Journal of the Nigerian Association of Mathematical Physics
***Volume 56* (March – May *2020 Issue*), pp19 – 24**
© J. of NAMP

STABILITY OF BACKWARD DIFFERENTIATION METHOD FOR THE NUMERICAL SOLUTION OF THE INITIAL VALUE PROBLEM

Aghama Evarista ALEGIEUNO

Mathematics Department, Faculty of Physical Sciences, University of Benin, Benin City, Edo State, Nigeria.

Abstract

This study was carried out to show that the backward differentiation method is stable for step numbers less than or equal to 6(k<= 6). The study also shows that the variable order step methods are self starting. The explicit methods are feasible for variable step size with Adam methods and Backward Differentiation formula. Orders two and one are respectively adopted to generate the starting values. It was found that, it is unstable for k=7, and k=8. This indeed corroborates earlier results.

INTRODUCTION

The general form of the ordinary differential equation can be written as

$A(y) = r(x)$ (1.1)

Where L is a differential operator and r (x) is a given function of the independent variable x. The order of the differential equation is the order of the highest order after the equation has been normalized. If the dependent variable y (x) and it's derivatives occurs in the first degree the equation is said to be linear, otherwise it is non – linear.

A linear differential equation of order can be expressed in the form

$d(y) = \sum_{p=0}^{n} f_p(x)\, y^{(p)}(x) = r(x)$ (1.2)

In which f p (x) are known functions. The general non – linear differential equation of order n can be written as;

$F(x, y, y' \ldots y^{(n-1)} y^{(n)}) = 0$ (1.3)

or

$y^{(n)}(x) = f(x, y, y' \ldots y^{(n-1)}$ (1.4)

The general solution of the nth order ordinary differential equation contains n independent arbitrary constants. In order to determine the arbitrary constants in the general solution, the n conditions are prescribed at one point these are called initial conditions.

The differential equation together with the initial condition is called the INITIAL VALUE PROBLEM. Thus the nth order initial value problem can be expressed as:

$y^{(n)}(x) = f(x, y, y' \ldots y^{(n-1)})$

$y^{(p)}(a) = y^{(p)}, p = 0, 1, \ldots n-1$ (1.5)

If the N conditions are prescribed at more than one point, these are called boundary conditions. The differential equation together with the boundary condition is known as the BOUNDARY – VALUE PROBLEM Perhaps the simplest boundary – value problem is expressed the conditions:

$y^{n} = f(x,y)$: $y(a) = A$, $y(b) = B$ (1.6)

Where b > a and A and B are given constants.

Fourth – order boundary – value problems

$y^{(iv)}(x) + ky(x) = q$ (1.7a)

$y^{(0)} = y^{1}(0) = 0$ (1.7b)

$y^{(1)}\ y^{11}(1) = 0$ (1.7c)

Here y may represent the deflection of a beam of length L which is subjected to a Uniform load q. Condition (1.7b) states that the end x = 0 is built in, while (1.7c) states that the end x = L is simply supported. Heat flow problems fall into this class when the temperatures or temperature gradient are given at two points. A special case of boundary value problem is involved in vibration problems. The theory of boundary value problems for ordinary differential equation relies rather heavily on initial – value problems.

The fact is that some of the most generally applicable numerical methods for solving boundary – value problems employ initial – value problems, hence we briefly review some of the basic result required. However we assume that the reader is familiar with numerical methods for solving initial – value problems

Corresponding Author**:** Aghama EA., Email: evalegieuno@yahoo.com, Tel: +2348054337303

