*Journal of the Nigerian Association of Mathematical Physics*
***Volume 55* (February *2020 Issue*), pp67 –82**
***© J. of NAMP***

# TRANSPORTATION MODELING AND ANALYSIS FOR URBAN TRIP DISTRIBUTION USING INTERVAL TYPE-2 FUZZY LOGIC

***Umoh U. A[1]., Asuquo D. E.[2], Eyoh I. J.[3] and Isuekpe E. C.[4]***

**[1,2,3,4]Department of Computer Science, Faculty of Science, University of Uyo, Uyo**

***Abstract***

***This paper explores the potential capabilities of Interval Type-2 Fuzzy Logic (IT2FL) in urban trip distribution modeling (TDM) to estimate the volume of trip interaction between selected zones. Some objectives and constraints of transport processes are often difficult to be measured by crisp values because of the inherent imprecision and uncertainties of transportation data. An IT2FL model is designed and applied to model inter-city passenger flow. Six (6) urban centers in Akwa Ibom State, Nigeria, were selected for the study, which includes; Uyo, Eket, Oron, Ikot Ekpene, Abak and Ikot Abasi. The data on passengers' trips generated, passengers trips attracted and the distance used in the study were obtained from travel survey. The IT2FL model for trip distribution involves the operation of fuzzification which Gaussian membership function is employed, knowledge base which comprises rules and data base built to support the fuzzy inference using Mandani inference mechanism, K-Mendel algorithm is explored for type reduction process and defuzzification.. The study investigates gravity model as applied in urban trip distribution for the purpose of comparison. For the purpose of performance measure and model evaluation in this analysis, four goodness-of-fit statistics are used to compare the model performance. The four performance metrics - mean absolute error (MAE), mean square error (MSE), root mean square error (RMSE) and mean absolute percentage error (MAPE) - are applied and the results indicate that IT2FL model gives a better performance with lower error values of 10.2, 1.18, 0.14 and 0.22 for MAE, MSE, RMSE and MAPE, respectively compared to 11.66, 2.25, 0.18 and 0.32, respectively given by the gravity model. Thus, IT2FLS serves as a better model in managing transportation problem by aiding good decision for both travelers and transport planners.***

***Keywords:*** *IntervalFuzzy-2 Logic, Transportation Planning, Trip Generation, Trip distribution, Trip attraction*

## 1 INTRODUCTION

Transportation planning process is a wide human-oriented field with diverse and challenging problems waiting to be solved. Transportation distribution model (TDM) is an aspect of transportation process. It is an essential part in social, industrial and economical process which helps shape an economic health and provide for the mobility of people and goods. Effective transportation systems lead to the efficient movement of goods and people, which significantly contribute to the quality of life in every society. However, some objectives and constraints of transport processes are often difficult to be measured by crisp values because of the inherent imprecision and uncertainties of transportation data.

The TDM distributes the generated trip (TG), which is the decision to travel for a given purpose from each zone to all other zones based on the choice of destination. After determining trip productions (TPs) and trip attractions (TAs), TDM links the production with attraction in order to quantify how the trips produced in a zone are distributed among all other zones. Trips are distributed among destinations by employing a number of traditional approaches such growth factor model, gravity model, opportunity models and stochastic behavioural models. Growth factor model is a method which responds only to relative growth rates at origins and destinations and this is suitable for short-term trend extrapolation. The gravity model starts from assumptions about trip making behavior and the way it is influenced by external factors. Opportunity model is based on statistical theory of probability [1,2,3]. However, all the traditional methods do not take into consideration the inherent uncertainties associated with trip distribution data.

Corresponding Author**:** Umoh U.A., Email: uduakumoh@uniuyo.edu.ng, Tel: +2348122064911

It also influences patterns of growth and economic activity. Trip distribution models are used to determine the number of trips between pairs of zones when the number of trips generated attracted by particular zones is known. Thus, the prediction of trip distribution involves the prediction of flows in a network regardless of a possible transportation mode or travel route [4]. In the real world, most of the trip distribution data are characterized by uncertainty, ambiguity or imprecise sets of information [5,6,7,8], which can be difficult to translate into numerical values, called crisp values whereby simplified mathematical models are not adequate for their analysis There is a need for more advanced methods and models in order to analyze the causality, coupling, feedback loops, and chaotic behaviour involved in transportation problem situations.

Techniques based on intelligent computing such as Fuzzy logic (FL) is useful in trip distribution modeling because it is flexible enough to deal with complex systems, offers acceptable and adequate approximate solutions. The use of fuzzy set theory in real-world problems is very promising in modeling spatial interactions in transport planning because of its ability to tolerate imprecision, uncertainty and lack of information to achieve robustness in decision making with low cost [9,10,11]. The interval type-2 fuzzy logic (IT2FL) has the ability to cope well with transportation data.

In this work, an IT2F intelligent framework is designed and applied to model inter-city passenger flow. Six (6) urban centers in Akwa Ibom State, Nigeria, are selected for the study, which includes; Uyo, Eket, Oron, Ikot Ekpene, Abak and Ikot Abasi. Uyo is the capital city of the state. It is located on the Northwest of the state, extending from latitude $7^047^1$ and $8^003^1$north and from longitude $4^052^1$ to $5^007^1$ East. Uyo is the foremost commercial center in Akwa Ibom State and it is also the transport nerve center of the state [6]. Arterial roads radiating from the city, link it with all corners of the state. The landscape of the city comprises industrial, commercial, recreation, education, transport and residential land uses. Abak urban is located on latitudes $4^053^1$ to $5^007^1$ north and longitude $7^040^1$ to $7^050^1$ East. Ikot Ekpeneis located on latitudes $5^07^1$ to $5^015^1$ north and longitude $7^041^1$ to $7^045^1$East.Eket urban center is located on latitudes $4^033^1$ to $4^040^1$ north and longitude $7^047^1$ to $8^002^1$ East. Oron Urban center is located on latitudes $4^045^1$ to $4^052^1$ north and between longitudes $8^010^1$ to $8^025^1$East. Ikot Abasi is located on latitudes $4^030^1$ to $4^040^1$ north and longitude $7^035^1$ to $7^045^1$East. It serves as administrative headquarters of Ikot Abasi Local Government Area.

The data on passengers trips generated, passengers trips attracted and the distance used in the study are obtained from travel survey. The IT2F intelligent framework involves the operation of fuzzification which Gaussian membership function is employed, knowledge base which comprises rules and data base which is built to support the fuzzy inference using Mandani inference mechanism, Karnik-Mendel algorithm is explored for type reduction process and defuzzification. IT2FLS as applied in the study gives satisfactory and better results when compared with the existing gravity model and can be applicable in current and future scenarios.

The remainder of this paper is as follows; Section 2 presents literature support while Section 3 gives research methodology. Result and discussion are presented in Section 4. Section 5 gives conclusion and references are presented in Section 6.

## 2 THEORITICAL ANALYSIS

Transportation planning process takes community needs and expectations into consideration and establishes a way of designing future transportation systems [12]. One of the most important stages of transportation planning process is forecasting future travel demand in a desired level of accuracy. A number of deterministic and stochastic models have been developed to understand travel behaviour better and to achieve more accurate forecasts. The planning processes requires developing strategies for operating, managing, maintaining, and financing the area's transportation system in such a way as to advance the area's long-term goals. The traditional urban travel demand modeling consists of a sequential procedure often referred to as the 'four-step' modeling process: *trip generation, trip distribution, modal choice and trip assignment* [13,14,15,16]. A study on trip distribution modeling using mobile phone data to derive country-wide mobility trends was carried out. A set of doubly constrained trip distribution models that integrates intra-zonal trips was estimated for 123-district-level traffic analysis zones trip generation in Budapest was studied. The paper describes the model structure and explains what observations were made, what the key findings were in terms of transportation characteristics and what decisions led to the applied model components[17,18].

A conceptual framework for trip forecasting problem to map social and demographic variables to total number of trips between Origin-Destination (OD) pairs is studied. The framework comprises two (2) input variables, trip generation and trip attraction, a transfer function for mapping trip attraction and generation variables to number of trips between two given regions and an output space. The study employs a fuzzy inference system to map input space to output space. A Poisson regression and gravity models were investigated to specify and estimate trip distribution models between railway stations by considering the most influential demand variables [19].

The type-1 fuzzy logic (T1FL) is developed to deal with reasoning that is approximate rather than fixed and exact. It has rapidly and successfully applied in sophisticated control systems and decision making systems because of its ability to handle vague, uncertain and imprecise information [20].The concept of IT2FLS was introduced [21]. AnIT2F set is characterized by a fuzzy membership value where each element of this set is a fuzzy number with an interval in [0, 1] and can deal with the uncertainties about the fuzzy membership value itself. The uncertainty in data can be represented by the footprint of uncertainty (FOU).The IT2FL has the ability to handle uncertainty adequately than its T1FL counterpart with ability, reliability, capability and robustness and has been applied to different fields of human existence.

