frac{24}{\pi}\right)\sin\left(\frac{\pi}{24}\right)$ is approximately 1 [11]. With this assumption, the Whillier and Liu and Jordan models almost gives same results. As a result, the reduced Whillier model is known as Whillier/ Liu and Jordan model abbreviated as WLJ and the radiation ratio is express as r_{WLJ}. This model has been successfully applied to estimate the magnitude of solar radiation of different location. There are several radiation ratio theoretical models developed in literatures for hourly estimation of global solar radiation and are also applied in different locations by using measured daily global solar radiation of the location as inputs [12] – [19].
Unfortunately, none of these hourly radiation models has been employed for hourly estimate of global solar radiation in Nigeria. The only available global solar radiation information available is the daily global solar radiation.
This study, therefore will compute monthly average hourly estimate of global solar radiation using radiation ratio model of Whillier/ Liu and Jordan with monthly average ground measured daily global solar radiation from meteorological station as input. Also, the relative performances of this model will be assessed by using Mean Bias Error (MBE), Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE), and t- statistical test.

STUDY AREA DESCRIPTION

The city of Kaduna is located in the northern Guinea savannah zone of Nigeria. It lies between latitudes 10^0 N and 11^0 N and longitude 7^0 and 8^0 E at an altitude of 645 m above sea level. The interface between the two, known as the Inter-tropical Convergence Zone, is a front which moves irregularly in March up to October when it retreats. After October, the Sahara system dominates the weather. The rainy season in the Kaduna city region starts around March and ends in October. Annual rainfall averages around 1200 mm. The rainfall pattern is traditionally characterized as
monomodal with peak precipitation between July and August. The mean monthly temperature generally varies between 26^0C and 34^0C with maximum temperatures occurring in February, March and April and minimum temperatures in the ‘‘Harmattan’’ months of November, December and January.

METHODOLOGY

Whillier/ Liu and Jordan (WLJ) model

The original Whillier radiation ratio model is defined as:

$$r_W = \frac{\pi}{24}\frac{\frac{24}{\pi}\sin\left(\frac{\pi}{24}\right)(\cos W_h - \cos W_s)}{\sin W_s - \frac{\pi}{180}W_s\cos W_s} \qquad (2)$$

The assumption of Liu and Jordan reduced the Whillier equation above to;

$$r_{WLJ} = \frac{\pi}{24}\frac{(\cos W_h - \cos W_s)}{\sin W_s - \frac{\pi}{180}W_s\cos W_s} \qquad (3)$$

The sunset hour angle W_s is defined as:

$$W_s = \cos^{-1}(-\tan\varphi\tan\delta) \qquad (4)$$

Where φ is the latitude of the research location in degrees as stated above.
And δ is the sun declination angle given in degrees and defined as:

$$\delta = 23.45\sin\left[\frac{360}{365}(284+n)\right] \qquad (5)$$

Where n is the day of the year; for example, January 20 means that n = 20.
And solar hour angle (or angle of the) W_h is defined as:

$$W_h = \frac{\pi}{12}(t_{sol} - 12) \qquad (6)$$

Where t_{sol} is the solar time at the mid-point of the hourly period of interest. It should be noted here that W_h is zero at solar noon, negative before solar noon and positive after solar noon in the northern hemisphere. The signs are reversed in the southern hemisphere.
Solar time (t_{sol}) can be calculated using:

$$t_{sol} = t_{st} + 4(L_{ST} - L_{LOC}) + E_{OT} \qquad (7)$$

Where t_{st} is the local standard time taken from 6 AM to 5 PM.
The local standard time meridian L_{ST} is the reference meridian used for a particular time zone and is similar to the prime meridian, used for Greenwich Mean Time of the local time zone and is given by;

$$L_{ST} = 15^0 \times Time\ zone\ in\ GMT \qquad (8)$$

L_{LOC} is the longitude of the research location measured to be 7.45^0E.

The equation of time E_{OT} (in minutes) is the difference between true solar time and mean solar time both taken from a known longitude at the same real instant of time given by:

$$E_{OT} = 9.87\sin(2B) - 7.53\cos(B) - 1.5\sin(B) \quad (9)$$

Where B is the day angle (in degrees), given by;

$$B = \frac{360}{365}(n - 81) \quad (10)$$

Where n is the day of the year as explained earlier.