Initial – value problems
A first order differential equations
$y^1 = f(x,y)$ (1.8)
May posses an infinite number of solutions for instance
$y(x) = (e^{\lambda x})$ (1.9)
For any value of constant (λ), is a solution of the differential equation
$y^1 = \lambda y$ (1.10)
Where λ is a given constant. Any particular solution is obtained by prescribing an initial condition y (a) = A
Hence the particular solution satisfying this initial condition is easily found to be
$y(x) = Ae^{\lambda(x-a)}$ (1.11)
The differential equation together with an initial value problem
$y^1 = f(x,y),\ y(a) = A$ (1.12)
In many applications, we are confronted with a single differential equation, but with a system of M simultaneous first order equation in M dependent variables.
$y^1, y^2 \ldots y^m$
If each of these variables satisfies a given condition at the same value x = a then we have an initial value problem for a first order system which we may write
$y_1^1 = f_1(x,y_1,y_2, \ldots, y_m) \quad y_1(a) = A_1$
$y_2^1 = f_2(x,y_1,y_2, \ldots, y_m)$
$y_2(a) = A_2$ (1.13)
$y^1_m = f_m(x,y_1,y_2, \ldots, y_m) \quad y_m(a) = A_m$
This IVP can be compactly expressed in a vector form
$y^1 = f(x,y) \quad y(a) = A$ (1.14)
Where y is a common vector and
$A = (A_1, A_2, \ldots A_m)^T$

Theoretical foundation
The existence and uniqueness properties of the solutions of initial – point (a, A). The following theorem states conditions on f (x,y) which guarantee the existence of a unique solution of the initial value problem.

Theorem 1.3
Let f (x,y) be defined and continuous for all point (x,y) in the region D defined by a < x < b, - , a and b finite, and, let there exist a constant L such that for every x, y, y such that (x, y) and (x,y) are both in D
If (x,y) - f (x,y*) 1≤ Ldy – y*1 (1.15)
The if a is any given number, there exists a unique solution y (x) of the IVP (1.12) where y (x\) is continuous and differentiable for all (x, y) in D.
The requirement (1.15) is called a lipschitz condition of order one and the constant 1 as a lipschitz constant.
Constitutes a discrete point set defined on the integration interval [a,b where h ≥ 0 is a suitable mesh size, possibly allowed to vary in accordance with certain desired criteria, except h is assumed to be fixed, provided the points
[(xn =j, yn=j) 1 j=0(1)k-1]
below to the solution space of IVP, the K – step linear multiple method (LMM) extrapolates the next point in the solution space with the following k^{-th} order finite difference equation
&(E)yn = h σ(E) fn,
fn = f(xn + j) (1.16)
Where E is the shift operator defined as
Eyn = yn + j
and σ(E) and σ(E) are respectively, the first and second characteristic polynomials of the LMM. These polynomials are defined respectively as:
&(E) $= \sum_{j=0}^{k} \alpha j\, E^j,\ \alpha_k \neq 0$ (1.17)
$\sigma(E) = \sum_{j=0}^{k} \beta j\, E^j$ (1.18)
Usually with the added assumption that
$\sigma_0^2 + \beta_0^2 > 0$
Degree (σ) < degree (S)
Implies that the method is explicit and the unknown quantity yn + K in (1.16) can be obtained directly from the solution of the difference equation. Otherwise if (1.16) is implicit, iteration processes have to be adopted. This introduce the problem of generating a good initial estimate and of ascertaining conditions which ensure the convergence of the iterative process.
A desired solution is generated from (1.16) provided the past values
$[y_n, y_{n+1}, y_{n+k-1}]$
are available. Unfortunately, at the initial stage, only the starting value y_o is known. Hence, we cannot apply the LMM (1.16) except when necessary additional starting values.