In [22], presents basic premises of fuzzy logic systems as well as a detailed analysis of fuzzy logic systems developed to solve various traffic and transportation planning problems. The study investigates the importance of fuzzy logic model as a universal approximator in solving complex traffic and transportation problems.

### 2.1 Overview of IT2FL

The structure of the interval type-2 fuzzy logic model is made up of five (5) components namely; Fuzzification- maps the crisp input to a type-2 fuzzy set using a Gaussian membership function. Inference Engine - evaluates the rules in a rule base against type-2 fuzzy set gotten from fuzzification to produce yet another type-2 fuzzy set. Type reducer - uses Karnik-Mendel algorithm to reduce an interval type-2 fuzzy set to type-1 fuzzy set. Defuzzification - maps the fuzzy set to a crisp output using center of gravity defuzzification method. Knowledge Base – comprises rule base and membership functions used by the inference engine. Membership Function - a mathematical equation that helps the fuzzification module converts the crisp input into a fuzzy set.
An interval type-2 fuzzy set (IT2 FS) Ã is characterized by a membership interval in the universe of discourse X as;

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (1)$$

$$\tilde{A} = \sum_{i=1}^{P}\left[\sum_{u \in Jx}[1/u]\right]/x_i \quad (2)$$

Where *x*, the *primary variable*, has domain *X*; *u*∈*U*, the *secondary variable*, has domain $J_x$at each *x*∈*X;*$J_x$ is called the primary membership of x and the secondary grades of Ã all equal 1. That is, $\mu_{\tilde{A}}(x, u)$ is a type-1 fuzzy set known as the secondary in which $0 \le \mu_{\tilde{A}}(x, u) \le 1$. $J_x \subseteq [0, 1]$ is the primary membership of x[19-20]. Uncertainty about Ã is conveyed by the union of all the primary memberships, which is called the *footprint of uncertainty* (FOU) of *A*, encompassing all the embedded primary membership functions $J_x$ of Ã as shown in 3.

$$\mu_{\tilde{A}}(x,u) = 1, \; FOU\,(\tilde{A}) = \cup_{\forall x \in X} J_x = \{(x,u): u \in Jx \subseteq [0,1]\} \quad (3)$$

FOU(Ã) is bounded by *upper membership function* (UMF) $\bar{\mu}_{\tilde{A}}(x)$ and *lower membership function* (LMF) $\underline{\mu}_{\tilde{A}}(x)$, $\forall x \in X$, respectively assuming minimum and maximum of the membership functions of the embedded T1FSs in the FOU

$$\bar{\mu}_{\tilde{A}}(x) \equiv \overline{FOU\,(\tilde{A})} \quad \forall x \in X \quad (4)$$

$$\underline{\mu}_{\tilde{A}}(x) \equiv \underline{FOU\,(\tilde{A})} \quad \forall x \in X \quad (5)$$

For IT2FS, $J_x = [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)], \forall x \in X$ (6)

Fuzzification maps inputs to fuzzy values using membership function (MF). The study uses IT2 Gaussian membership function (GMF) because of its suitability for a highly dynamic random system, example in transportation planning with fixed mean, *c* and uncertain standard deviation, $\sigma$ to calculate the degree of membership of the input variables.

$$\mu_A(x) = \exp\left(-\frac{(x-w)^2}{2\sigma^2}\right) \; \sigma \in [\sigma_1, \sigma_2] \quad (7)$$

The upper and lower membership functions are calculated using:

$$\bar{\mu}_{\tilde{A}_{im}}(x_i) = \exp\left(-\frac{(x_i - w_{im})^2}{2\bar{\sigma}_{2,im}^2}\right), \; \bar{\mu}_{\tilde{A}}(x) = N(w, \sigma_2; x) \quad (8)$$

$$\underline{\mu}_{\tilde{A}_{im}}(x_i) = \exp\left(-\frac{(x_i - w_{im})^2}{2\underline{\sigma}_{1,im}^2}\right), \; \underline{\mu}_{\tilde{A}}(x) = N(w, \sigma_1; x) \quad (9)$$

Where *w* is the center (mean) of the MF, σ is the standard deviation of the MF and *x* is the input vector. The variables $\bar{\sigma}_{2,im}$ and $\underline{\sigma}_{1,im}$ are premise parameters that define the degree of membership of each element to the fuzzy set Ã and FOUs of the IT2FS. The detail description is found in [19,21].

For an IT2FLS with *m* inputs and *one* output have the if-then rules specified as in equation (10).

$$IF x_i is \tilde{D}_i^l AND, \ldots, AND x_m is \tilde{D}_m^l THEN y is \tilde{E}^l \quad (10)$$

Where$x_i$, *i = 1,…, m* are the antecedents, $y_j$, *j = 1,..., n* are the consequents of the *lth* rule, *l* = 1,…, P of IT2FLS. The $\tilde{D}^i$'s are the MFs $\mu_{\tilde{D}_i^l}(x_i)$ of the antecedent part assigned to the *i*th input $x_i$, The $E^l$'s are the MFs $\mu_{\tilde{E}_j^l}(y_j)$ of the consequent part assigned to the *j*th output $y_j$.
The inference engine combines the fired rules and gives a mapping from input IT2FSs to output IT2FSs. The firing intervals for lower and upper membership functions are evaluated using [22] as;

$$F^i(x^i) = \left[\underline{\mu}_{\tilde{f}_1^i}(x_1') * \ldots * \underline{\mu}_{\tilde{f}_m^i}(x_1')\right], [\bar{\mu}_{\tilde{f}_1^i}(x_1') * \ldots * \bar{\mu}_{\tilde{f}_m^i}(x_1')] \equiv \left[\underline{f}^i, \bar{f}^i\right], \; i = 1, 2, .., M] \quad (11)$$

Where $F^i(x^i)$ is the antecedent of rule *i* and $\mu_{F1}{}^i(x')$ is the degree of membership of *x* in F. $\bar{\mu}_{\tilde{f}_1^i}(x)$ and $\underline{\mu}_{\tilde{f}_m^i}(x)$ are upper and lower MFs of $\mu_{f^i}$.
In IT2FLS, an exact iterative method of type reduction is performed by combining $F^i(x')$ and it consequent to compute the centroid of anIT2FSto produce type-reduced sets (T1FS) using (11) (12) and (13) respectively.

$$Y_{TR}(x') = [y_l(x'), y_r(x')] \equiv [y_l, y_r] = \cup_{\substack{f^i \in F^i(x') \\ y^i \in Y^i}} \frac{\sum_{i=1}^N f^i y^i}{\sum_{i=1}^N f^i} \quad (12)$$

$$y_l = \min_{L\in[1,N-1]} l \frac{\sum_{n=1}^{L} \bar{f}^n \underline{y}^n + \sum_{n=L+1}^{N} \underline{f}^n \underline{y}^n}{\sum_{n=1}^{L} \bar{f}^n + \sum_{n=L+1}^{N} \underline{f}^n} = \frac{\sum_{i=1}^{N} f_l^i y_l^i}{\sum_{i=1}^{N} f_l^i} \quad (13)$$

$$y_r = \max_{R\in[1,N-1]} r \frac{\sum_{n=1}^{R} \underline{f}^n \bar{y}^n + \sum_{n=R+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{R} \underline{f}^n + \sum_{n=R+1}^{N} \bar{f}^n} = \frac{\sum_{i=1}^{N} f_r^i y_r^i}{\sum_{i=1}^{N} f_r^i} \quad (14)$$