The monthly average hourly global solar radiations was calculated using equation 11 for each day and month of the year as indicated in Table 2.

$$\overline{H_h} = \overline{r_0}\,\overline{H_d} \quad (11)$$

Where $\overline{H_d}$ is the measured monthly average daily radiation collected from any meteorological station and $\overline{r_0}$ is the radiation ratio from WLJ.

RESULTS

EXPERIMENTAL DATA

The measured data of hourly global solar radiation of Kaduna was obtained from Nigeria Meteorological Agency (NIMET) with latitude of 10.60^0 N and longitude of 7.45^0E spanning from 2014 to 2018. The monthly- mean hourly global solar radiation was computed by averaging individual hourly values for each month for five years. The long term monthly-mean daily global solar radiation is obtained as the sum of hourly global solar radiation for that day.

Table 1. Measured Average Hourly Global Solar Radiations in (W/m^2) for Kaduna.

Time → / Month ↓	6 a.m	7 a.m	8 a.m	9 a.m	10 a.m	11 a.m	12 p.m	13 p.m	14 p.m	15 p.m	16 p.m	17 p.m
January	4.2	175	337.22	497.58	592.6	671.61	683.76	625.79	507.75	433	233.6	48.3
February	13.9	176.1	358.075	551.075	699.108	797.475	820.375	756.208	627.43	496.641	314	91.775
March	35.87	175	323.7667	470.983	592.766	663.05	673.733	620.783	511.533	369.733	216.667	82.9333
April	69.14	173.89	288.54	392.94	472.74	515.14	508.54	463.54	379.34	271.14	161.34	63.34
May	89.14	202	308.54	384.94	441.74	480.14	481.54	440.54	389.34	290.14	200.34	69.34
June	76.88	190	312.2	426.6	515.4	562.6	561.4	511.8	421.4	308.4	185.6	41.6
July	44.65	128.4	220.1	302.775	369.775	408.775	411.975	380.275	317.075	235.875	144.175	61.275
August	40.2	115.1	196.4	273.4	333	369.7	375.7	342.1	283.85	205.3	122.74	46.313
September	67.2	190.7	346	458	546	593.8	591	530.3	426.2	391.8	76.2	25.2
October	31.6	88.2	148.41	199.2	246.4	260.1	251.9	223.3	163	113.5	66.6	6.4
November	41.2	129.96	226.46	315.1	380.6	410.93	400.45	352.1	261.8	182.01	101.6	20.6
December	18.6	145	280.8	351.2	460.1	498.3	503.6	463.1	363	247.7	87.6	36.2

I. Computation of Day of the Month.

The estimation of monthly average hourly global radiation on a horizontal surface, for simplicity, was computed on an average day of each month starting from January to December as recommended [20].

Table 2 Average day for each month

Months	n for jth Day of the Month	Average day of the month	
		Date (j)	Day of the year (n)
January	J	17	17
February	31 + j	16	47
March	59 + j	16	75
April	90 + j	15	105
May	120 + j	15	135
June	151 + j	11	162
July	181 + j	17	198
August	212 + j	16	228
September	243 + j	15	258
October	273 + j	15	288
November	304 + j	14	318
December	334 + j	10	344

II. Computation of WLJ Radiation Ratio model and Monthly Average Hourly Global Solar Radiation.

The WLJ radiation ratio model $r_{WLJ}$was computed by means of equation (3). To successfully compute the radiation ratio r_{WLJ} of equation (3), equation (4) is first solved for the sunset hour angle with latitude of the location (10.60^0 N) and declination angle of equation (5) as inputs and solar hour angle of equation (6) with solar time as the input. The declination angle of equation (5) was evaluated using the values of the day of year (n) from Table 2. The sunset hour angle was computed using equation (6) with the solar time (t_{sol}) as the input. Using Equation (7) to (10), the solar time was calculated.

Note: this process is employed to convert local standard times over which hourly global solar radiation data were recorded to local solar times.
Finally, the monthly mean hourly global solar radiations in equation (11) was computed with known values of radiation ratio in equation (3) and monthly mean daily global solar radiation from Table 1.