$Y_1\ y_2\ y_{k-1}$
are available. These values can be generated with a Runge – Kutta method whose order of accuracy is at least as high as that of the LMM under consideration. Explicit R - schemes are recommended for the non stiff phase. While the implicit R – K schemes might be adopted in the stiff phase. The variable order step methods are self starting. The explicit methods are after feasible for variable step size with Adams methods and Backward differentiation formula.
The (K – 1) additional starting values are generated by the single step (K – 1) stage R- K process of the form.
$Yj = Yo + n\sum_{r=1}^{k-1} Y_{jr} Y_r^1$ j = 1(r) *K – 1*
the slopes $Y_r = hf\ (x_o + h\alpha_r^1 Yo + h\sum_{s=1}^{k-1}\beta_{reYs}$ = r = 1(1) *K - 1*
with the constant
$\alpha_x = \sum_{s=1}^{k} - 1\ \beta_{re}$
[6] proposed a fourth order six – stage explicit R – K process for non- stiff IVPS, while he proposed (K – 1)th order (K-1) stage R-K process to generate the K-1 points simultaneously.

METHODS BASED ON NUMERICAL INTEGRATION
Explicit Linear multistep methods
Adams – Bashforth, and related methods
The linear multistep method is essentially a polynomial interpolation procedure whereby the solution or it's derivative is replaced by a polynomial of appropriate degree in the independent variable x, whose derivative or integral is readily obtained. This accounts for the poor performance of the LMM when dealing with IVPS whose solutions contains singularities, or those with large lipschitz constant as polynomials approximations are in appropriate for these classes of problems.
Table 1.: Co – efficient of ABF [1] [3]

K	K	0	1	2	3	4
Y_k	Y_k	1	½	5/12	3/8	251/720
K	5	6	7	8	9	10
Y_k	95/288	19087/60480	5257/17280	1070017/3628800	25713/89600	26842258/95800320

The influence function approach can be applied to bound the right hand side of (1.19).
An alternative expression for the integration formular (1.19) can be obtained by inserting the identity.

♦$^r f_{y=0} = {}^r\sum(-1)^j{}_j{}^r\ f_{n-j}$ (1.20)
In (1.19). This yields
$Y_{n+1} - y_n{}^k\ h\ {}_{j=0}\sum(T)_{kj} f_{n-j}$ (1.21)
With
$Yf_{kj} - (-1)^{j-r}\sum(j-1)y_i$ (1.22)
The Adams – Bashforth [1][3] formula are really captivating because being explicit they involve a straight forward computation of y_{n+1}. However, it will be shown later that such procedures are handicapped by lower attainable order compared with the corresponding implicit formulars. Besides, their stability is quite, inferior to that of the corresponding implicit process. In practical applications the explicit Adams are used to generate the necessary additional starting values.
The multistep formulas, known as backward differentiation formulas or the B.D.F. – methods] are widely used for the integration of stiff differential equations. They were introduced by [5] [8] call them standard step by step methods
For sake of completeness we give these formulas also in the equation in which the backward differences are expressed in terms of the followings;
When K = 1
$$\sum_{j=1}^{k}\frac{1}{j}\,\Box^j\, y_{n+1} = hf_{n+1} \qquad where\ 1\ fn+1 = 0$$
$1\ \Box^j\ y_{n+1} = hf_{n+1}$
Recall
$y_{n+1} = hf_{n+1} - y_n$
$y_{n+1} - y_n = hf_{n+1}$
$= y_{n+1} - y_n = 0$
When K = 2
$$\sum_{j=1}^{2}\frac{1}{j}\,\Box^j\, y_{n+1} = hf_{n+1}$$
$y_{n+1} - y_n + {}^1/_2\ \Box\ 2y_{n+1} = hf_{n+1}$
Recall
$y_{n+1} = y_{n+1} = y_n$
$\Box^2\ y_{n+1} = (y_{n+1} - y_n)$
$= y_{n+1} - y_n -= (y_n - y_{n-1})$