Where $y_l^i$ and $y_r^i$ are the left and right end points of the centroid of the consequent of the *i*th rule while $\underline{f}^i$ and $\overline{f}^i$ are the lower and upper firing degrees of the *i*th rule and *N* is the number of fired rules.
Defuzzification of the interval set is performed using the average of $y_{lk}$ *and* $y_{rk}$, and the defuzzified crisp output for each output *k* is achieved;

$$Y(X) = \frac{y_l + y_r}{2} \quad (15)$$

## 3. EXPERIMENTAL WORK

### 3.1 Data Collection and Analysis

In this paper, six urban centers in Akwa Ibom State are selected for the study. They are Uyo (UY), Abak (AB), Ikot Ekpene (IE), Oron (OR), Eket (EK) and Ikot Abasi (IA). These are notable important urban centers in the State not only because they are local government headquarters administratively accorded the status of urban centers but many development factors have been observed to be the underlying forces responsible for their physical and spatial development. These development factors include; high population and greater number of socio-economic activities centers.
Field assistants are trained and used for the survey exercise. The survey is conducted simultaneously in the six urban centers for a period of six days, for a week (Monday to Saturday), during the hours of 7am-9am; 12-2pm and 4-6pm. These hours are determined as peak hours during the pilot survey. The survey is conducted precisely during the week beginning Monday 10th October 2016 to Saturday 15th October, 2016. These periods were considered most suitable because it falls within the transition period between the hot dry season and the wet rainy season. Temperature and other related climatic factors are quite conducive for trip makers. It is ensured that the period of the survey is devoid of any local or national festivals and holidays.
A survey is carried out using one thousand eight hundred and fifty-six (1,856) copies of questionnaire coded. The parameters used in the survey are *total trip production, total trip Attraction, Distance* and *Trip Interaction*. The dependent variables are the types of trips generated/attracted in the six selected urban centers and the distance between zones. The list of dependent(Trip production) and independent(Trip Attraction) variables used in this study is presented as follows: *Work trip, School trip, Recreation Trip, Religion Trip, Medical Trip, Wedding Trip, Burial Trip, Market Trip, Personal Business Trip, Visit to friends and relations, Others Trip.* Attraction variables are socio-economic development indicators in the six(6) selected urban centers namely: *Number of tertiary institutions, Number of major Market, Number of Hospitals, Number of Industries, Number of tourist sites.*
Data collected from daily trip generation in the six (6) zones are analyzed using frequency distribution model. The Trip Distribution Matrix for Urban Distribution Modeling is presented in table 1.The summary of trip generation/production indicating the purpose and volume of trips generated with the corresponding values in percentage is summarized is in Table 2.The summary of analysis of the data collected for trip generation with their volume and the trip attraction including distance for all the six(6) selected zones is presented in Table 3.

**Table 1: The Trip Distribution Matrix for Urban Distribution Modeling**

| | | Destination | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Origin | | Uyo | Abak | Ikot Ekpene | Oron | Eket | Ikot Abasi | Total |
| | UY | - | 150 | 142 | 60 | 155 | 56 | 565 |
| | AB | 155 | - | 52 | 8 | 72 | 32 | 319 |
| | IE | 137 | 29 | - | 35 | 50 | 28 | 283 |
| OR | | 143 | 0 | 34 | - | 88 | 23 | 288 |
| EK | | 170 | 53 | 64 | 26 | - | 32 | 345 |
| IA | | 35 | 2 | 0 | 0 | 20 | - | 57 |
| | Total | 644 | 234 | 292 | 129 | 385 | 171 | 1856 |

**Table 2: Summary of Trip Purpose and Volume of Trip**

| S/N | Purpose of Trips | Volume of Trips | Percentage Total |
|---|---|---|---|
| 1 | School Trip | 373 | 20% |
| 2 | Work Trip | 254 | 13.70% |
| 3 | Visit to Friends | 248 | 13.40% |
| 4 | Business Trips | 234 | 12.60% |
| 5 | Recreation Trips | 179 | 9.60% |
| 6 | Medical Trips | 146 | 8.00% |
| 7 | Religious Trips | 117 | 6.30% |
| 8 | Burial Trips | 104 | 5.60% |
| 9 | Market Trips | 97 | 5.20% |
| 10 | Wedding Trips | 89 | 4.80% |
| 11 | Other Trips | 15 | 0.80% |
| | Total | 1856 | 100.00% |

**Table 3: Summary of Data Analysis Collected for Production and Attraction Variables**

| | | Data Obtained for Generation and Attraction Variables | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Purpose of Trip | | | | | | | | | | | | | | | | Socioeconomic Developm | | | | | |
| S/N | Origin/ Destination of Inter-City Trip | Work | School | Recreation | Medical | Market | Business | Visit To Friend | Religion | Wedding | Burial | Total | % | Others | Distance (Km) | Trip Population | No. of Sch. | No. of Market | No of Hosp. | No of Indus. | Tourism | Church | Total |
| 1 | UY-AB | 45 | 61 | 24 | 27 | 17 | 32 | 40 | 20 | 17 | 22 | 305 | 16.4 | 0 | 20.1 | 139 | 7 | 6 | 4 | 4 | 4 | 10 | 177.6 |
| **2** | **UY-IK** | **44** | **85** | **17** | **15** | **20** | **18** | **20** | **10** | **9** | **10** | **250** | **15.0** | **2** | **28.2** | **176** | | | | | | | |
| 3 | UY-OR | 22 | 31 | 16 | 17 | 14 | 19 | 27 | 22 | 13 | 16 | 203 | 11.0 | 6 | 56.8 | 173 | | | | | | | |
| 4 | UY-IA | 8 | 31 | 8 | 4 | 8 | 7 | 18 | 3 | 10 | 4 | 101 | 5.0 | 0 | 72.7 | 127 | | | | | | | |
| 5 | UY-EK | 33 | 37 | 45 | 18 | 23 | 27 | 42 | 30 | 24 | 22 | 325 | 17.5 | 4 | 59.0 | 160 | | | | | | | |
| 6 | AB-IE | 6 | 26 | 5 | 9 | 2 | 9 | 15 | 4 | 1 | 5 | 82 | 4.42 | 0 | 21.3 | 70 | | | | | | | |
| 7 | AB-OR | 0 | 3 | 0 | 1 | 0 | 3 | 1 | 0 | 0 | 0 | 8 | 0.43 | 0 | 37.2 | 76 | | | | | | | |
| 8 | AB-IA | 11 | 0 | 4 | 3 | 1 | 5 | 8 | 2 | 0 | 0 | 34 | 1.80 | 0 | 37.2 | 40 | | | | | | | |
| 9 | B-EK | 15 | 12 | 18 | 11 | 5 | 18 | 18 | 10 | 5 | 12 | 125 | 6.7 | 1 | 93.7 | 63 | | | | | | | |
| 10 | IK-OR | 8 | 16 | 3 | 6 | 2 | 18 | 10 | 3 | 2 | 1 | 69 | 3.7 | 0 | 48.2 | 113 | | | | | | | |
| 11 | IE-IA | 8 | 2 | 3 | 1 | 2 | 6 | 3 | 2 | 0 | 1 | 28 | 1.5 | 0 | 82.2 | 77 | | | | | | | |
| 12 | IE-EK | 25 | 23 | 12 | 10 | 1 | 12 | 21 | 4 | 6 | 9 | 124 | 6.2 | 1 | 84.7 | 100 | | | | | | | |
| 13 | OR-EK | 30 | 17 | 10 | 9 | 4 | 38 | 8 | 3 | 2 | 1 | 123 | 6.2 | 1 | 42.3 | 97 | | | | | | | |
| 14 | OR-IA | 6 | 0 | 3 | 2 | 1 | 5 | 1 | 2 | 2 | 1 | 23 | 1.2 | 0 | 87.6 | 74 | | | | | | | |
| 15 | IA-EK | 12 | 0 | 11 | 3 | 1 | 17 | 10 | 2 | 0 | 0 | 56 | 3.0 | 0 | 45.3 | 61 | | | | | | | |
| **TOTAL** | | **254** | **373** | **179** | **146** | **97** | **234** | **248** | **117** | **89** | **104** | **1856** | | | | | | | | | | | |
| % | | 13.7 | 20.0 | 9.6 | 8.0 | 5.2 | 12.6 | 13.4 | 6.3 | 4.8 | 5.6 | 100 | | | | | | | | | | | |

## 3.2 Gravity Model for Urban Trip Distribution

The study investigates gravity model as applied in urban trip distribution for the purpose of comparison. In gravity model, trip making start from assumptions and it is influenced by external factors. The general form of gravity model is given in (16):

$$T_{ij} = P_i \frac{A_i F_{ij} K_{ij}}{\sum A_j F_{ij} K_{ij}} \qquad (16)$$

where, $T_{ij}$ is the total number of trips between zones,*i* and *j*, $P_i$ and $A_j$ are the total number of trips produced in and attracted to zones *i* and *j*, $F_{ij}$ is the friction function that exist between zones (usually travel time, distance or), and $K_{ij}$ is socioeconomic adjustment factor for trips produced in *i* and attracted to *j*. (The socioeconomic factor, K, can be a matrix of values rather than just one value).

Before the gravity model can be used for prediction of future travel demand, it must be calibrated. Calibration is accomplished by adjusting the various factors within the gravity model until the model can duplicate a known base year's trip distribution. The number of trips between 2 zones is directly proportional to the number of trip attractions at the destination zone and inversely proportional a function of the travel time. In this paper, the number of trips between each zone is determined using (16) and presented in Table 4.The travel time in Table 4 is represented in a matrix form as shown in Table 5.Trip production and trip attraction variables are extracted from Table 4 and displayed in a matrix form as shown Table 6.The trip interaction between the 6 zones is predicted using gravity model and presented in Table 7.

