

IMPACT OF HYDRAULIC CONDUCTIVITY AND DIFFUSION COEFFICIENT ON ATTENUATION OF CONTAMINANT IN FRESHWATER COASTAL AQUIFERS

O. O. Faluyi¹ and F. R. Amodu²

¹Department of Physics and Materials Science, Kwara State University, Malete Kwara State Nigeria.

²Physics Unit, Department of Science Laboratory Technology, Federal Polytechnic Ede, Osun State.

Abstract

Freshwater coastal aquifers can be contaminated by influx of seawater. The study investigated the effect of geophysical parameter such as hydraulic conductivity and effective diffusion coefficient determined empirically on the attenuation coefficient of mass flux (J) of contaminant through the coastal aquifers. Porosities of the grains were determined and a tagged sample A to E. Porosity of each grain is a factor that would determine hydraulic conductivity and effective diffusion coefficient. Results showed that porosity of the samples ranged between 0.250 to 0.420 while the corresponding values of hydraulic conductivity and effective diffusion coefficient ranged between 0.570×10^{-5} to $17.500 \times 10^{-5} \text{ ms}^{-1}$ and 3.310 to $4.680 \times 10^{-5} \text{ m}^2\text{s}^{-1}$ respectively. Therefore, the attenuation coefficients λ of J decreased from -2.84 to -34.89 with increased in hydraulic conductivity and effective diffusion coefficient the samples.

Keywords: porosities, hydraulic conductivity, attenuation coefficient, freshwater and coastal aquifers

1. INTRODUCTION

Coastal aquifers are vital sources of fresh water throughout the world. Population growth and agricultural development in coastal areas has increased the demand for freshwater. As a result of this demand more stress is being placed on coastal aquifers. Saltwater intrusion is arguably the most common contamination problem in aquifers, and a major constraint imposed on groundwater utilisation [1]. Saline contamination of freshwater resources can cause significant social, economic and environmental costs.

The position of the saline interface is dynamic and depends on the geological formation, hydraulic gradient, topography, and the quantity of freshwater moving through the aquifer system [2]. A reduction in groundwater recharge, or an increase in abstraction, can reduce the hydraulic gradient, the freshwater hydrostatic pressure, and the movement of freshwater. This can cause the saline interface to move landwards resulting in saline intrusion and a reduction in water quality. As seawater has a chloride concentration of approximately 19 g/l, very little seawater is required to contaminate freshwater [3]. Therefore, proactive management of coastal aquifers is essential to ensure sustainable water resource development.

Groundwater flow in an aquifer is defined as Darcy's Law. It expresses the velocity of groundwater flow as a function of the hydraulic gradient and hydraulic conductivity of the aquifer medium.

$$V = -K\left(\frac{dh}{dl}\right) \quad (1)$$

where:

V = Groundwater flow velocity (m/day)

K = Hydraulic conductivity (m/day)

$\frac{dh}{dl}$ = Hydraulic gradient [5]

Corresponding Author: Faluyi O.O., Email: oludotunfaluyi@gmail.com, Tel: +2348033772597

Journal of the Nigerian Association of Mathematical Physics Volume 55, (February 2020 Issue), 115 – 122

The hydraulic gradient determines the direction, and relative velocity of groundwater flow. It is the measure of the change in hydraulic head (dh) across a given distance (dl). The hydraulic head represents the potential energy of the subsurface water at any given point and is determined from the water level pressure and elevation of the point relative to a specified datum [6]. Groundwater flows from a high hydraulic head to a lower hydraulic head [4].

Hydraulic conductivity is the specific discharge per unit of hydraulic gradient. It expresses the ease with which a fluid can move through a porous matrix [5]. The properties of the medium matrix that determine hydraulic conductivity are mainly grain/pore size distribution, shape of grains, tortuosity, specific surface area and porosity [4]. In a saturated zone, the hydraulic conductivity will largely depend on the geometry and distribution of the pore spaces. A highly permeable unit, such as sand or gravel will have a large hydraulic conductivity. While less permeable units, for example clay or silt, will have a relatively small value [2].