$= y_{n+1} - 2y_n + y_{n-1}$
$y_{n+1} - y_n \, ^1/_2 \, y_{n+1} - {}^1/_{2.} 2y_n + {}^1/_2 \, y_{n-1}$
$y_{n+1} - {}^1/_2 \, y_{n+1} - 2y_n + {}^1/_2 \, y_{n-1} = hf_{n+1}$
${}^3/_2 \, y_{n+1} - 2y_n + {}^1/_2 \, y_{n-1} = hf_{n+1}$
${}^1/_2 \, y_{n+1} - 2y_n + {}^1/_2 \, y_{n-1} = 0$
When K = 3

$$\sum_{j=1}^{3} \frac{1}{j} \quad \nabla^j y_{n+1} = hf_{n+1}$$

${}^3/_2 \, y_{n+1} - 2y_n + {}^1/_2 \, y_{n-1} + {}^1/_3 \, \nabla^3 y_{n+1} = hf_{n+1}$
$\nabla^3 y_{n+1} = \nabla(y_{n+1} - y_n)$
$= {}^3/_2 \, y_{n+1} - 2y_n + {}^1/_2 \, y_{n-1} + {}^1/_3 \, y_{n+1} - y_{n-1} - {}^1/_3 \, y_{n-2}$
$= {}^{11}/_6 \, y_{n+1} - 3y_n + {}^3/_2 \, y_{n-1} - {}^1/_3 \, y_{n-2} = 0$
When K = 4

$$\sum_{j=1}^{4} \frac{1}{j} \quad \nabla^j y_{n+1} = hf_{n+1}$$

${}^{11}/_6 \, y_{n+1} - 3y_n + {}^3/_2 \, y_{n-1} - {}^1/_3 \, y_{n-2} + {}^1/_4 \, \nabla^4 y_{n+1} = hf_{n+1}$
$\nabla^4 y_{n+1} \, \nabla(y_{n+1} - 3y_n + 3y_{n-1} - y_{n-2}$
$= y_{n+1} - yn - 3yn + 3y_{n-1} + 3y_{n-1} - 3y_{n-2} - y_{n-2} + y_{n-3}$
$= y_{n+1} - 4yn + 6y_{n-1} - 4y_{n-2} + 3y_{n-3}$
${}^{11}/_6 \, y_{n+1} - 3yn + {}^3/_2 \, y_{n-1} - {}^1/_3 \, y_{n-2} + {}^1/_4 \, y_{n+1} - y_n + {}^6/_4 \, y_{n-1} - y_{n-2} + {}^1/_4 \, y_{n-3}$
${}^{25}/_{12} \, y_{n+1} - 4y_n 3y_{n-1} - {}^4/_3 \, y_{n-2} + {}^1/_4 \quad y_{n-3} = hf_{n+1}$
${}^{25}/_{12} \, y_{n+1} - 4y_n + 3y_{n-1} - {}^4/_3 \, y_{n-2} + {}^1/_4 \quad y_{n-3} = 0$
When K=5

$$\sum_{j=1}^{5} \frac{1}{j} \quad \nabla^j y_{n+1} = hf_{n+1}$$

${}^{25}/_{12} \, y_{n+1} - 4y_n + 3y_{n-1} - {}^4/_3 \, y_{n-2} + {}^4/_3 \, \nabla \quad y_{n-2} = \frac{1}{4} yn - 3$
$+ \frac{1}{5} \nabla 5 \; yn + 1 = hfn + 1$
$\nabla^5 y \; n + 1 = \nabla(y_{n+1} - 4y_n + 6yy_{n-1} - 4yy_{n-2} + y_{n-1}$
$= y_{n+1} - yn - 4yn + 4y_{n-1} + 6y_{n-1} - 6y_{n-2} - 4y_{n-3} + y_{n-3} - y_{n-4}$
$= y_{n+1} - 5yn + 10y_{n-1} - 10y_{n-2} + 5y_{n-3} - y_{n-4}$
$= {}^{25}/12y_{n+1} - 4y_n + 3y_{n-1} - 4/3y_{n-2} + 1/4y_{n-3} + 1/5y_{n+1} - 1/5.5yn + 1/5\ 10y_{n-1} - 1/5.10y_{n-2} + 1/5.5y_{n-3} - 1/5y_{n-4}$
$= {}^{25}/12y_{n+1} - 4yn + 3y_{n-1} - 4/3y_{n-2} + 1/4y_{n-3} + 1/5y_{n+1} - yn + 2yn - 2y_{n-2} + y_{n-3} - 1/5y_{n-4}$
$= {}^{137}/60y_{n+1} - yn + 5y_{n-1} - {}^{10}/3y_{n-2} + {}^5/4y_{n-3} - 1/5y_{n-4} = hfn + 1$
${}^{137}/60y_{n+1} - 5yn + 5y_{n-1} - 10/3y_{n-2} + 5/4y_{n-3} - 1/5y_{n-4} = 0$