**Table 4: Observed Inter-City Trip Production, Trip Attraction, Trip Distance**

| S/N | Urban Centers (ij) | Distance (Km) | Travel Time (miniute)$F_{ij}$ | Trip Production | Trip Attraction | Friction Factor($K_{ij}$) |
|---|---|---|---|---|---|---|
| 1 | UY-AB | 20.1 | 25 | 305 | 177.6 | 1.0 |
| 2 | UY-IK | 28.2 | 30 | 250 | 288.2 | 1.0 |
| 3 | UY-OR | 56.8 | 63 | 203 | 254.8 | 1.0 |
| 4 | UY-IA | 72.7 | 84 | 101 | 231.7 | 1.0 |
| 5 | UY-EK | 59.0 | 58 | 325 | 244.5 | 0.3 |
| 6 | AB-IE | 21.3 | 16 | 82 | 99.8 | 1.0 |
| 7 | AB-OR | 79.8 | 87 | 8 | 122.9 | 1.0 |
| 8 | AB-IA | 67.4 | 75 | 34 | 74.7 | 1.0 |
| 9 | B-EK | 93.7 | 87 | 125 | 167.7 | 0.3 |
| 10 | IK-OR | 82.5 | 85 | 69 | 176.2 | 1.0 |
| 11 | IE-IA | 82.2 | 87 | 28 | 171.2 | 1.0 |
| 12 | IE-EK | 84.7 | 81 | 124 | 200.2 | 0.5 |
| 13 | OR-EK | 42.3 | 39 | 123 | 155.8 | 1.0 |
| 14 | OR-IA | 87.6 | 89 | 23 | 174.6 | 1.0 |
| 15 | IA-EK | 45.3 | 50 | 56 | 119.8 | 3.0 |

**Table 5: The Travel Time zone (Min) $F_{ij}$Matrix**

| Urban Centers | Uyo | Abak | Ikot Ekpene | Oron | Eket | Ikot Abasi |
|---|---|---|---|---|---|---|
| Uyo | - | 25 | 30 | 63 | 58 | 84 |
| Abak | 25 | - | 16 | 34 | 87 | 34 |
| Ikot Ekpene | 30 | 16 | - | 55 | 81 | 87 |
| Oron | 63 | 34 | 55 | - | 39 | 89 |
| Eket | 58 | 87 | 81 | 39 | - | 50 |
| Ikot Abasi | 84 | 34 | 87 | 89 | 50 | - |

**Table 6: Trip production and Attraction in the studied zone ($P_i$, $A_j$)**

| Parameter | Uyo | Abak | Ikot Ekpene | Oron | Eket | Ikot Abasi | Total |
|---|---|---|---|---|---|---|---|
| Zone | 1 | 2 | 3 | 4 | 5 | 6 | |
| Trip Production | 565 | 319 | 283 | 288 | 345 | 57 | 1856 |
| Trip Attraction | 644 | 234 | 292 | 129 | 385 | 171 | 1856 |

**Table 7: Observed Inter-City Gravity concept prediction estimate**

| S/N | Urban Centers (ij) | Gravity Model-Predicted Volume |
|---|---|---|
| 1 | UY-AB | 56 |
| 2 | UY-IK | 83. |
| 3 | UY-OR | 63 |
| 4 | UY-IA | 68 |
| 5 | UY-EK | 63 |
| 6 | AB-IE | 23 |
| 7 | AB-OR | 21 |
| 8 | AB-IA | 29 |
| 9 | B-EK | 50 |
| 10 | IK-OR | 26 |
| 11 | IE-IA | 55 |
| 12 | IE-EK | 58 |
| 13 | OR-EK | 59 |
| 14 | OR-IA | 63 |
| 15 | IA-EK | 27 |

### 3.2 Interval Type-2 Fuzzy Logic Model for Urban Trip Distribution

In this paper, an urban trip distribution modeling is proposed using interval type-2 fuzzy logic approach in order to improve trip interaction between zones. The choice of IT2FLS lies in its ability in handling adequately the inherent uncertainties associated with trip distribution variables. The problem considered in this paper is the control of urban trip distribution parameters in transportation planning in order to achieve a better trip interaction between zones. Designing an interval type-2 FLS for TDM includes collecting the knowledge, setting the rules, choosing and defining antecedent and consequent membership functions, choosing type-reduction, and extracting decision boundaries.

In order to achieve our objective, a survey is carried out using one thousand eight hundred and fifty-six (1,856) copies of questionnaire coded and the analyzed data is presented in Table 3.The universe of discourse is defined and presented in Table 8. Fuzzy sets of the inputs and output variables and their associated values and labels are defined as shown in Table 9. The structure of interval type-2 fuzzy for urban trip distribution modeling is shown in Figure 1.In this stage, firstly, fuzzification is carried based on equations (4) (5) and (6) respectively.

Table 8: Domain Intervals of Input and Output Variables

| Variables | Lower Bound | Upper Bound |
|---|---|---|
| Trip production | 0 | 350 |
| Trip attraction | 0 | 300 |
| Distance | 0 | 100 |
| Trip Interaction | 0 | 100 |

**Table 9: IT2FL Input's/Output's Universe of Discourse for Urban trip Distribution**

| Variable | | Mean Center (c) | Standard Deviation (σ) | |
|---|---|---|---|---|
| | | | Upper | Lower |
| Production 0-350 | VL | 39.06 | 12.4 | 7.33 |
| | L | 94.64 | 14.1 | 9.47 |
| | M | 151.4 | 13.8 | 8.02 |
| | H | 214.8 | 15.9 | 10.14 |
| | VH | 276.0 | 14.07 | 9.17 |
| Attraction 0-300 | VL | 29.72 | 8.97 | 4.26 |
| | L | 74.47 | 12.7 | 7.55 |
| | M | 122 | 10.7 | 5.89 |
| | H | 166 | 12.15 | 7.13 |
| | VH | 216.9 | 12.2 | 7.58 |
| Distance 0-100% | S | 14.95 | 4.648 | 2.76 |
| | M | 39.17 | 6.53 | 4.51 |
| | L | 68.8 | 10.72 | 7.77 |
| Interaction(Output) 0-100% | VL | 4 | 10.62 | 7.4 |
| | L | 25 | 10.62 | 7.61 |
| | M | 50 | 10.62 | 7.18 |
| | H | 75 | 10.62 | 7.66 |
| | VH | 100 | 10.62 | 7.31 |

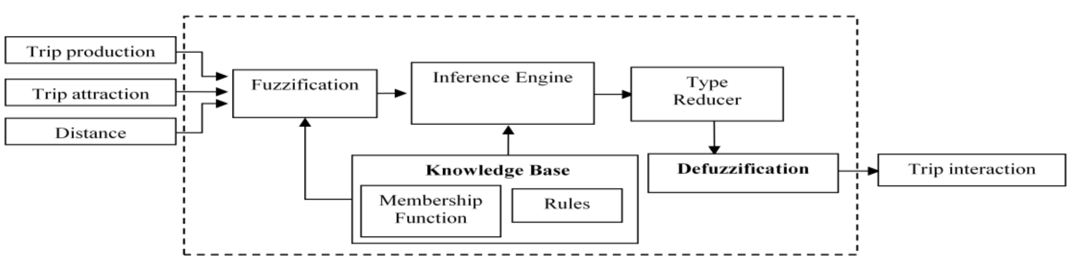


Figure 1: Interval Type-2 Fuzzy Logic Model for Urban Trip Distribution

The fuzzification is carried out based on equations (7) (8) and (9) respectively. In this stage, for each input and output variable selected, four membership functions (MFs) are defined namely, total trip production, total trip attraction, distance and trip interaction. A qualitative category is defined for each of the variable. These categories are called fuzzy term which are; very low (VL), low (L), medium (M), high (H), very high (VH) for trip production, trip attraction and trip interchange. Also, Short (S), Medium (M) and Long (L) are defined for trip distance. Gaussian membership function (MF) is employed in this paper to determine MFs (both upper and lower MFs values). For this reason, at least two points (σ, c) are needed to define one MF of one variable. The mathematical evaluations of IT2FL MF for the input variables are presented as follows:

Membership function for Production

$$\mu_{vL}(x,[7.33,12.4],[39.06,39.06]) = e^{\frac{-1}{2}\left(\frac{x-[39.06,39.06]}{[7.33,12.4]}\right)^2}$$
$$\mu_{L}(x,[9.47,14.1],[94.64,94.64]) = e^{\frac{-1}{2}\left(\frac{x-[94.64,94.64]}{[9.47,14.1]}\right)^2}$$
$$\mu_{M}(x,[8.02\ ,13.8],[157.4,157.4]) = e^{\frac{-1}{2}\left(\frac{x-[157.4,157.4]}{[8.02\ ,13.8]}\right)^2} \quad (17)$$
$$\mu_{H}(x,[10.14,15.9],[214.8,214.8]) = e^{\frac{-1}{2}\left(\frac{x-[214.8,214.8]}{[10.14,15.9]}\right)^2}$$
$$\mu_{VH}(x,[9.17,14.07],[276.0,276.0]) = e^{\frac{-1}{2}\left(\frac{x-[276.0,276.0]}{[9.17,14.07]}\right)^2}$$