Diffusion coefficient of mass flux is less in soil than in free solution because solid particles in soil occupy some of the cross-sectional area. Due to the reduced cross-sectional area of flow in soil; diffused mass flux can be modified in soil as follows

$$J = D\theta \frac{dC}{dX} \quad (2)$$

where D is diffusion coefficient in free solution and $\theta = \phi S_r$ the volumetric water content defined as

$$\theta = \phi S_r \quad (3)$$

where ϕ = the total soil porosity and S_r = the degree of saturation of the soil, expressed as a decimal. Therefore, the maximum flux for liquid phase diffusion will occur when the soil is saturated ($S_r = 1.0$). Diffusion coefficient of the saturated soil depends on the porosity of the soil as a result of high volumetric water content.

This work investigated the effect of hydraulic conductivity and effective diffusion coefficient determined empirically on the attenuation coefficient of mass flux (J) of contaminant through the coastal aquifers.

2. THEORETICAL BACKGROUND.

The mass flux of saltwater intrusion in coastal aquifer can be modeled by applying some constraints on Darcy's law and Fick's laws.

According to Darcy's law, the volumetric flow rate per unit area (volume flux) is directly proportional to the hydraulic gradient [7]

$$q = -K \frac{dh}{dl} \quad (4)$$

and Seepage velocity,

$$V = \frac{q}{\theta} \quad (5)$$

where K is the hydraulic conductivity and dh/dl is the hydraulic gradient (i).

According to first Fick's law of diffusion, the amount of mass of saltwater passing through a unit area per unit time is called mass flux in $\text{kgm}^{-2}\text{s}^{-1}$.

The mass flux is directly proportional to the gradient of concentration [8]

$$J = -D_o \frac{dC}{dx} \quad (6)$$

Where J is the mass flux of substance ($\text{Kg/m}^2\text{s}$), D_o is the effective diffusion coefficient of soil and dc/dx is gradient of concentration.

According to one dimensional second Fick's law of diffusion, the rate of change of concentration with time is directly proportional to the second order of concentration gradient

$$\frac{dC}{dt} = D_o \frac{d^2C}{dx^2} \quad (7)$$

Where C is the concentration of saltwater in porous medium, t is the time of diffusion and x is the distance (length) of diffusion.

The concentration of contaminant can be expressed by exponential function of time [9].

$$C = C_o \exp(-\lambda_t t) \quad (8)$$

Also, it can be expressed by exponential function of distance [10, 11 and 12].

$$C = C_o \exp(-\lambda_x x) \quad (9)$$

Where C_o is the concentration at the interface between freshwater and saltwater when $t=0$ and $x=0$ with magnitude 1025 kgm^{-3} .

where λ_t and λ_x is the coefficient of attenuation of contaminant as function time and position.

$$\lambda_x = -\frac{V}{D_o} \quad (10)$$

$$\frac{dc}{dx} = \lambda_x C_o \exp\left(-\lambda_x x\right) \quad (11)$$

$$\frac{dc}{dx} = -\frac{V}{D_o} C_o \exp\left(-\frac{V}{D_o} x\right) \quad (12)$$

Concentration gradient $\left(\frac{dc}{dx}\right)$ is directly proportional to the mass flux of contaminant which is the function of seepage velocity and diffusion coefficient that depend on hydraulic conductivity.

Hence,

$$J = -D_o \frac{dc}{dx} = -VC_o \exp\left(-\frac{V}{D_o} x\right) \quad (13)$$

3. METHODOLOGY

The sample was sieved into five different grains with sizes of 125, 250, 350, 400 and 450 μm and the grain sizes were tagged samples A, B, C, D and E, respectively. The porosity (\emptyset) of the samples A, B, C, D and E were determined using volumetric method.

