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COMPUTATIONAL AND MATHEMATICAL MODELING OF METRIC SPACE PROPERTIES ON THE PREDICTION OF BIODIVERSITY

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Abstract

The dynamical system that defines the interaction between two competing legumes, for a limited resource is defined. By using Ordinary Differential Equation of order 45 (ODE 45) numerical simulation we have studies metric space properties on a differential effect of the intrinsic growth rate on the biodiversity scenario. The full results of this study are presented and discussed quantitatively.

Keywords*: Metric space, Numerical simulation, logistic, model equation biodiversity, intrinsic growth rate.*

Introduction
Following [1] a metric space is a pair (X,d),where X is a set and d is a metric on X (or distance function on X), that is, a function defined on $X \; x \; X$ such that for all $x, y, z, \varepsilon X$ we have:

1. $d(x, y) \geq 0,$ (i.e is real valued, finite and non-negative).
2. $d(x, y) = 0,$ if and only if (Iff) $x = y$
3. $d(x, y) = d(y, x)$ (symmetry)
4. $d(x, y) \leq d(x,z) + d(z, y)$ (Triangle inequality,

From axiom (4) we obtain by induction the generalized triangular inequality $d(x_1, x_n) \leq d(x_1, x_2) + d(x_2, x_3) + \ldots d(x_{n-1} x_n)$
A subspace (Y,d) of (x,d) is obtained if we take a subset
$Y \subset X$ and restrict d to $Y \; x \; Y$ this metric on Y is the restriction $d = d / YxY$
For the purpose of this simulation Study, we shall consider a Real line R, the set of all real numbers, taken with the usual metric and is defined by:
$d(a,b) = |a-b|$
For the proof see [1].
One of the processes of understanding the interaction between two legumes such as cowpea and groundnut depend on the construction of a deterministic mathematical model that has the structure of a logistic model formulation. This model is defined by two intrinsic growth rate parameter values, two intra-species coefficient; initial condition and the length of the growing season. Following and [2-8] the study of the effect of a model parameter variation on biodiversity is an ongoing active area of research.
In this study, we are interested to find how we link metrics space properties and numerical analysis in tackling problems of biodiversity.
Following the recent application of numerical simulation to model biodiversity, we have come to observe that the mathematical technique of a numerical simulation which is rarely been applied to interpret the extent of biodiversity loss and biodiversity gain is an important short term and long term quantitative scientific process. We will expect the application of numerical simulation to model biodiversity considering the metrics space properties. However, something is missing, we find a gap on the limited aspect. How are the two data set responding to metric formation? They must satisfy symmetric property before using it to make predictions. Predictions deem not to be complete without it depending on the property of functional analysis. For the effect or impact to be deem correct it must depend on something; So the effect of differential parameter values which have occurred will hold if the property of the metric are satisfied for a stronger argument on the link between functional analysis and model predictions. If the effect is only calculated without the inclusion of a metric function parameterization, it may not lead to a useful philosophy and mathematical thinking on the link between functional analysis and numerical simulation result which have been neglected in most mathematical literature. It is against this background therefore, we want to take up this challenge.

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Method of Analysis
To provide an empirical proof of the link between a metric function and numerical simulation, we shall utilize Ordinary Differential Equation 45 (ODE 45).

Mathematical Formulation
Following [10], we shall consider a continuous dynamical system of non-linear first order ordinary differential equations having the following logistic mathematical model structure.