Membership function for Attraction

$$\mu_{vL}(x,[4.36,8.97],[29.72,29.72]) = e^{\frac{-1}{2}\left(\frac{x-[29.72,29.72]}{[4.36,8.97]}\right)^2}$$
$$\mu_{L}(x,[7.55,12,7],[74.47,74.47]) = e^{\frac{-1}{2}\left(\frac{x-[74.47,74.47]}{[7.55,12,7]}\right)^2}$$
$$\mu_{M}(x,[5.89,10.2],[122,122]) = e^{\frac{-1}{2}\left(\frac{x-[122,122]}{[5.89,10.2]}\right)^2} \quad (18)$$
$$\mu_{H}(x,[7.13,12.15],[166,166]) = e^{\frac{-1}{2}\left(\frac{x-[166,166]}{[7.13,12.15]}\right)^2}$$
$$\mu_{VH}(x,[7.55,12.2],[216.9,216.9]) = e^{\frac{-1}{2}\left(\frac{x-[216.9,216.9]}{[7.55,12.2]}\right)^2}$$

Membership function for Distance

$$\mu_{s}(x,[2.76,4.64],[14.95,14.95]) = e^{\frac{-1}{2}\left(\frac{x-[14.95,14.95]}{[2.76,4.64]}\right)^2}$$
$$\mu_{M}(x,[4.51,6.53],[39.17,39.17]) = e^{\frac{-1}{2}\left(\frac{x-[39.17,39.17]}{[4.51,6.53]}\right)^2} \quad (19)$$
$$\mu_{L}(x,[7.77,10.72],[68.8,68.8]) = e^{\frac{-1}{2}\left(\frac{x-[68.8,68.8]}{[7.77,10.72]}\right)^2}$$

Fuzzy logic toolbox in 7.5.0 is used to plot the MFs for the linguistic variables for total trip production, total trip attraction, trip distant, and trip interaction respectively. Membership Matrix is evaluated to show the degree of membership of crisp inputs at various levels in the interval type-2 input MF defined on the three (3) input variables, for total production, total attraction and the distance between paired zones for the IT2FS. Tables 10 -12 present membership Matrix for total production, total attraction and distance respectively.74 fuzzy rules are defined by the experts using (10) and are shown in part in Figure 2, where, total trip production, total trip attraction, distance and trip Interaction are linguistic variables. The input and output linguistic term) are determined by fuzzy sets on the defined universe of discourse as Very Low, Low, Medium, High, Very High, Short, Medium and Long respectively. Inference Engine mechanism is performed (11).Given the crisp input vector v = [250, 107, 40], their degree of membership are calculated from respective Gaussian MFs and the fuzzified values for the three (3) input parameters are shown in Table 13. Inference engine evaluates rules 59, 34, 35, 10 against the fuzzy set in Table 6and the result is given in Table 14.A type-reduction is computed using (12) (13 and (14) respectively. Finally, we perform defuzzification using the average of the two end points, $y_l$and $y_r$ in (15) and obtain crisp value (Trip Interaction).

**Table 10: Membership matrix for production**

| Fuzzy Set | Crisp Input | | | | | | |
|---|---|---|---|---|---|---|---|
| $[\overline{\mu}^1, \mu^1]$ | 10 | 50 | 100 | 150 | 200 | 250 | 300 |
| VL | [0.0, 0.06] | [0.3, 0.67] | [0.0, 0.0] | [0.0,0.0] | [0.0,0.0] | [0.0,0.0] | [0.0, 0.0] |
| L | [0.0, 0.0 ] | [0.0,0.006] | [0.85,0.93] | [0.0,0.0] | [0.0,0.0] | [0.0,0.0] | [0.0, 0.0] |
| M | [0.0, 0.0] | [0.0,0.0] | [0.0,0.001] | [0.98,0.99] | [0.0,0.001] | [0.0, 0.0] | [0.0, 0.0] |
| H | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.34, 0.64] | [0.002,0.086] | [0.0, 0.0] |
| VH | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.018, 0.18] | [0.032,0.23] |

**Table 11: Membership Matrix for Total Attraction**

| Fuzzy Set $[\overline{\mu}^2, \mu^2]$ | Crisp Input | | | | | |
|---|---|---|---|---|---|---|
| | 10 | 50 | 107 | 150 | 200 | 250 |
| VL | [0.0, 0.014] | [0.0, 0.04] | [0.0, 0.0] | [0.0,0.0] | [0.0,0.0] | [0.0,0.0] |
| L | [0.0, 0.0 ] | [0.005,0.15] | [0.42,0.68] | [0.0,0.0] | [0.0,0.0] | [0.0,0.0] |
| M | [0.0, 0.0] | [0.0,0.0] | [0.00,0.33] | [0.0,0.023] | [0.0,0.0] | [0.0, 0.0] |
| H | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.08, 0.42] | [0.0, 0.019] | [0.0, 0.0] |
| VH | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.0, 0.0] | [0.08, 0.38] | [0.0, 0.025] |

Table 12: Membership Matrix for Distance

| Fuzzy Set $[\overline{\mu}^3, \mu^3]$ | Crisp Input | | | | | |
|---|---|---|---|---|---|---|
| | 10 | 20 | 40 | 60 | 80 | 100 |
| S | [0.2, 0.567] | [0.19, 0.55] | [0.0, 0.0] | [0.0,0.0] | [0.0,0.0] | [0.0,0.0] |
| M | [0.0, 0.0 ] | [0.0,0.01] | [0.98,0.99] | [0.0,0.006] | [0.0,0.0] | [0.0,0.0] |
| L | [0.0, 0.0] | [0.0,0.0] | [0.001,0.027] | [0.52,0.71] | [0.35,0.579] | [0.02, 0.14] |

1 IF total Trip production of the origin is **VERY LOW,** ***AND*** the total trip attraction of the destination is **VERY LOW,** ***AND*** the friction factor (Distance) between corresponding origin and destination is **SHORT,** ***THEN*** the interaction/trip between Origin and Destination is **VERY LOW**

2 IF total Trip production of the origin is **LOW,** ***AND*** the total trip attraction of the destination is **VERY LOW,** ***AND*** the friction factor (Distance) between corresponding origin and destination is **MEDIUM,** ***THEN*** the interaction/trip between Origin and Destination is **LOW**

3 IF total Trip production of the origin is **MEDIUM,** ***AND*** the total trip attraction of the destination is **VERY LOW,** ***AND*** the friction factor (Distance) between corresponding origin and destination is **LONG,** ***THEN*** the interaction/trip between Origin and Destination is **MEDIUM**

4 IF total Trip production of the origin is **HIGH,** ***AND*** the total trip attraction of the destination is **VERY LOW,** ***AND*** the friction factor (Distance) between corresponding origin and destination is **SHORT,** ***THEN*** the interaction/trip between Origin and Destination is **LOW** .. ..

.. ... ..

74 IF total Trip production of the origin is **HIGH,** ***AND*** the total trip attraction of the destination is **VERY HIGH,** ***AND*** the friction factor (Distance) between corresponding origin and destination is **MEDIUM,** ***THEN*** the interaction/trip between Origin and Destination is **VERY LOW**

75 IF total Trip production of the origin is **VERY HIGH,** ***AND*** the total trip attraction of the destination is **VERY HIGH,** ***AND*** the friction factor (Distance) between corresponding origin and destination is **LONG,** ***THEN*** the interaction/trip between Origin and Destination is **VERY LOW**

2: IT2FL-TDM Fuzzy Rules

Table 13: Fuzzified Values for Production, Attraction and Distance

| Linguistic Variable | | |
|---|---|---|
| Production$[\underline{\mu}^1, \overline{\mu}^1]$ | Attraction $[\underline{\mu}^2, \overline{\mu}^2]$ | Distance$[\underline{\mu}^3, \overline{\mu}^3]$ |
| µVL[0.0,0.0] | µVL[0.0,0.00] | µS [0.0,0.0] |
| µL[0.0,0.0] | µL[0.42,0.68] | µM [0.98,0.99] |
| µM[0.0,0.0] | µM[0.0,0.33] | µL [0.001,0.027] |
| µH[0.002,0.08] | µH[0.0,0.0] | µH[0.0,0.0] |
| µVH[0.018,0.18] | µVH[0.0,0.0] | µVH[0.0,0.0] |