Experimental Procedure

The experimental setup consisted of two arms of calibrated cylinder glass with diameter $1.2 \times 10^{-2}\text{m}$ erected vertically and joined together with a similar glass cylinder in the horizontal position which housed the samples. Saturated sample A of length x with air completely eliminated was inserted into the horizontal glass and screened at both ends before commencement of the experiment. Freshwater was filled in the arm labeled M to 27 cm mark and saltwater which was coloured with red dye (neutral pigment) was filled in the arm labeled N to 15 cm mark. The controls were opened at the same time to allow flow to occur. The volume of freshwater displaced volume of saltwater in the setup because of the difference in the hydraulic heads of freshwater and saltwater. The flow occurs from high hydraulic head to low hydraulic head. The hydraulic gradient between freshwater and saltwater was obtained by determining the difference between hydraulic heads of freshwater and saltwater and then divided by the flow length X between the two heads. The volumetric flow rate was computed from volume of fluid displaced between the two arms per time taken and it was denoted as Q with unit of m^3s^{-1} . Darcy flux (volume flux) was obtained by dividing the volumetric flow rate with cross sectional area of the tube and it was denoted as v_x with unit of ms^{-1} , $v_x = Q/A$ (Where A is cross sectional area of the cylindrical glass which is $\frac{\pi d^2}{4}$ and d is the diameter of the cylindrical glass, given as $1.2 \times 10^{-2}\text{m}$). The flow length X was determined from the distance of the movement of the red dye from the interface of freshwater and saltwater. The diffusion coefficient (D) of soil was determined from the slope of plot of square of flow length X (X^2) against time (t) in second. The seepage velocity (V) which is the average velocity of water flow through the pores was determined by dividing the volume flux (v_x) by porosity (\emptyset). The mass flux of saltwater contaminant in the controlled experiment was determined using the following steps:

- (i) by multiplying the volumetric flow rate Q with time (t) in second to give volume of contaminated fluid displaced within the columns.
- (ii) the mass of contaminated water displacement was obtained from the density (concentration) formula i.e mass of contaminated water displacement = density (concentration) of contaminated water \times volume displacement
- (iii) The mass flux (J) of saltwater contaminant within the columns was arrived at by dividing the mass of contaminated water displacement of saltwater with cross sectional area (A) in m^2 and time (t) taken in second i.e unit of J is $\text{kgm}^{-2}\text{s}^{-1}$.

The procedure was repeated for samples B, C, D and E. The hydraulic gradient i , volumetric flow rate Q , volume flux v_x , diffusion coefficient D , seepage velocity V and mass flux J for each sample were determined.

4. RESULTS AND DISCUSSION

The hydraulic conductivity, diffusion coefficient and porosity of the samples increase with the mass of the contaminant through the samples. In Tables 4.1 to 4.5, mass flux at a particular time of 360 seconds increases with the hydraulic conductivity and diffusion coefficient of samples A to E. The hydraulic conductivity of each sample was obtained from the plot volume flux q against hydraulic gradient i . The slope of the graph is the hydraulic conductivity as shown in Figure 4.1 for sample A. The same pattern of graph was plotted for samples B to E and the hydraulic conductivities for samples were obtained from the slopes of the graphs. The slope of graph of square of flow length (x^2) against time is the diffusion coefficient of sample A as shown in Figure 4.2. Determination of diffusion coefficients for samples B to E follows the same pattern. Table 4.6 shows the porosities, hydraulic conductivities and diffusion coefficients for samples A to E. Mass flux of contaminant at time 60, 120, 180, 240, 300 and 360 seconds each sample respectively from Tables 4.1 to 4.5 was plotted against hydraulic conductivity. It shows that the hydraulic conductivity of the samples is a function mass flux of contaminant through the samples as in Figure 4.4 to Figure 4.6 with a linear graph. The slopes of the graphs obtained were concentration of the contaminant through all the samples from 60 seconds to 360 seconds as presented in Table 4.7.