$$\frac{dc(t)}{dt} = \alpha_1 c(t) - B_1 c^2(t) \quad (1)$$

$$\frac{dg(t)}{dt} = \alpha_2 g(t) - B_2 g^2(t) \quad (2)$$

with the initial conditions

$$c(0) = c_0 > 0 \quad (3)$$

$$g(0) = g_0 > 0 \quad (4)$$

where:
c(t) defines the biomass of the cowpea legume at time (t) in the unit of days.
g(t) defines the biomass of groundnut at legume at time (t) in the unit of days.
α_1 defines the intrinsic growth rate of the cowpea legume
α_2 defines the intrinsic growth rate of groundnut
$B_1 B_2$ defines the intra-competition coefficients.
Following [10], we have utilized the following parameter values
α_1 = 0.0225
α_2 = 0.0446
B_1= 0.006902
B_2= 0.0133
The length of the growing season is in the unit of days.
The models are computing the biodiversity gain and biodiversity loss due to two legumes under study (cowpea and groundnut). If a variation of a model parameter value produces a new biomass which is smaller than the old biomass for any interacting legumes, such as cowpea and groundnut, then a biodiversity loss has occurred and can be quantified as we have done in this study.
On the other hand, if a variation of a model parameter value produces a new biomass which outweighs the old biomass, then a biodiversity gain has occurred.
The method of solution used in this study is Matlab programming, Ordinary Differential Equation 45 (ODE 45) as most application problems in mathematics do not admit closed form solution, hence computational method is required. Biodiversity loss due to cowpea and groundnut is defined as follows:

$$BLc(\%) = \left(\frac{cowpea\ (biomass)old - cowpea\ (biomass)new}{cowpea\ (biomass)old}\right)100, \text{provided cowpea(biomass)old } \neq 0$$

$$BLg(\%) = \left(\frac{groundnut\ (biomass)old - groundnut\ (biomass)new}{groundnut\ (biomass)old}\right)100,$$

provided $groundnut$ (biomass)old $\neq 0$
Similarly, the biodiversity gain due to cassava and yam biomasses is defined as follows:

$$BGc(\%) = \left(\frac{cowpea\ (biomass)old - Cowpea\ (biomass)new}{cowpea\ (biomass)old}\right)100, \text{provided } cowpea\text{(biomass)old } \neq 0$$

$$Hence, -BGc(\%) = \left(\frac{cowpea\ (biomass)old - cowpea\ (biomass)new}{cowpea\ (biomass)old}\right)100 = BLc(\%),$$

provided $Cowpea$(biomass)old $\neq 0$

$$BGg(\%) = \left(\frac{groundnut\ (biomass)old - groundnut\ (biomass)new}{groundnut\ (biomass)old}\right)100, \text{provided } groundnut\text{(biomass)old } \neq 0.$$

$$Hence, -BGg(\%) = \left(\frac{groundnut\ (biomass)old - groundnutYam(biomass)new}{groundnut\ (biomass)old}\right)100 = BLg(\%),$$

provided $groundnut$(biomass)old $\neq 0$.
In the event of environmental perturbation in which climate change is one, it is the intrinsic growth rates that are more vulnerable to the impact of climate change [2]. We have two biomasses: biomass for cowpea and biomass for groundnut.

Scenario 1: If by the process of simulation, the effect of climate change on cowpea biomass produces a new cowpea biomass and is less than the old cowpea biomass when the model parameters are all fixed, then a biodiversity loss has occurred.
Similarly, if by the process of simulation, the effect of climate change on groundnut biomass produces a new groundnut yam biomass that is less than the old groundnut yam biomass when the model parameters are all fixed, then a biodiversity loss has occurred.

Seenario 2: If by process of simulation, the effect of climate change on cowpea biomass produces a new cowpea biomass that outweighs the old cowpea biomass when the model parameters are all fixed, then a biodiversity gain has occurred.

Results

The full results of the link between metric properties (symmetry) and numerical simulation are presented as displayed on Table 1-Table 3. The first column data represent the cowpea biomass when all the model parameter values are fixed.

The second column data represent t the cowpea biomass when only the intrinsic growth rate model parameter values are varied.

The third column data represent the effect of the second column data otherwise called the perturbed data on the first column data otherwise called the original data.

The fifth column data represent the groundnut biomass when only the intrinsic growth rate model parameter values are varied.

The sixth column data represent the effect of the fifth column data otherwise called the perturbed data on the fourth column data otherwise called the original data.

Table 1: Quantifying the effect of functional analysis properties on decreasing the intrinsic growth rates of two interacting cowpea and groundnut by 10 percent on biodiversity loss: length of growing season is ten (10) months.