Table14: Inference Engine Result

| Rule No. | Firing Interval | Consequent |
|---|---|---|
| R59 | $[\underline{f}^1, \overline{f}^1]$ = [0.002*0.42*0.98], , [0.086*0.68*0.99] = [ 0.00082,0.05789 ] | $[\underline{y}^1, \overline{y}^1]$ = M[1.0,1.0] |
| R34 | $[\underline{f}^2, \overline{f}^2]$ = [0.002*0.42*0.001],[0.086*0.68*0.027] = [0.0000008, 0.0015] | $[\underline{y}^2, \overline{y}^2]$ = L[0.3,0.21] |
| R35 | $[\underline{f}^3, \overline{f}^3]$ = [0.017*0.42*0.98], , [0.18*0.68*0.099] = [0.00699, 0.012] | $[\underline{y}^3, \overline{y}^3]$ = M[0.68,0.84] |
| R10 | $[\underline{f}^4, \overline{f}^4]$ = [0.017*0.42*0.001], [0.18*0.68*0.027] = [0.00000714,0.33048] | $[\underline{y}^4, \overline{y}^4]$ = M[1.0,1.0] |

Type reduction is computed for L=3 and R = 1,

$y_l = \frac{\bar{f}^1\underline{y}^1+\bar{f}^2\underline{y}^2+\bar{f}^3\underline{y}^3+\bar{f}^4\underline{y}^4}{\bar{f}^1+\bar{f}^2+\bar{f}^3+\bar{f}^4} = y_l = 0.98$; $y_r = \frac{\underline{f}^1\overline{y}^1+\bar{f}^2\overline{y}^2+\bar{f}^3\overline{y}^3+\bar{f}^4\overline{y}^4}{\underline{f}^1+\bar{f}^2+\bar{f}^3+\bar{f}^4} = y_r = 0.99$

Hence, the deffuzified crisp output;

$y_k(x) = 0.99$(Trip Interaction with 99% possibility)

Steps taken in the design of the IT2FL for TDM is presented in Figure 3.

The study employs Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE) for the purpose of performance measure and model evaluation. Figure 3 presents the steps taken in the design of IT2FL for TDM.

Step 1: Assign linguistic labels, TP, TA, TD and TI.
Step 2: Assign membership functions to each linguistic label, as defined in the equations. The membership function used is the Gaussian Type-2 FL MF .
Step 3: Define the rules for the rule base and obtain a set of firing rules for each range of inputs based on the linguistic labels and membership functions of input variables.
Step 4: Obtain the non-zero minimum of the firing rules with their consequence from the composed rules.
Step 5: Perform inference mechanism using Mamdani's MAX-MIN method on the linguistic labels on the consequence part of the rule base for a given range.
Step 6: Perform type reduction using KM Algorithm
Step 7: Defuzzify the result of (step 5) to obtain the crisp output.
Step 8: Stop.

Fig. 3: Steps taken to Design the IT2FL for TDM

**4.0 RESULTS AND DISCUSSION**

In this paper, an IT2F is designed and applied to model inter-city passenger flow in six (6) urban centers in Akwa Ibom State, Nigeria. The data on passengers' trips generated, trips attracted and the distance used in the study are obtained from travel survey conducted across the selected six (6) urban centers (Uyo, Abak, Ikot Ekpene, Oron, Eket and Ikot Abasi) in Akwa Ibom State. Trip production variables used in this paper were obtained from, work trip, school trip, recreation trip, religious trip, medical trip. While the trip attraction variables were obtained from socio-economic development factors in the six selected urban centres using National Population commission (NPC) Government annual report and year book 2013 and they include number of tertiary institutions, number of major markets, population size, number of hospital, number of industries, number of tourist sites, number of financial institutions and distance.

The input data are generated based on the three variables; *trip production*, *trip attraction* and trip *distance* while *trip interaction* is the desired output. The graph of the result of the analysis of the data collected for trip generation with their volume and the trip attraction including the distance in all the six (6) selected zones in Table 2 is presented in Figure 4. This result indicates that school has the largest volume of trip. The summary of result of trip generation/production as analyzed and observed from the survey is shown in Table 3. Fuzzy logic toolbox in Matlab 7.5.0 is used for the input and output membership functions plots as presented in Figures 4 - 7 respectively. Figure 8 shows the inputs and output interface of a case where the total trip production of305 and total attraction of 177.6 with distance 20.1km are selected to obtain 74.2% strong interaction possibility and the interaction surface plot is presented in Figure 9. Figure 10 shows the control surface of IT2FL trip interaction between Uyo-Oron. The control surface ofIT2FL trip interaction between Uyo-Ikot Abasi is presented in Figure 11 while Figure 12 gives the control surface of trip interaction between Uyo-Eket. The control surface ofIT2FL trip interaction between Abak-Ikot Ekpene is presented in Figure 13.

The results of comparison of the IT2FL and the gravity model for urban trip distribution applied in the six (6) zones is presented in Table 15 and the graph is shown in Figure 10. Figure 10 indicates that the volume of interaction between Uyo and Abak ranks fourth.

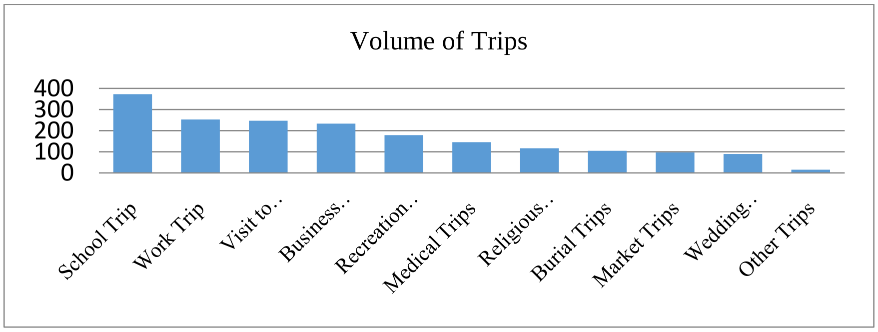


Fig. 3: The graph of the result of the analysis of the data collected for trip generation with their volume and the trip attraction including the distance in all the six (6) selected zones