<u>When k = 6</u>

$$\sum_{j=1}^{6} \frac{1}{j} \qquad \nabla^j y_{n+1} = hf_{n+1}$$

$137/60y_{n+1} - 5yn + 5y_{n-1} - /3y_{n-2} + 5/4y_{n-3} - 1/5y_{n-4} = hf_{n+1}$
$\nabla^6 y_{n+1} - \nabla(y_{n+1} - 5yn + 10y_{n-2} + 5y_{n-3} - y_{n-4})$
$= y_{n+1} - yn - 5y_{n-1} + 10y_{n-1} - 10y_{n-2} - 10y_{n-2} + 10y_{n-3} + 5y_{n-3} - 5y_{n-4} - y_{n-4} + y_{n-5})$
$= y_{n+1} - 6yn + 15y_{n-1} - 20y_{n-3} + 15y_{n-3} - 6y_{n-4} \, y_{n-5}$
$137/60y_{n+1} - 5yn + 5y_{n-1} - {}^{10}/3y_{n-2} + 5/4y_{n-3} - 1/5y_{n-4}$
$+ 1/6y_{n+1} - 1/6.6yn + 1/6.15y_{n-1} - 1/6.20y_{n-2}$
$+ 1/6. \, {}^{15}y_{n-3} - 1/6.^{6yn-4} + 1/6yn\text{-}5.$
$147/60y_{n+1} - 6yn + {}^{15}/2y_{n-1} - {}^{20}/6y_{n-2} + {}^{15}/4yn_{n-3} - 6/5y_{n-4} + 1/6y_{n-5} = hf_{n+1}$
$147/60y_{n+1} - 6yn + {}^{15}/2y_{n-1} - {}^{20}/6y_{n-2} + {}^{15}4y_n - 6/5y_{n-4} + 1/6y_{n-5} = 0$

When k = 7

$$\sum_{j=1}^{7} \frac{1}{j} \quad \nabla^{j} y_{n+1} = hf_{n+1}$$

$47/60y_{n+1} – 6yn + 15/2y_{n-1} – 20/6y_{n-2} + 15/4y_{n-3} – 6/5y_{n-4} + 1/6y_{n-5} = hfn + 1$
$\nabla 7y_{n+1} = \nabla(y_{n+1} – 6yn + 15y_{n-1}- 20y_{n-2} + 15y_{n-3}\, 6y_{n-4} + y_{n-5}$
$= y_{n+1}\; yn – 6yn + 6y_{n-1} + 15y_{n-1} – 15y_{n-2} – 20y_{n-2} + 20y_{n-3} + 15y_{n-3} -15y_{n-4} – 6y_{n-4} + 6y_{n-5} + y_{n-5} – y_{n-6}.$
$= y_{n+1} – 7yn + 21y_{n-1} – 35y_{n-2} + 35y_{n-3} – 21y_{n-4} + 7y_{n-5} – y_{n-6}$
$= {}^{147}/60y_{n+1} – 6yn + 15/2y_{n-1}\, 20/6y_{n-2} + 15/4y_{n-3} – 6/5y_{n-4} + 1/6y_{n-5} +$
$1/7.7yn + 1/7.21y_{n-1} – 1/7.^{35}y_{n-2} + 1/7y.^{21}y_{n-4} + 1/7.^{7}y_{n-6}$
$147/60y_{n+1} – 6yn + 15/2y_{n-1} – 20/6y_{n-2} + 15/4y_{n-3} – 6/5y_{n-4} + 1/6y_{n-5} + 1/7y_{n+1} – yn +$
$3y_{n-1} – 5y_{n-2} + 5y_{n-3} – 3y_{n-4} + y_{n-5} – 1/7yn-6$
$108/420y_{n+1} – 7yn + 21/2y_{n-1} – 25/3y_{n-2} + 35/4y_{n-3} – 21/5y_{n-4} + 7/6y_{n-5} – 1/7y_{n-6} = 0$