Example	C(t) = a	Co(t) = b	d(a,b) =(a-b)	b-a	d(a-a) =(b-a)	BL(%)	g(t) =a	gm(t) =b	(a-b)-d(a,b)	b-a	d(b,a)	BL(%)
1	0.4800	0.4800	0	0	0	0	0.4800	0.4800	0	0	0	0
2	0.4893	0.4795	0.0098	-0.0098	0.0098	2.0013	0.4986	0.4791	0.0195	-0.0195	0.0195	3.9221
3	0.4987	0.4790	0.0197	-0.0197	0.0197	3.9561	0.5179	0.4782	0.0397	-0.0397	0.0397	7.6659
4	0.5083	0.4785	0.0298	-0.0298	0.0298	5.8653	0.5377	0.4773	0.0604	-0.0604	0.0604	11.2398
5	0.5180	0.4780	0.0400	-0.0400	0.0400	7.7301	0.5581	0.4764	0.0817	-0.0817	0.0817	14.6515
6	0.5279	0.4775	0.0504	-0.0504	0.0504	9.5516	0.5792	0.4755	0.1037	-0.1037	0.1037	17.9084
7	0.5379	0.4770	0.0609	-0.0609	0.0609	11.3307	0.6009	0.4746	0.1263	-0.1263	0.1263	21.0176
8	0.5481	0.4765	0.0716	-0.0716	0.0716	13.0684	0.6232	0.4737	0.1495	-0.1495	0.1495	23.9859
9	0.5584	0.4760	0.0824	-0.0824	0.0824	14.7658	0.6461	0.4729	0.1732	-0.1732	0.1732	26.8199
10	0.5689	0.4755	0.0934	-0.0934	0.0934	16.4237	0.6697	0.4720	0.1977	-0.1977	0.1977	29.5255

Table 2: Quantifying the effect of functional analysis properties on decreasing the intrinsic growth rates of two interacting cowpea and groundnut by 99.8 percent on biodiversity loss: length of growing season is ten (10) months.

Example	Co(t) =a	Cn(t) =b	d(a,b) =(a-b)	b-a	d(a-a) =(b-a)	BL(%)	g(t) = a	gm(t) =b	d(x,y)	y-x	d(y,x)	BL(%)
1	0.4800	0.4800	0	0	0	0	0.4800	0.4800	0	0	0	0
2	0.4893	0.4893	0	0	0	0.0000	0.4986	0.4986	0	0	0	0.0089
3	0.4987	0.4987	0	0	0	0.0000	0.5179	0.5178	0.0001	-0.0001	0.0001	0.0177
4	0.5083	0.5082	0.0737	-0.0737	0.0737	0.0134	0.5377	0.5376	0.0001	-0.0001	0.0001	0.0265
5	0.5180	0.5179	0.3599	-0.3599	0.3599	0.0179	0.5581	0.5579	0.0002	-0.0002	0.0002	0.0352
6	0.5279	0.5278	0.0001	-0.0001	0.0001	0.0223	0.5792	0.5789	0.0003	-0.0003	0.0003	0.0438
7	0.5379	0.5378	0.0001	-0.0001	0.0001	0.0267	0.6009	0.6006	0.0003	-0.0003	0.0003	0.0523
8	0.5481	0.5479	0.0002	-0.0002	0.0002	0.0311	0.6232	0.6228	0.0004	-0.0004	0.0004	0.0608
9	0.5584	0.5582	0.0002	-0.0002	0.0002	0.0355	0.6461	0.6457	0.0004	-0.0004	0.0004	0.0692
10	0.5689	0.5687	0.0002	-0.0002	0.0002	0.0398	0.6697	0.6692	0.0005	-0.0005	0.0005	0.0774

Table 3: Quantifying the effect of functional analysis properties on increasing the intrinsic growth rates of two interacting cowpea and groundnut by 150 percent on biodiversity loss: length of growing season is ten (10) months.

Example	Co(t) = a	Cn(t) = b	a-b	d(a,b)	d(b,a)	BL(%)	g(t) =x	gm(t) = y	x-y	d(x,y)	d(y,x)	BL(%)
1	0.4800	0.4800	0	0	0	0	0.4800	0.4800	0	0	0	0
2	0.4893	0.4948	-0.0055	0.0055	0.0055	1.1294	0.4986	0.5098	-0.0112	0.0112	0.0112	2.2475
3	0.4987	0.5100	-0.0113	0.0113	0.0113	2.2676	0.5179	0.5413	-0.0234	0.0234	0.0234	4.5293
4	0.5083	0.5256	-0.0477	0.0477	0.0477	3.4144	0.5377	0.5745	-0.0368	0.0368	0.0368	6.8438
5	0.5180	0.5417	-0.0237	0.0237	0.0237	4.5696	0.5581	0.6094	-0.0513	0.0513	0.0513	9.1893
6	0.5279	0.5582	-0.0303	0.0303	0.0303	5.7330	0.5792	0.6462	-0.0670	0.0670	0.0670	11.5640
7	0.5379	0.5751	-0.0372	0.0372	0.0372	6.9044	0.6009	0.6848	-0.0839	0.0839	0.0839	13.9656
8	0.5481	0.5924	-0.0443	0.0443	0.0443	8.0835	0.6232	0.7254	-0.1022	0.1022	0.1022	16.3916
9	0.5584	0.6102	-0.0518	0.0518	0.0518	9.2701	0.6461	0.7679	-0.1218	0.1218	0.1218	18.8394
10	0.5689	0.6285	-0.0596	0.0596	0.0596	10.4639	0.6697	0.8124	-0.1427	0.1427	0.1427	21.3060