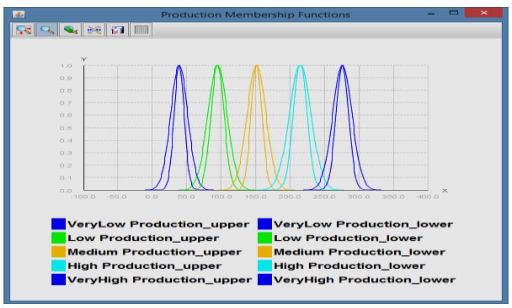


Figure 4: Membership plots for Total Trip Production

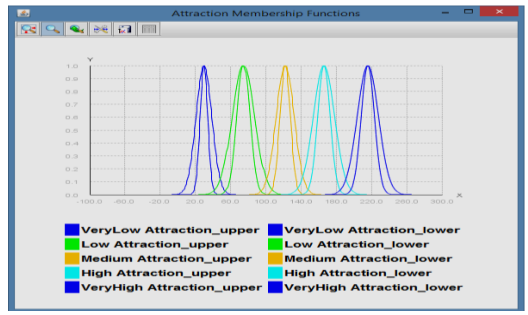


Figure 5: Membership plots for Total Trip Attraction

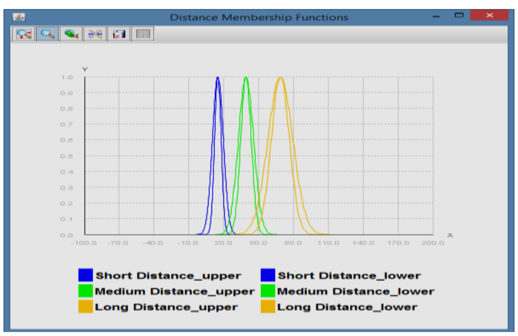


Figure 6: Membership plots for Total Trip Distance between Pair of Zones

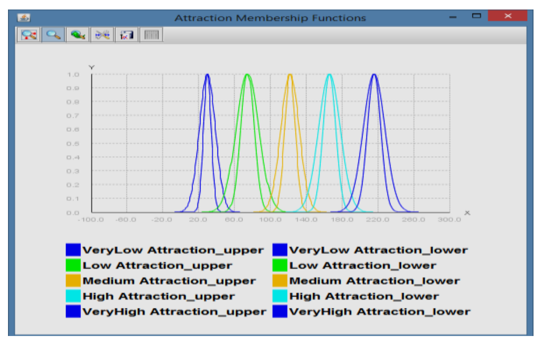


Figure 7: Membership plots for Total trip Interaction

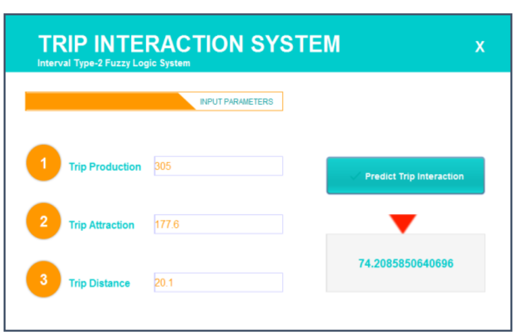


Figure 8: Input and output interface between Uyo and Abak

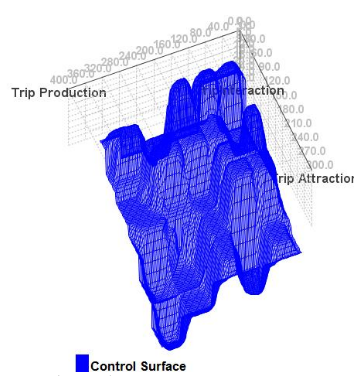


Figure 9: Control Surface of IT2FL Trip Interaction between Uyo and Abak

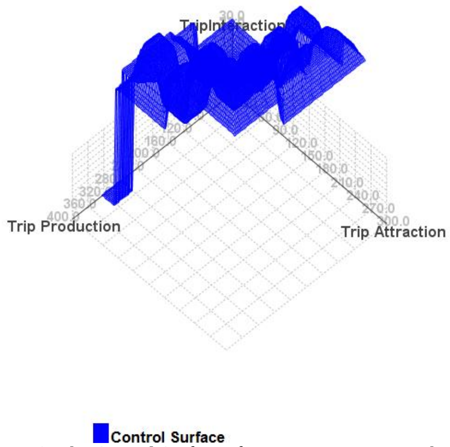


Fig. 10: The Control Surface ofIT2FL Trip Interaction between Uyo-Oron

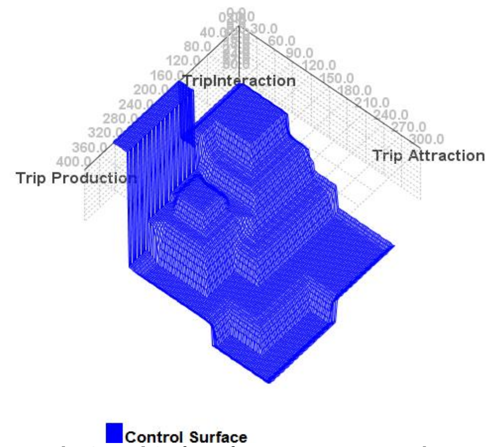


Fig. 11: The Control Surface ofIT2FL Trip Interaction between Uyo-Ikot Abasi

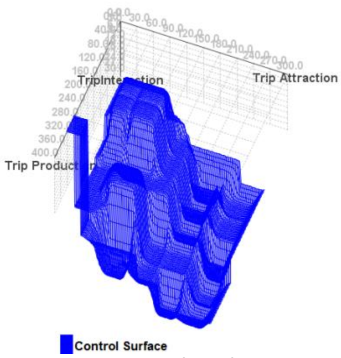


Fig. 12: The Control Surface ofIT2FL Trip Interaction between Uyo-Eket Ekpene

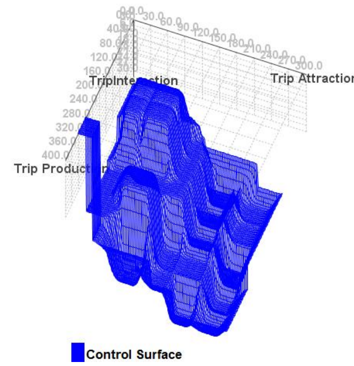


Fig. 13: The Control Surface ofIT2FL Trip Interactionbetween Abak-Ikot

The trip interaction for the six zones is shown in Table 15. The percentage of volume of trip in each inter-urban link was calculated based on the observed total regional volume of trips and illustrated in Figure 14. The upper limit of the 15 interaction links was 86. The mean value obtained was 43 as shown in Figure 4.7. Within the upper limit of 86, mean of 43 and 0.0 origin, 5 classes of interaction were derived as follows:0-17.2 (very weak interaction), 7.3 – 34.4 (weak interaction), 34.5-51.6 (moderate interaction), 51.7 – 68.8 (strong interaction) and 68.9–86 (very stronginteraction) respectively.

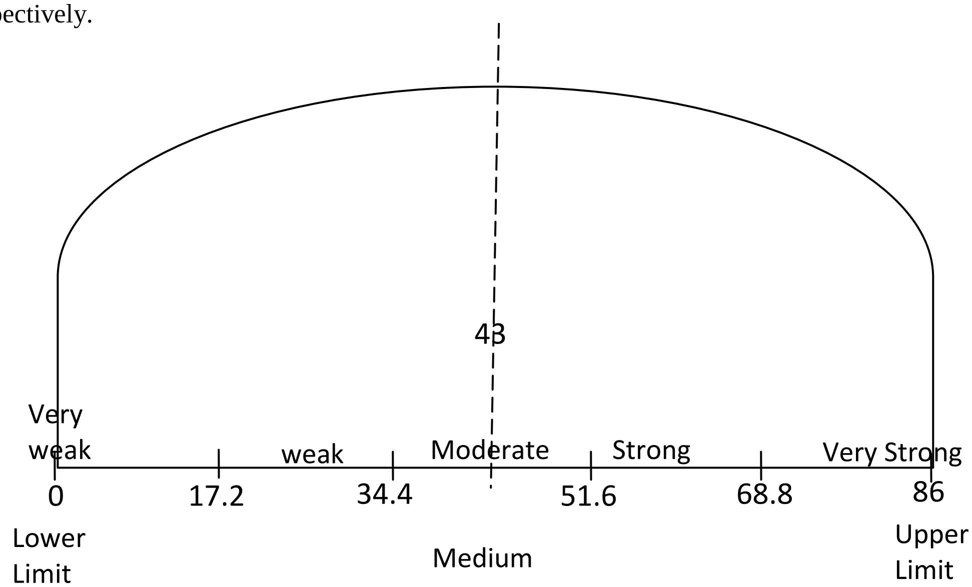


Figure 4.7 Derived classes of interaction used in the study

Table 4.1: Trip Interaction for IT2FLs with their rank

| S/N | Origin/ Destination of Trips | Trip Interaction IT2FL Model | Ranking |
|---|---|---|---|
| **1** | Uyo-Abak | 74 | 4 |
| **2** | Uyo-Ikot Ekpene | 83 | 1 |
| **3** | Uyo-Oron | 68 | 6 |
| **4** | Uyo-Ikot Abasi | 75 | 3 |
| **5** | Uyo-Eket | 79 | 2 |
| **6** | Abak-Ikot Ekpene | 34 | 12 |
| **7** | Abak-Oron | 52 | 7 |
| **8** | Abak-IkotAbasi | 25 | 13 |
| **9** | Abak-Eket | 73 | 5 |
| **10** | Ikot Ekpene-Oron | 48 | 10 |
| **11** | Ikot Ekpene-IkotAbasi | 75 | 3 |
| **12** | Ikot Ekpene-Eket | 45 | 9 |
| **13** | Oron-Eket | 43 | 11 |
| **14** | Oron- Ikot Abasi | 74 | 5 |
| **15** | IkotAbasi-Eket | 50 | 8 |

From Figure 9, the output surface indicates a very strong trip interaction between Uyo and Abak giving the interaction volume of 74%. Also, From Table 6, it is shown that the volume of interaction between Uyo and Ikot Ekpene ranksthe highest.When the total trip productionof 250, total trip attraction of 228.2 and distance of 28.2km are selected,a very strong trip interaction is achieved with of 82.7 (83%) possibility.

From Table 3it is observed that the IT2FL in urban trip distribution modeling demonstrates certain superiority over gravity model.Four performance metrics namely, mean absolute error (MAE), mean square error (MSE), root mean square error (RMSE) and mean absolute percentage error (MAPE) were applied and results indicate that IT2FL model gives a better performance with lower error values of 10.2, 1.18, 0.14 and 0.22 for MAE, MSE, RMSE and MAPE, respectively compared to 11.66, 2.25, 0.18 and 0.32, respectively for the gravity model. Thus, IT2FLS can be applicable in transportation problem to aid good decision making for both travelers and transport planners.