When K = 8

$$\sum_{j=1}^{8} \frac{1}{j} \quad \nabla^{j} y_{n+1} = hf_{n+1}$$

$1089/420y_{n+1}\; 7yn + 21/2y_{n-1} – y_{n-1}\; 25/3y_{n-2} + 35/4y_{n-4} + 21/y_{n-5} + 7/6y_{n-5} – 1/7y_{n-5} –$
$1/7y_{n-6} = hf_{n+1}$
$\nabla^{8}y_{n+1} = \nabla(y_{n+r} – 7yn + 21y_{n-1} – 35y_{n-2} + 35y_{n-3} + 35y_{n-3} – 21y_{n-4} + 7y_{n-5} – y_{n-6}$
$= y_{n+1} – yn – 7yn + 7y_{n-1} + 21y_{n-1}\, 21y_{n-2} + 35y_{n-2} + 35y_{n-3} + 35y_{n-3} – 35y_{n-4} –$
$21y_{n-5} + 7y_{n-5} – 7y_{n-6} – y_{n-6} + y_{n-7}$
$= y_{n+1} – 8yn + 28y_{n-1} – 56y_{n-2} + 70y_{n-3} – 56y_{n-4} + 28y_{n-5} – 8y_{n-6} + y_{n-7}$
$= 1089/420y_{n+1} – 7yn + 21/2y_{n-1} – 25/3y_{n-2} + 35/4y_{n-3} – 21/5y_{n-4} + 7/6y_{n-5} – 1/7y_{n-6} +$
$1/8y_{n+1} – 1/8.^{8}yn + 1/8.28y_{n-1} – 1/8.^{56}y_{n-2} – 1/8.^{79}y_{n-3} – 1/8.^{28}y_{n-5} – 1/8.^{8}y_{n-6} + 1/8yn – 7$
$1089/420y_{n-1} – 7yn + 21/2y_{n-1} – 25/3y_{n-2} + 34/4y_{n-3}\; 21/5y_{n-4} + 7/6y_{n-5} – 1/7y_{n-6} +$
$1/8y_{n+1} – yn + 7/2y_{n-1} – 7y_{n-2} + 35/4y_{n-4} + 7/2y_{n-5} – y_{n-6} + 1/8y_{n-7} = hfn + 1$
$= {}^{761}/280y_{n+1} – 8yn + 14y_{n-2}\; 46/3y_{n-2} + 35/2y_{n-3} + 56/5y_{n-4} + 14/3y_{n-5} – 8/7y_{n-6} + 1/8y_{n-7}$
$= 0$