Similarly, Mass flux of contaminant at time 60, 120, 180, 240, 300 and 360 seconds of each sample respectively from Tables 4.1 to 4.5 was plotted against diffusion coefficient in Table 4.6. Also, it shows that the diffusion coefficient of the samples is a function mass flux of contaminant through the samples as in Figure 4.7 to Figure 4.9 with a linear graph. The slopes of the graphs obtained were concentration gradient of the contaminant through all the samples from 60 seconds to 360 seconds as presented in Table 4.7. Natural logarithm of mass flux of contaminant was plotted against flow length of contaminant through the samples from Tables 4.1 to 4.5. It was a linear graph with negative slope as shown in Figure 4.3 and the slope was the attenuation coefficient of sample A. The same pattern of graphs was obtained for the other samples. The attenuation coefficient obtained for samples A, B, C, D and E were -2.60, -14.72, 24.89, -29.43 and -35.53 m^{-1} respectively. Therefore, attenuation coefficient decreases with increase in hydraulic conductivity and this means mass flux of contaminant increases with increase in hydraulic conductivity.

Table 4.1: Experimentally determined values of hydraulic gradient i, specific discharge q, seepage velocity, mass flux J and flow length x for sample A (Ø=0.25).

Time (Sec.)	Hydraulic head of saltwater (cm)	Hydraulic Head of freshwater (cm)	Hydraulic gradient I	Flow length X(m) ×10 ⁻²	length X ² (m ²) ×10 ⁻³	Volumetric flow Rate, Q(m ³ s ⁻¹) ×10 ⁻⁸	Volume flux, q (ms ⁻¹) ×10 ⁻⁴	Seepage Velocity q/Ø (ms ⁻¹) ×10 ⁻⁴	Mass Flux J (Kg m ⁻² s ⁻¹) ×10 ⁻¹	In J
60	17.30	24.70	1.430	5.175	2.916	8.672	4.313	14.325	4.420	-0.817
120	18.10	23.90	0.832	6.975	4.865	7.793	3.875	12.917	3.972	-0.923
180	18.55	23.45	0.614	7.988	6.380	6.693	3.328	11.094	3.411	-1.076
240	18.85	23.15	0.496	8.663	7.504	5.807	2.888	9.625	2.960	-1.217
300	19.05	22.95	0.428	9.113	8.304	5.090	2.531	8.438	2.595	-1.349
360	19.20	22.80	0.381	9.450	8.930	4.525	2.250	7.500	2.306	-1.467

Table 4.2: Experimentally determined values of hydraulic gradient i, specific discharge q,

Time (Sec.)	Hydraulic head of saltwater (cm)	Hydraulic Head of freshwater (cm)	Hydraulic gradient i	Flow length X(m) ×10 ⁻²	length X ² (m ²) ×10 ⁻³	Volumetric flow Rate, Q(m ³ s ⁻¹) ×10 ⁻⁸	Volume flux, q (ms ⁻¹) ×10 ⁻⁴	Seepage Velocity q/Ø (ms ⁻¹) ×10 ⁻⁴	Mass Flux J (Kg m ⁻² s ⁻¹) ×10 ⁻¹	In J
60	16.10	25.90	3.960	2.475	0.613	4.148	2.063	8.252	2.114	-1.554
120	16.60	25.40	2.444	3.600	1.300	4.022	2.000	8.000	2.050	-1.585
180	17.10	24.90	1.651	4.725	2.233	3.959	1.969	7.875	2.018	-1.601
240	17.60	24.40	1.162	5.850	3.422	3.922	1.950	7.800	1.999	-1.610
300	18.00	24.00	0.889	6.750	4.556	3.771	1.875	7.500	1.922	-1.649
360	18.30	23.70	0.727	7.425	5.513	3.555	1.768	7.072	1.812	-1.708

Table 4.3: Experimentally determined values of hydraulic gradient i, specific discharge q, seepage velocity, mass flux J and flow length x for sample C (Ø=0.330).