**. Table 3: The results IT2FL and gravity model for urban trip distribution applied in the six (6) zones**

| S/N | Origin/ Destination of Trips | Trip Interaction Gravity Model | Trip Interaction IT2FL Model |
|---|---|---|---|
| **1** | UY-AB | 56 | 74 |
| **2** | UY-IK | 80. | 83 |
| **3** | UY-OR | 77 | 68 |
| **4** | UY-IA | 68 | 75 |
| **5** | UY-EK | 63 | 79 |
| **6** | AB-IE | 23 | 34 |
| **7** | AB-OR | 21 | 52 |
| **8** | AB-IA | 29 | 25 |
| **9** | B-EK | 50 | 73 |
| **10** | IK-OR | 26 | 48 |
| **11** | IE-IA | 55 | 75 |
| **12** | IE-EK | 58 | 45 |
| **13** | OR-EK | 59 | 43 |
| **14** | OR-IA | 63 | 74 |
| **15** | IA-EK | 27 | 50 |

**Table 4: Results of Performance Metrics Evaluation on Gravity and IT2FL Models**

| Metrics | Gravity Model | IT2FL Model |
|---|---|---|
| MAE | 11.66 | 10.21 |
| MSE | 2.25 | 1.18 |
| RMSE | 0.18 | 0.14 |
| MAPE | 0.32 | 0.22 |

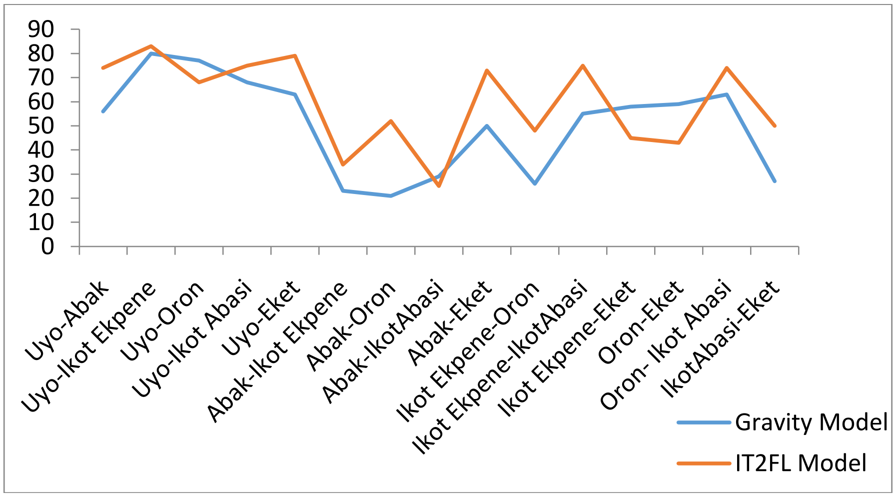


Fig:The graph of IT2FL and Gravity Model for urban Trip Distribution applied in the six (6) zones

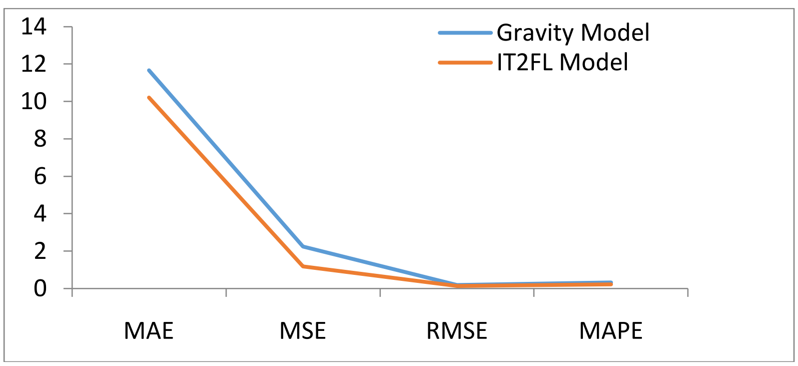


Fig: The Results of Performance Metrics Evaluation on Gravity and IT2FL Models

## 14 CONCLUSION

Trip distribution modeling (TDM) is one of the most active parts of travel demand analysis in transportation planning (TP). Most of the transport decisions take place under subjectivity, ambiguity, uncertainty and imprecision and partial truth. In this study, IT2FL is applied to model urban trip distribution problem in transportation planning in order to predict the trip interaction among the various urban centres in Akwa Ibom State. Four performance metrics namely, mean absolute error (MAE), mean squared error (MSE), root mean squared error (RMSE) and mean absolute percentage error (MAPE) were applied and results indicate that IT2FL model gives a better performance with lower error values of 10.2, 1.18, 0.14 and 0.22 for MAE, MSE, RMSE and MAPE, respectively compared to 11.66, 2.25, 0.18 and 0.32, respectively for the gravity model. Thus, IT2FLS can be applicable in transportation problem to aid good decision making for both travelers and transport planners. Generally, it is observed that fuzzy logic could be used successfully to model situations in which people make decisions in an environment that is so complex that it is very hard to develop a mathematical model.
In the future, the work can be implemented with more input variables and more zones. Also, the performance of the proposed system can be improved by integrating IT2FL with ant colony optimization tool.

**REFERENCES**

[1] Kanafani, A. (1983) *Transportation Demand Analysis* (McGraw Hill, New York). de Ortuzar,

[2] Carpio, L. and Gomes, L. (1991) 'Modeling trip distribution through game-theory', *Systems Analysis Modelling Simulation* **8**, 515–521.

[3] J.D. and Willumsen, L.G. (1990) *Modelling Transport* (John Wiley &Sons, New York).

[4] Kalic, M. & Teodorovic, D. (2003). Trip distribution modelling using fuzzy logic and a genetic algorithm. *Transportation Planning and Technology*, 26, (3) 213-238p.

[5] Smith, D.P. & Hutchinson, B.G. (2000). Goodness of fit statistics for trip distribution models. *Transportation Research Part A: General*, 15, (4) 295-303p.

[6] Williams, H.C.W.L. (1977). On the formation of travel demand models and economic evaluation measures of user benefit. *Environment and Planning A*, 9, (3) 285-344p.

[7] Wilson, A.G. (1970). A statistical theory of spatial distributions. *Transportation Research*, 1, 253-269p.

[8] Wilson, A.G. (1974). *Urban and regional models in geography and planning* London - New York, John Wiley & Sons.

[9] Jassbi, F. and L. G. Willumsen (2011), *Modelling Transport*, Fourth edition, Chichester, West Sussex, United Kingdom: John Wiley and Sons, 2011.

[10] Cordera, R. Sanudo, R.Dell'Olio, L. Ibeas, A (2018) Trip distribution model for regional railway services considering spatialeffects between stations. Transport Policy 67 (2018) 77–84.

[11] Ioannis K., Kaczmarek M. (2016), *Fuzzy Group Model of Traffic Flow in Street Networks, Transportation Research Part C:* Emerging Technologies, 13(2), 93-105p.

[12] Boyce, D. (2002) The sequential travel forecasting paradigm .*Journal of Urban Planning and Development*, 128, (4) 169-183p.

[13] Casey, H.J. (1955) Applications to traffic engineering of the law of retail gravitation. *Traffic Quarterly*, 9, 23-25p.

[14] Taaffe, E.J. and Gauthier, H.L., (2006). *Geography of transportation*, 2nd ed. New Jersey, Prentice Hall.

[15] Kalic, M. and Teodorovic, D. (1996). Solving the trip distribution problem by fuzzy rules generated by learning from examples (in Serbian), *In Proceedings of the XXIII Yugoslav Symposium on Operations Research*, 777-780p.

[16] Teodorovic D. (1994), *Fuzzy Sets Theory Applications in Traffic and Transportation*, European Journal of Operational Research, 74, 379-390p.

[17] Demissie, M. G., S. Phithakkitnukoon and L. Kattan, (2019) Trip Distribution Modeling Using Mobile Phone Data: Emphasis on Intra-Zonal Trips. *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 7, pp. 2605-2617.

[18] Berki, Z., Habil. Janos Monigl (2017) Trip generation and distribution modelling in Budapest. 20th EURO Working Group on Transportation Meeting, EWGT 2017, 4-6 September 2017, 1-9, Buda-pest, Hungary.

[19] Black, W.R. (2003) *Transportation: a geographical analysis* New York - London, The Guilford Press.

[20] Zadeh L. A. (1964). "Fuzzy sets," Inf. Contr., *8*, 338–353.

[21] Zadeh L. A. (1975). "The concept of a linguistic variable and its application to approximate reasoning—1," Inform. Sci., *8*, 199–249.

[22] Sahoo, A. S. G. and Sahoo, U.C., (2001). Application of Fuzzy Logic in TransportPlanning. International Journal on Soft Computing (IJSC) Vol.3, No.2, 1-21.

[23] Mendel J. M. (2001). "On the importance of interval sets in type-2 fuzzy logic systems," in Proc. Joint 9th IFSA World Congr. 20th NAFIPS Int. Conf., Vancouver, BC, Canada, Jul. 25–28, 1647–1652.

[24] Mendel, J., Hagras, H. and R. John Standard Background Material About Interval Type-2 Fuzzy Logic Systems. google scholar, 2006.

[25] Liang, Q. and Mendel,J. M. (2012) "Interval type-2 fuzzy logic systems: theory and design", IEEE Transactions on Fuzzy systems, vol. 8, no. 5, (2012), pp. 534-550.

[26] H. Hagras and C. Wagner, "Towards the wide spread use of type-2 fuzzy logic systems in real world applications," *IEEE Computational Intell. Mag.*, vol. 7, no. 3, pp. 14–24, 2012

[27] N. N. and Mendel, J. M. (2001). *Operations on Interval Type-2 Fuzzy Sets. Fuzzy Sets and Systems*. 122, 327-348p.

[28] Karnik, N. N. and J. M. Mendel (1999), *Applications of Type-2 Fuzzy Logic Systems to Forecasting of Time-Series, Information Sciences*, vol. 120, 89-111p.