4.2 Discussion of Results

Since the length of the growing season is consistent with ten (10) months, the decreased variations of the intrinsic growth rates of the cowpea and groundnut have predicted biodiversity loss whereas the increased variation of the intrinsic growth rates of the cowpea and groundnut has predicted biodiversity gain. In obtaining these results, the functional analysis property (symmetric property of a metric) was satisfied; hence providing an empirical prove on the link between metric space and numerical simulation.

Conclusion

In this study, we have applied the method of a numerical method (ODE 45 simulation modeling) on metrics space properties (symmetric property of a metric space) on to predict biodiversity loss due to a decreased variation of the intrinsic growth rates together and also predict biodiversity gain due to an increased variation of the intrinsic growth rates together. The effective calculation of the interval of the growth rates where biodiversity loss changes to a biodiversity gain remains to be an open problem that we would consider in our next research investigation. The modeling of the summitry property of a metric space is very important for from out results it helps to determine the extent of biodiversity loss. The smaller the metric between two pairs of data the nearer from biodiversity loss to biodiversity gain.

Reference

[1] Chidume, C.E. (1984). *Functional analysis, an introduction to metric space.* University Press, University of Nigeria Nsukka.
[2] Ekaka-a, E.N. (2009). *Computational and mathematical modeling of plant spaces interactions in a Harsh climate*. Dissertation, University of Liverpool.
[3] Ekaka-a, E.N, Osahogulu D.J, Atsu J.U and Isibor L.A. (2018). Numerical and statistical quantification of Biodiversity: Two at A-Time Equal variations. *International Journal of Advanced Engineering management, and science, Vol*. 4 bttps://dx.doi-org/10.22161/Ijaems.4.12
[4] Atsu, J.U. and Ekaka-a, E.N. (2017). Modeling the policy implication of biodiversity loss A case study of the cross River national park, South – South Nigeria *International Journal of pure and applied science Cambridge Research and publications.* Vol 10 No. 1; pp 30-3
[5] De Mazancourt, C, Isbell, F. Larocque, E, Grace, J. B. et al. (2013) predicting ecosystem stability from community composition and biodiversity. *Ecology Lellers,* D.O.I:10.111/ele.12088
[6] Ekaka-a, E.N, Asor, V.E, Lale N.E.S. and Wokocha, C.C. (2014) A literature Review of Deterministic stability of a system of Nonlinear first order ordinary differential equations: Theory and application in Agriculture. 10SR Journal of *Mathematics* (10SR-JM) e-ISSN: 2278-5728
[7] Troost, B.W., Kooi, S.A.L, Kooijam, M., (2007). Bifuration Analysis of ecological and evolu-tionary processes' in ecosystem, ecological modeling 204, , 253-268.
[8] Ford N.J., Lumb P.M, and Ekaka-a E.N. (2010). Mathematical modeling of plant species interactions is a harsh *climate Journal of computational and applied mathematics,* 2732 – 2744
[9] Ekaka-a, E.N, Eke, Nwagrabe and Atsu, J.U. (2018). Modelling the effect of decreasing the inter competition coefficients on biodiversity loss. *International Journal of Advanced Engineering management and science Vol. 4 https//*dx.dor.org/10.22161/ijaems.4.1.3
[10] Ekaka-a, E.N. and Atsu, J.U. (2017). Quantifying the impact of changing intrinsic growth rate on the biodiversity of the forest resource biomass's implication for the cross River State forest resource at the Cross River National Park, South – South, *Nigeria African scholar Journal of pure and applied Science, 7(1); 1*17-130.