Time (Sec.)	Hydraulic head of saltwater (cm)	Hydraulic Head of freshwater (cm)	Hydraic gradient i	Flow length X(m) ×10 ⁻²	X (m) ×10 ⁻³	Volumetric flow Rate, Q(m s ⁻¹) ×10 ⁻⁸	Volume flux, q (ms ⁻¹) ×10 ⁻⁴	Seepage Velocity, q/Ø (ms ⁻¹) ×10 ⁻⁴	Mass Flux, J Kg m ⁻² s ⁻¹ ×10 ⁻¹	In J
60	18.50	23.50	0.635	7.875	6.202	13.192	6.560	19.879	6.724	-0.397
120	19.10	22.90	0.412	9.225	8.510	10.316	5.130	15.546	5.258	-0.643
180	19.50	22.50	0.296	10.125	10.252	8.484	4.219	12.784	4.324	-0.838
240	19.80	22.20	0.222	10.800	11.664	7.240	3.600	10.909	3.690	-0.997
300	19.95	22.05	0.190	11.138	12.404	6.222	3.094	9.375	3.171	-1.149
360	20.05	21.95	0.167	11.363	12.911	5.441	2.705	8.198	2.773	-1.283

Table 4.4: Experimentally determined values of hydraulic gradient i, specific discharge q, seepage velocity, mass flux J and flow length x for sample D (Ø=0.375).

Time (Sec.)	Hydraulic head of saltwater (cm)	Hydraulic Head of freshwater (cm)	Hydraulic gradient i	Flow length X(m) ×10 ⁻²	X (m) ×10 ⁻³	Volumetric flow Rate, Q(m s ⁻¹) ×10 ⁻⁸	Volume flux, q (ms ⁻¹) ×10 ⁻⁴	Seepage Velocity, q/Ø (ms ⁻¹) ×10 ⁻⁴	Mass Flux, J Kg m ⁻² s ⁻¹ ×10 ⁻¹	In J
60	18.96	23.04	0.480	8.889	7.901	14.896	7.407	19.752	7.592	-0.276
120	19.60	22.40	0.307	10.126	10.252	11.317	5.628	15.008	5.768	-0.550
180	19.86	22.14	0.218	10.914	11.912	9.148	4.549	12.131	4.663	-0.763
240	20.11	21.89	0.161	11.475	13.168	7.695	3.827	10.205	3.922	-0.936
300	20.26	21.74	0.132	11.815	13.959	6.603	3.283	8.755	3.365	-1.089
360	20.34	21.66	0.116	11.990	14.376	5.737	2.853	7.608	2.925	-1.229

Table 4.5: Experimentally determined values of hydraulic gradient i, specific discharge q, seepage velocity, mass flux J and flow length x for sample E (Ø=0.420).

Time (Sec.)	Hydraulic head of saltwater (cm)	Hydraulic Head of freshwater (cm)	Hydraulic gradient i	Flow length X(m) ×10 ⁻²	X (m) ×10 ⁻³	Volumetric flow Rate, Q(m s ⁻¹) ×10 ⁻⁸	Volume flux, q (ms ⁻¹) ×10 ⁻⁴	Seepage Velocity, q/Ø (ms ⁻¹) ×10 ⁻⁴	Mass Flux, J Kg m ⁻² s ⁻¹ ×10 ⁻¹	In J
60	19.40	22.60	0.323	9.900	9.801	16.591	8.250	19.643	8.456	-0.168
120	19.90	22.10	0.200	11.025	12.155	12.317	6.125	14.583	6.278	-0.466
180	20.20	21.80	0.137	11.700	13.369	9.804	4.875	11.607	4.997	-0.693
240	20.40	21.60	0.099	12.150	14.762	8.145	4.050	9.643	4.151	-0.879
300	20.55	21.45	0.072	12.488	15.594	6.976	3.469	8.259	3.556	-1.034
360	20.60	21.40	0.063	12.600	15.876	6.033	3.000	7.143	3.075	-1.179

Table 4.6: Values of porosities, hydraulic conductivities and diffusion coefficients for homogeneous samples A,B, C, D and E.