When k = 9

$$\sum_{j=1}^{9} \frac{1}{j} \quad \nabla^{j} y_{n+1} = hfn+1$$

$761/280\; y_{n+1} – 8y_{n} + 14y_{n-1} – 46/3y_{n-2} + 35/2y_{n-3} – 56/5y_{n-4} + 14/3y_{n-5} - 8/7y_{n-6} +$
$1/8y_{n-7}$
$\nabla 9y_{n+1} = \nabla\; (y_{n+1} – 8yn + 28y_{n-1-46y}n-2 + 60y_{n-3} + 56y_{n-4}\; 28y_{n-4} - 8y_{n-6+y}n-7$
$y_{n+1} – yn – 8yn + 8y_{n-1} + 28y_{n-1}\; 28y_{n-2} - 46y_{n-2} + 46y_{n-3} + 60y_{n-3} - 60y_{n-4} - 56y_{n-4} +$
$56y_{n-5} + 28y_{n-5} – 28y_{n-6} – 8y_{n-6} + 8y_{n-7} – y_{n-8}$
$= y_{n+1} – 9yn + 36y_{n-1}\; 74y_{n-2} + 106y_{n-3} – 166y_{n-4} + 84y_{n-5} - 36y_{n-6} + 9y_{n-7} - y_{n-8}$
$= {}^{761}/280y_{n+1} – 8yn + 14y_{n-1} – 46/3y_{n-2} + 35/2y_{n-3} – 56/5y_{n-4} + 14/3y_{n-5} – 8/7y_{n-6} +$
$1/8y_{n-7} – 1/9y_{n} + 1/9.^{36}y_{n-1} – 1/9.^{36}y_{n-6} + 1/9.^{9}y_{n-7} – 1/9.^{y}n-8$
$= {}^{761}/280y_{n+1} – 8yn + 14y_{n-1} – 46/3y_{n-2} + 35/2y_{n-3} – 56/5y_{n-4} + 14/3y_{n-5} – 8/7y_{n-6} +$
$1/8y_{n-7} + 1/9y_{n+1} – yn = 4y_{n-1} – 74/9y_{n-2} + {}^{106}/9y_{n-3} – {}^{116}/9y_{n-4} + 84/9y_{n-5} – 4y_{n-6} +$
$y_{n-7} – 1/9y_{n-8}$
$= \frac{7129}{2520}y_{n+1} – 9yn + 81y_{n-1} – \frac{212}{9}y_{n-2} + \frac{527}{18}y_{n-3}\frac{1084}{45}y_{n-4} + 14y_{n-5} – \frac{36}{7}y_{n-6} + 9/8y_{n-7}$
$– 1/9y_{n-8} = 0$

When K = 10

$$\sum_{j=1}^{10} \frac{1}{j} \quad \nabla^{j} y_{n+1} = hf_{n+1}$$

$\nabla^{10} = \nabla(y_{n+1} – 9yn + 36y_{n-1} – 74y_{n-2} + 106y_{n-3} – 116y_{n-4} + 84y_{n-6} – 36y_{n-6} + 9y_{n-7} – y_{n-8})$
$Y_{n+1} – yn\; 9yn + 9y_{n-1} + 36y_{n-1} – 36y_{n-2}\; 74y_{n-2} + 74y_{n-3} + 106y_{n-3} – 106y_{n-4} – 116y_{n-4} – 116y_{n-5} +$
$84y_{n-5} – 84y_{n-6} – 36y_{n-6} + 36y_{n-7} + 9y_{n-7} – 9y_{n-8} – y_{n-8} + y_{n-9}$
$= y_{n+1} – 10yn + 45y_{n-1} – 110y_{n-2} + 180y_{n-3} – 222y_{n-4} + 200y_{n-5} – 110y_{n-6} – 45y_{n-7} – 10y_{n-8} + y_{n-9}$
$= {}^{7129}/2520y_{n+1} – 9yn + 18y_{n-1} – {}^{212}/9y_{n-2} + {}^{527}/18y_{n-3} – {}^{1084}/45y_{n-4} + 14y_{n-5} – 36/7y_{n-6} + 9/8yn-7$
$- 1/9y_{n-8} + 1/10y_{n+1} – 1/10.^{10}yn + 1/10.^{45}yn-1-1/10.^{110}yn-2 + 1/10.^{180}yn-3 – 1/10.^{222}yn-4$