III. Evaluation and Performance of the Models

In this study, the performance evaluation of the model was obtained based on the statistical analysis test, namely; Mean Absolute Bias Error (MABE), Root Mean Square Error (RMSE), Mean Bias Error (MBE) and t-statistical test as described in equation 12 to 15.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(c_i - m_i)^2} \quad (12)$$

$$MBE = \frac{1}{n}\sum_{i=1}^{n}(c_i - m_i) \quad (13)$$

$$MABE = \frac{1}{n}\sum_{i=1}^{n}(|c_i - m_i|) \quad (14)$$

$$t - statistic = \sqrt{\frac{(n-1)MBE^2}{RMSE^2 - MBE^2}} \quad (15)$$

Where c_i is the i^{th} estimated global solar radiation data, m_i is the i^{th} measured global solar radiation data and n is the number of data. The models are evaluated from January to December and the results are presented in Table 3.
The aim of statistical error analysis is to assess the performance of the studied model. That is, to determine how well measured and estimated monthly average hourly global solar radiation values agree with each other.
Generally, statistical test methods results should be closer to zero for better performance of the models considered in any study Özge and Ümmühan, (2017) [1].

Table 3 statistical error assessments of the model

Statistical Test → Month ↓	MBE	RMSE	MABE	t- Stat.
January	-10.4804	25.07727	20.06989	2.095921
February	0.291168	43.10826	35.50763	0.000547
March	-0.74701	32.25947	27.39426	0.006435
April	-2.46088	30.89844	25.31356	0.076118
May	0.621665	25.48513	21.9102	0.00714
June	2.733454	32.19943	27.41872	0.086479
July	-0.12238	20.65205	17.78966	0.000421
August	2.559653	17.73957	15.24095	0.249837
September	-2.01752	47.73651	35.31453	0.021435
October	-0.55376	12.73214	10.5965	0.0227
November	-5.27438	19.84461	16.74803	0.847695
December	-8.83096	30.90551	25.58202	0.979773

DISCUSSION

Four statistical error test parameters were employed in this study; namely, Mean Bias Error (MBE), Root Mean Square Error (RMSE), Mean Absolute Bias Error (MABE) and t- statistic analysis (t-stat.).
The MBE analysis provides information on the long-term performance of the model. A low MBE results is desired. If at all possible a zero value of MBE should be obtained for better performance. A positive MBE result gives the average amount of over-estimation in the estimated value while negative MBE result gives the average amount of under-estimation in the estimated value. Table 3 showed that the studied model has negative MBE results in January, March, April, July, September, October, November and December while in February, May, June and August the MBE results were positive. This simply explains under-estimation and over-estimation from the WLJ model estimates as earlier discussed.
The RMSE analysis provides information on the short-term performance of the model by allowing a term by term comparison of the actual deviation between the estimated value and the measured value. RMSE results should be positive, but a zero value is ideal.
Table 3 indicates that the RMSE results of the considered model are all positive from January to December with the lowest value in October and the highest value in February.
Also, the MABE statistical analysis was carried out to assess the accuracy of the considered model. Results in Table 3 indicates that the WLJ model have minimum MABE values in October and maximum MABE value in February.
Finally, t-statistical analysis was done to assess the statistical importance of the results of the considered model with least value in July. This indicates agreement between the measured and estimated values. The highest value was recorded in January which indicates the deviation away between the measured and estimated values.

Consequently, results closer to zero in the statistical analysis has the best accuracy while results farther away from zero is least accurate. The WLJ model performance is also illustrated graphically in Figure 1 and Figure 2 for two months (March and September), showing how the measured and estimated monthly average hourly global solar radiation varied with solar time.

In summary, the results from the statistical error analysis of Table 3and Figure 1 and 2 indicate that;

i. The WLJ model performs significantly well to estimate hourly mean global solar radiation of the chosen location.
ii. The WLJ model can be considered for further study and comparison with other radiation ratio models.
iii. Also the WLJ model can be used to create a database for hourly global solar radiation of other locations in Nigeria where daily ground measurement data are available but hourly global solar radiation data are not available.