Samples	Porosity	Hydraulic Conductivity (m/s) $\times 10^{-5}$	Diffusion Coefficient (D) (m ² /s) $\times 10^{-5}$
A	0.250	0.570	1.600
B	0.300	1.870	1.900
C	0.330	7.960	2.200
D	0.375	11.700	2.300
E	0.420	17.500	2.800

Table 4.7: Values of Concentration of contaminant and Concentration gradient at different given time

Time (Sec.)	Concentration of contaminant (Kg/m ³) $\times 10^3$	Concentration gradient (Kg/m ⁴) $\times 10^3$
60	0.344	5.431
120	0.218	3.516
180	0.153	2.471
240	0.112	1.803
300	0.080	0.138
360	0.060	0.107

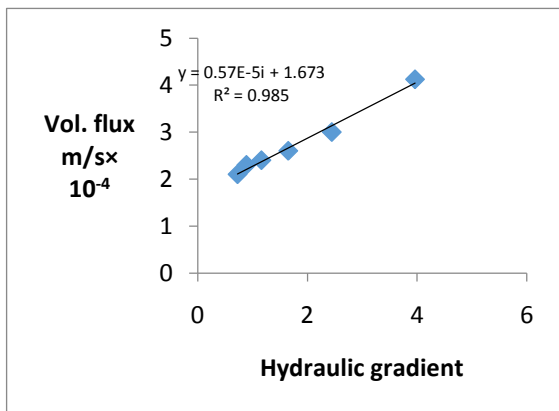


Figure 4.1: volume flux, q against hydraulic gradient, i (sample A)

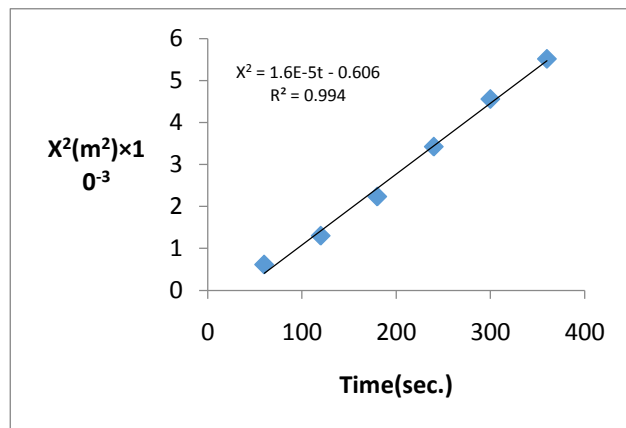


Figure 4.2: Square of flow length (x^2) against time T(Sec.) for sample A

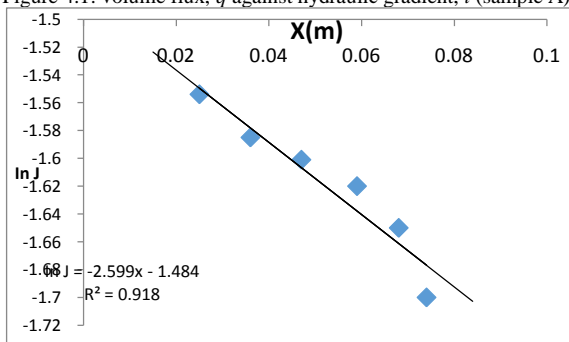


Figure 4.3: Natural log of mass flux (lnJ) against flow length X for sample A.

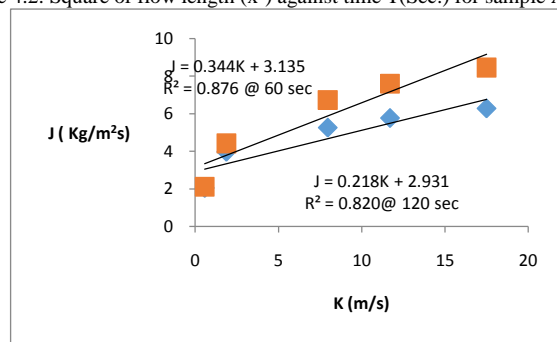


Figure 4.4: Mass flux (J) against the hydraulic conductivity (K) for all samples @ 60 sec. and 120 sec.

