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EXISTENCE AND UNIQUENESS OF INFECTIOUS DISEASE WITH QUARANTINE

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Abstract

A deterministic differential equation model for endemic malaria with quarantine in a non-constant population is formulated and analyzed. We determine the existence and uniqueness of the system to better understanding the solution of the model.

Keyword: quarantine, endemic, existence, uniqueness

1. Introduction

Epidemics of infectious diseases have happened all through history [1]. One intervention procedure to control the spread of infectious diseases is to isolate some infective, in order to reduce transmissions of the infection to susceptible [2]. Quarantines have successfully been used as one such control [3]. The fundamental dilemma associated with the implementation of isolation and quarantine is how to predict the population level efficacy of individual quarantine: which and how many individuals need to be quarantine to achieve effective control at the population level?

The word quarantine has evolved to mean forced isolation or stoppage of interactions with others over the centuries quarantine has been used to reduce the transmission of human diseases such as leprosy, tuberculosis, measles etc [4].

In order to study the effects of quarantine on endemic infectious diseases, we consider SEIQS model. In this model, susceptibles in the S class become infected and move to exposed E class and after which the infection is manifested, it moves to infectious class I [5]. Some infected individuals remain in the I class for their entire infectious period before they return to the susceptible class, while other infected individuals are transferred into a quarantined class Q and remain there until they are no longer infectious, at which time they return to the susceptible class [6] .

2. The model equation

The total population $N(t)$ is divided into four compartments with $N(t)=S(t)+E(t)+I(t)+Q(t)$, where S is the number of individuals in the susceptible class, E is the number of people who are exposed (the latent period, in which the person is infected but not yet infectious) but not quarantined, I is the number of people who are infectious but not quarantined, Q is the number of individuals who are quarantined and it is assumed that an infection dos not confer immunity [7]. This model is called an SEIQS model. The differential equations for this model are:

$$\frac{dS}{dt}=\Lambda+\rho Q-\alpha S-\mu S \qquad (1)$$

$$\frac{dE}{dt}=\alpha S-\lambda E-\mu E$$

$$\frac{dI}{dt}=\lambda E-\beta I-\mu I$$

$$\frac{dQ}{dt}=\beta I-\rho Q-\mu Q$$

$$\frac{dN}{dt}=\Lambda-\mu N \qquad (2)$$

3. Basic properties of the model

Since the model monitors human population, all the associated parameters and state variables are non-negative. It can easily be shown that the state variables of the model remain non-negative for all non-negative initial conditions[8]. Consider the biological feasible region

$$\Phi=\left\{(S,E,I,Q)\in\Re^4{}_+ : N\to\frac{\Lambda}{\mu}\right\} \qquad (3)$$

Lemma 1 [9]. The closed Φ is positively invariant and attracting.

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Proof

$$\frac{dN}{dt}=\Lambda-\mu N$$

This implies that the total population is bounded by $\frac{\Lambda}{\mu}$, if $\frac{dN}{dt}=0$ and $N(t)=\frac{\Lambda}{\mu}$.

$$\frac{dN}{dt}+\mu N=\Lambda \quad (4)$$

$$Ne^{\mu t}=\frac{\Lambda}{\mu}+ke^{-\mu t} \quad (5)$$

$$N(t)=\frac{\Lambda}{\mu}+\left(N_0-\frac{\Lambda}{\mu}\right)e^{-\mu t} \quad (6)$$

Then $N(t)=\frac{\Lambda}{\mu}$ if $N(0)=\frac{\Lambda}{\mu}$. (7)

Hence, the region Φ is positively invariant and attracts all solutions in $\Re^4$.

3. Existence and Uniqueness of Solution for the Model

If a mathematical model is to predict the future from its current state at time t_0, the initial value problem (IVP)

$$x^1=f(t,x), \qquad x(t_0)=x_0 \quad (8)$$

Must have a solution that exist and unique [10].

In this section, we shall establish conditions for existence and uniqueness of solution for the model of equations. Let

$$\begin{aligned} f_1(t,x)&=\Lambda+\rho q-\alpha s-\mu s \\ f_2(t,x)&=\alpha s-\lambda e-\mu s \\ f_3(t,x)&=\lambda e-\beta i-\mu i \\ f_4(t,x)&=\beta i-\rho q-\mu q \end{aligned} \quad (9)$$

So that

$$x^1=f(t,x)=f(x) \quad (10)$$

Definition1 [11]. Suppose function $f(t,x)$ has domain D in (t,x)-space and suppose there exists a constant $k>0$ such that if $(t,x^1),(t,x^2)\in D$ then

$$\left|f(t,x^1)-f(t,x^2)\right|\le k\left|x^1-x^2\right| \quad (11)$$

Then f satisfies a Lipchitz condition with respect to x in D, and k is a Lipchitz constant for f.