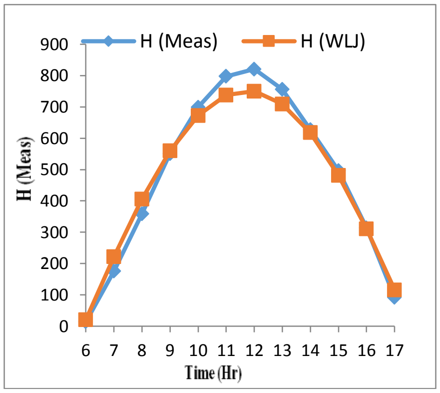

Figure 1: February

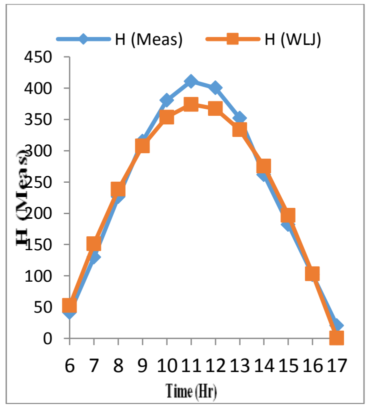

Figure 2: November

REFERENCES

[1] Özge Ayvazoğluyüksel and Ümmühan Başaran Filik. (2017). Estimation of Monthly Average Hourly Global Solar Radiation from the Daily Value in Çanakkale, Turkey. Journal of Clean Energy Technologies: Vol. 5, No. 5.
[2] Boluwaji M. Olomiyesan., and Onyedi, D. Oyedum. (2016). Comparative Study of Ground Measured, Satellite-Derived, and Estimated Global Solar Radiation Data in Nigeria. Journal of Solar Energy Volume 2016, Article ID 8197389.
[3] Angstrom, A. (1924). Solar and terrestrial radiation. Quarterly Journal of the Royal Meteorological Society, vol. 50, no. 210, pp. 121–125, 1924.
[4] Prescott, J. A. (1940). Trans. R. Soc of Australia. 64, 114–125.
[5] Page, J. K. (1964). Proceedings of UN Conference on New Sources of Energy, Rome (Agriculture and Food Organization, AFO United Nations), 4, 378.
[6] Mfon, D. U. and Sunday, O. U. (2010). Estimation of global solar radiation from sunshine hours for Warri, Nigeria. Global Journal of Environmental Sciences. Volume 9.1&2, 51-56.
[7] Gana, N. N and D. O. Akpootu, D. O. (2013). Estimation of global solar radiation using four sunshine based models in Kebbi, North-Western, Nigeria. Adv. Appl. Sci. Res., 4(5):409-421.
[8] Garba, M. B, Muhammad, A, Musa M, and Mohammed, A. G. (2018). Assessing the performance of global solar radiation empirical equations in Sokoto, Nigeria using meteorological parameters. Nigerian Journal of Technology (NIJOTECH) Vol. 37, No. 2, April 2018, pp. 358 – 364.
[9] Whillier A. (1953) Solar energy collection and its utilization for house heating. Sc.D. thesis, Dept. of Mech. Eng., M.I.T., Cambridge (also published by Arno Press, New York, 1979).
[10] Duffie, J. A., and Beckman, W. A. (2006). Solar Engineering of Thermal Processes, JohnWiley & Sons, New York, NY, USA, 3rd edition.
[11] Liu B. Y. H. and Jordan R. C. (1960). The interrelationship and characteristic distribution of direct, diffuse and total solar radiation. *Solar Energy* **4**, 1–12.
[12] Whillier A. (1956). The determination of hourly values of total solar radiation from daily summations. *Arch. Met. Geophys. Biokl*. B8, 197–204.
[13] Collares-Pereira M. and Rabl A. (1979). The average distribution of solar radiation: Correlations between diffuse and hemispherical and between daily and hourly insolation, values. *Solar Energy* **22**, 155–164.
[14] Newell T. A. (1983) Simple models for hourly to daily radiation ratio correlations. *Solar Energy* **31**, 339–342.
[15] Jain P. C. (1984) Comparison of techniques for the estimation of daily global irradiation and a new technique for the estimation of hourly global irradiation. *Solar Wind Technol*. **1**, 123–134.
[16] Gueymard C. (1986). Mean daily averages of beam radiation received by tilted surfaces as affected by the atmosphere. *Solar Energy* **37**, 262–267.

[17] Baig A., Akhter P. and Mufti A. (1991). A novel approach to estimate the clear day global radiation. *Renew*. *Energy* **1**, 119–123.

[18] Gueymard, C. (2000). Prediction and performance assessment of mean hourly global radiation. *Solar Energy* Vol. 68, No. 3, pp. 285–303.

[19] De Souza, K. (2018). Temperature-based model for monthly average hourly global solar radiation for the Caribbean island of Trinidad. Journal of Renewable Sustainable Energy 7, 013132.

[20] Jamil. A. M. and Tiwari, G. N. (2008). Evaluation and comparison of hourly solar radiation models. International Journal of Energy Research, vol. 33, no. 5, pp. 538–552.