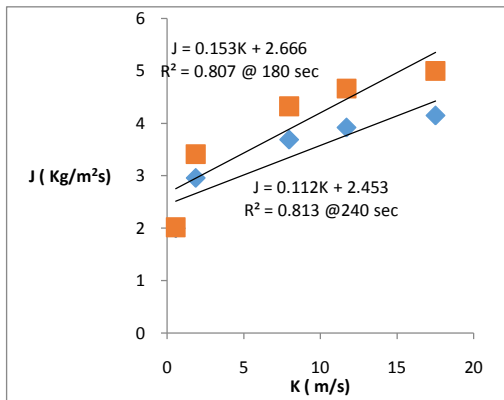


Figure 4.5: Mass flux (J) against the hydraulic conductivity (K) for all samples @ 180 sec. and 240 sec.

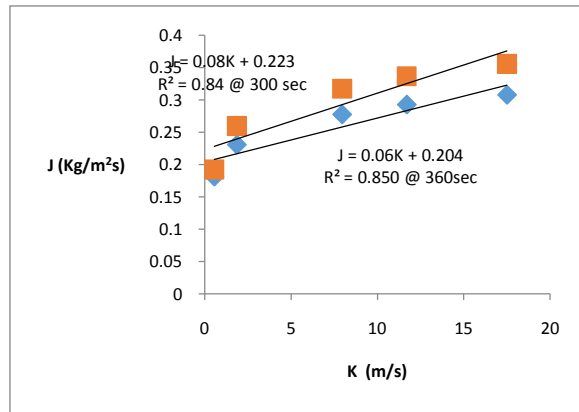


Figure 4.6: Mass flux (J) against the hydraulic conductivity (K) for all samples @ 300 sec. and 360 sec.

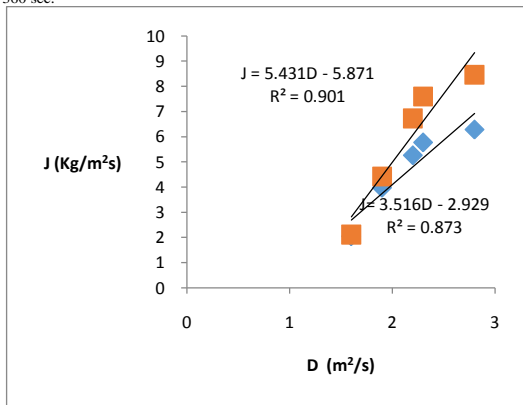


Figure 4.7: Mass flux (J) against the diffusion coefficient (D) for all samples @ 60 sec. and 120 sec.

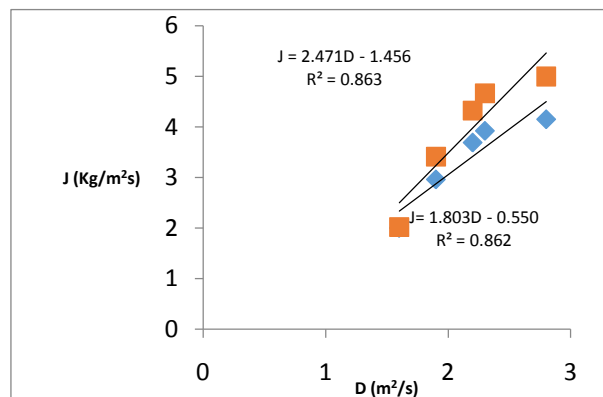


Figure 4.8: Mass flux (J) against the diffusion coefficient (D) for all samples @ 180 sec. and 240 sec.