Theorem1 [12]. Let D be an open set in (t,x)-space. Let $(t_o,x^o)\in D$ and let a,b be positive constants such that the set

$$R=\{(t,x)|t-t_o|\le a,\left|x-x^o\right|\le b\} \quad (12)$$

Is contained in D. Suppose function f is defined and continuous on D and satisfies a Lipchitz condition with respect to x in R. Let

$$Max=\max_{(t,x)\in R}\left|f(t,x)\right| \quad (13)$$

$$A=\min\left[a,\tfrac{b}{M}\right]$$

Then the differential equation

$$x^1=f(t,x) \quad (14)$$

has a unique solution $x(t,t_o,x^o)$ on (t_o-A,t_o+A) such that $x(t_o,t_o,x^o)=x^o$. This solution $x(t,t_o,x^o)$ is such that $\left|x(t,t_o,x^o)-x\right|\le MA$ for all $t\in(t_o-A,t_o+A)$.

Lemma 2 [13]. If $f(,x)$ has a continuous partial derivative $\frac{\partial f_i}{\partial f_j}$ on a bounded closed convex domain R, then it satisfies a Lipchitz condition in R.

We are interested in the region

$$1\le\varepsilon\le R\ . \quad (15)$$

We look for a bounded solution of the form

$$0<R<\infty. \quad (16)$$

We shall prove the following existence theorem.

Theorem2 [14]. Let D denote the region defined in (11) such that(15) and(16) hold. Then there exists a solution of model (9)-(10) which is bounded in the region D.

Proof. Let

$$\begin{aligned} f_1 &= \Lambda + \rho q - \alpha s - \mu s \\ f_2 &= \alpha s - \lambda e - \mu s \\ f_3 &= \lambda e - \beta i - \mu i \\ f_4 &= \beta i - \rho q - \mu q \end{aligned} \tag{17}$$

It suffices to show that $\frac{\partial f_i}{\partial f_j}$, I,j = 1,2,3,4 are continuous.

Consider the partial derivatives

$$\frac{\partial f_1}{\partial s} = -\alpha - \mu, \qquad \left|\frac{\partial f_1}{\partial s}\right| = |-\alpha - \mu| < \infty,$$

$$\frac{\partial f_1}{\partial e} = 0, \qquad \left|\frac{\partial f_1}{\partial e}\right| = |0| < \infty,$$

$$\frac{\partial f_1}{\partial i} = 0, \qquad \left|\frac{\partial f_1}{\partial i}\right| = |0| < \infty,$$

$$\frac{\partial f_1}{\partial q} = \rho, \qquad \left|\frac{\partial f_1}{\partial q}\right| = |\rho| < \infty.$$

Also,

$$\frac{\partial f_2}{\partial s} = \alpha - \mu, \qquad \left|\frac{\partial f_2}{\partial s}\right| = |\alpha - \mu| < \infty,$$

$$\frac{\partial f_2}{\partial e} = -\lambda, \qquad \left|\frac{\partial f_2}{\partial e} = |-\lambda|\right| < \infty,$$

$$\frac{\partial f_2}{\partial i} = 0, \qquad \left|\frac{\partial f_2}{\partial i}\right| = |0| < \infty,$$

$$\frac{\partial f_2}{\partial q} = 0, \qquad \left|\frac{\partial f_2}{\partial q}\right| = |0| < \infty.$$

$$\frac{\partial f_3}{\partial s} = 0, \qquad \left|\frac{\partial f_3}{\partial s}\right| = |0| < \infty,$$

$$\frac{\partial f_3}{\partial e} = \lambda, \qquad \left|\frac{\partial f_3}{\partial e}\right| = |\lambda| < \infty,$$

$$\frac{\partial f_3}{\partial i} = -\beta - \mu, \qquad \left|\frac{\partial f_3}{\partial i}\right| = |-\beta - \mu| < \infty,$$

$$\frac{\partial f_3}{\partial q} = 0, \qquad \left|\frac{\partial f_3}{\partial q}\right| = |0| < \infty.$$

Lastly,

$$\frac{\partial f_4}{\partial s} = 0, \qquad \left|\frac{\partial f_4}{\partial s}\right| = |0| < \infty,$$

$$\frac{\partial f_4}{\partial e} = 0, \qquad \left|\frac{\partial f_4}{\partial e}\right| = |0| < \infty,$$

$$\frac{\partial f_4}{\partial i} = \beta, \qquad \left|\frac{\partial f_4}{\partial i}\right| = |0| < \infty,$$

$$\frac{\partial f_4}{\partial q} = -\rho - \mu, \qquad \left|\frac{\partial f_4}{\partial q}\right| = |-\rho - \mu| < \infty.$$

Since all these partial derivatives are continuous and bounded, then by theorem (2), there exist a unique solutions of (9)-(10) n the region D.

4 Conclusion

In this paper, we have considered the variable population SEIQS epidemic model. We showed that there exists a positive invariant, where the system is biologically meaningful.

Also, the existence and uniqueness of the model is established using Lipchitz's condition

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