+ 1/10.^{200}yn-5-1/10.^{110}yn-6 + 1/10.^{45}yn-7-1/10.^{10}yn-8+1/10.^{y}n-9

$\frac{7129}{2520}y_{n+1} - 9yn + 18y_{n-1} - \frac{212}{9}y_{n-2} + \frac{527}{18}y_{n-3} - \frac{1084}{45}y_{n-4} - 14y_{n-5} - \frac{36}{7}y_{n-6} + 9/8y_{n-7} - 1/9y_{n-8} +$

$1/10y_{n+1} - yn + 9/2y_{n-1} - y_{n-8} + 1/10y_{n-9} = hf_{n+1}$

$= \frac{73810}{25200}y_{n+1} - 10ynn + \frac{45}{2}y_{n-1} - \frac{311}{9}y_{n-2} + \frac{851}{18}y_{n-3} - \frac{2083}{45}y_{n-4} + 34y_{n-5} - \frac{113}{7}y_{n-6} + \frac{45}{8}y_{n-7} - \frac{10}{9}y_{n-8}$

$+1/10y_{n-9} = 0$, h=0

Conclusion

The explicit Adams method, Implicit Adams method, explicit Nystrom methods, recurrent relations for [I] and[2] the Milline simpson method are classical linear multistep method based on Numerical integration.

The implicit formula

J=0 $Y_{n+1} = hf_{n+1}$

With coefficients

$\delta_j = (-1)^j \left. \frac{d}{ds} \begin{bmatrix} -s+1 \\ j \end{bmatrix} \right|_s = 1$

Using the definition of th inomial co-efficient

(-1) $[^{-s+1}]$= 1/j (s-1) s(s+1) …………… (s+j-2)

The co-efficient of S_j are obtained by direct differentiation

δ_0= 1/j (s-1) s(s+1) …………… (s+j-2)

The formula

$\sum_{j=0}^{k} \delta_j \blacktriangledown Y_{n+1} = hf_{n+1}$

Therefore becomes

$\sum_{j=0}^{k} \frac{1}{J}^j \blacktriangledown Y_{n+1} = jf_{n+1}$

The multistep formulas, known as backward differentiation formulas or the B.D.F methods which are widely used for the integration of stiff differential equations, were introduced by[5] and[8] which they call standard step by step methods.

In determining distribution of the roots of a polynomial, we used [7] theorem as found in [3]

Finally, is was shown that the Backward Differentiation method is stable for step number less than or equal to 6 ($K \leq 6$), out unstable for K=7 and K=8. This results corroborate earlier results of [1][7] and [9].

REFERENCES

[1] C. W. Cryer (1972): On the instability of high order backward difference multistep methods. BIT, VOL. 12 Pages 17-25.

[2] E. Hairer and G. Wanner * (1983): On the instability of the B. D. F. Formulas SIAMJ, numer Anal., VOL. 20, No. Pages 1206-1209.

[3] S. O. Fatunla, (1988): Numerical methods of initial value problems in O. D. Es. Academic Press, Boston, USA.

[4] E. Haireer, S. P. Norsett and G./ Wanner, (1980): Solving Ordinary Differtial Equations I, Non -stiff problems Springer-verlay, New York.

[5] C.F. Curch and J.O Hirschfelder(1985);Continuous BDF of Stability of numerical analysis of multistep methods for Differential equations

[6] C. W. Gear, (1971): Numerical initial value problems in ODEs, Prentice-Hall.

[7] J. J. H. Miller, (1971):On the location of zeros of certain classes of polynomials with applications to numerical analysis J. Inst. Maths Application 5, 8, Pages 397-406.

[8] Mitched and craggs;(1953)Backward Differentiation for numerical analysis for IVPs

[9] Herner(1983) ; Stability of B.D.F for IVP maths Application 3,4 Pages 267-293.

[10] G. Wanner(1980); Solving ODE, Non-stiff problems, Springer-verlag, New York.