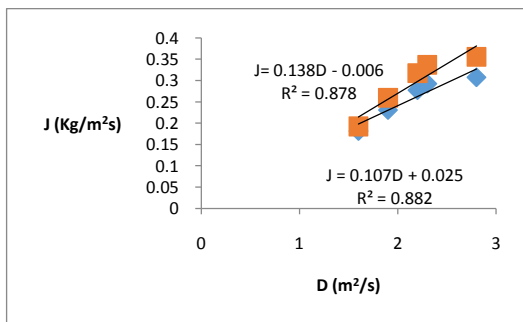


Figure 4.9: Mass flux (J) against the diffusion coefficient (D) for all samples @ 300 sec. and 360 sec.

5. CONCLUSION

The hydraulic conductivity and diffusion coefficient influence the flow of contaminant through the unit area of the coastal aquifer per unit time known as mass flux of contaminant. As the hydraulic conductivity and diffusion coefficient increase the attenuation coefficient of contaminant through the aquifer decreases. This means the flow of contaminant through the aquifer increases with high hydraulic conductivity as well as high diffusion coefficient. The concentration gradient and the concentration of contaminant decrease with time increase which means the driven force of contaminant known as concentration gradient decreases with increase in time. This leads to decrease in concentration of contaminant through the aquifer with time. Therefore, the aquifer with low hydraulic conductivity and diffusion coefficient will serve as a better attenuated material for flow of contaminant.

6. REFERENCES

- [1] Bear, J., Cheng, A.H.-D., Sorek, S., Ouazar, D. and Herrera, I. 1999. Seawater Intrusion in coastal aquifers - concepts, methods and practices. Kluwer Academic Publishers, Dordrecht, 640 pp.
- [2] Schwart, F.W. and Zhang, H 2003. Fundamentals of Groundwater. John Wiley & Sons, Inc., New York, 583 p.
- [3] McDonald R. J., Russill N. R. W., Miliorizos M. and Thomas J. W., 1998. A geophysical investigation of saline intrusion and geological structure beneath areas of tidal coastal wetland at Langstone Harbour, Hampshire, UK. In: Robins, N. S. (ed.) Groundwater Pollution, Aquifer Recharge and Vulnerability. Geological Society, London, Special Publications. 130: 77-94.
- [4] Bear, J. 2007. Hydraulics of groundwater. Dover Publications, Mineola, 569p.
- [5] Freeze, R.A. and Cherry, J.A. 1979: Groundwater. USA; Prentice Hall, Inc. 375 – 379.
- [6] Younger, P. L. 2007. Groundwater in the Environment: an introduction. Malden, M.A. Blackwell Publishing. 318p.
- [7] Adegoke, J.A. 2004. The effects of porosity, angle of hydrostatic equilibrium, volume flux and rate on groundwater flow through riverbed sand. Ph. D Thesis. Department of Physics. University of Ibadan (pages 25 and 26).
- [8] Nag, P.K. (2008). Heat and mass transfer, published by Tata McGraw-Hill publishing company limited.
- [9] Pekdeger A., Matthess G. and Schroter J. 1985; Protection of Groundwater against Pathogenic Bacteria and Viruses. Hydrogeology in the service of Man, Memoires of the 18th Congress of the International Association of Hydrogeologists, Cambridge, 1985.
- [10] Floppen J. W. A, M. van Herwerden and Schijven J. F., 2007. Transport of Escherichia coli in saturated porous media: dual mode deposition and intra-population heterogeneity. Water resources. DOI: 10.1016/j.watres.2006.12.041
- [11] Iwasaki, I., 1937: Some Notes on sand filtration, J. Amer. Water Works Assoc., 29: 1591-1609, Baltimore.
- [12] Ison C. R. and Ives K. J., 1969. Removal mechanisms in deep bed filtration. *Journal of Chemical Engineering Science*, Vol. 24, Issue 4, Pp.717 – 729.
- [13] Adegoke, J. A., Popoola, O. I. and Faluyi, O. O. (2019). Effect of geophysical parameters on attenuation of contaminant in freshwater coastal aquifers. *Journal of Engineering Studies and Research – Volume 25 No. 1*, 7-15.