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EFFECT OF NON-FOURIER HEAT EQUATION ON NATURAL CONVECTION FLOW IN A VERTICAL CHANNEL WITH PERIODIC THERMAL BOUNDARY CONDITION

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Abstract

This work investigates the natural convection flow of viscous incompressible fluid in a channel formed by two infinite vertical parallel plates. Fully developed laminar flow is considered in a vertical channel with steady–periodic temperature regime on the boundaries. Separating the velocity and temperature fields into steady and periodic parts, the resulting second order ordinary differential equations are solved to obtain the expressions for velocity and temperature. The amplitudes and phases of temperature and velocity are also obtained as well as the rate of heat transfer and the skin-friction on the plates. During the course of investigation, it is found that the amplitudes and phases of temperature and velocity exert strong influence on the temperature and velocity, and when the dimensionless relaxation time (λ) was set to be zero, there is an excellent agreement between this work and other works in the literature.

***Keywords**: heat source/sink, natural convection, periodic temperature, non-Fourier*

Nomenclature

A	steady velocity	
B	unsteady velocity	
Bi	Biot number	
C	specific heat at constant pressure	
E	Eckert number	
F	steady temperature	
g	acceleration due to gravity	[ms^{-2}]
G	periodic temperature	
h	half width of the channel	[m]
H	heat source/sink parameter	
k	thermal conductivity	[W/mK]
Nu	Nusselt number	
Pr	Prandtl number	
St	Strouhal number	
t	time	[s]
T_0	initial temperature of fluid	[K]
T_1	steady temperature of wall	[K]
T_2	periodic temperature of wall	[K]
T	temperature of fluid	[K]
u	velocity of the fluid	[ms^{-1}]
x	vertical axis (direction of flow) [m]	
y	co-ordinate perpendicular to the plate	[m]

Greek Alphabets

α	thermal diffusivity	[m^2s^{-1}]
β	coefficient of thermal expansion	[K^{-1}]
η	dimensionless horizontal coordinate	
λ	dimensionless relaxation time	
μ	coefficient of viscosity	[$kgm^{-1}s^{-1}$]
ν	kinematic viscosity	[m^2s^{-1}]
τ	skin friction	
ω	frequency of periodic heating	[s^{-1}]

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1. Introduction

Conduction occurs not only within a body but also between two bodies if they are brought into contact. If one of the substances is a liquid or a gas, then fluid motion will almost certainly occur. This process of conduction between a solid surface and a moving liquid or gas is called convection. The motion of the fluid may be natural or forced. If a liquid or gas is heated, its mass per unit volume generally decreases. If the liquid or gas is in a gravitational field, the hotter, lighter fluid rises while the colder, heavier fluid sinks. This kind of motion, due solely to non-uniformity of fluid temperature in the presence of a gravitational field, is called natural convection. Forced convection is achieved by subjecting the fluid to a pressure gradient and thereby forcing motion to occur according to the law of fluid mechanics. In classical heat conduction, energy equation is derived from Fourier's law of heat conduction, which assumes an infinite speed of heat propagation. Maxwell [1] generalized the Fourier's heat law for the dynamical theory of gases and his heat flux equation contains a term proportional to the time derivative of the heat flux vector. Since the constant of proportionality τ_t had a very small magnitude in Maxwell's work, he took to be zero.

A mathematical modification to Fourier's theory of heat conduction was developed by Cattaneo [2] using a second order approximation and aspects from the kinetic theory of gases. His modification of Fourier's theory allows the existence of thermal waves which propagate at finite speed and are means by which heat flows in gases. The derivation of non classical heat conduction equation in [2] does not make reference to the work of [1] or to other prior works. Also, the work was restricted to gases as no mention was made of heat propagation in liquids and solids. Cattaneo's equation has been derived in different ways by different authors including [1,2 and 3]. Then [4] described the equation in [1] as the most obvious and simple generalization of Fourier's law that will give rise to finite speeds of heat propagation.

The authors [5], [6], and [7] studied the effect ofperiodic heating of a single vertical plate with no edge on the boundary layer development, [8] studied the fully developed convection between two periodically heated parallel plates; a study that derived its relevance from the miniaturization of electrical and electronic panels. In 1988, [9] investigated the effect of the Strouhal number on the development of boundary layers between two periodically heated parallel plates and concluded that an increase in the Strouhal number results in a decrease in unsteady temperature and unsteady velocity of the fluid. Also, the effect of Strouhal number on unsteady temperature and velocity reduces as the Strouhal number increases, and the effect diminishes to zero for high Strouhal number.[10,11 and 12], generalized equation in [2] and other similar equation were postulated for other forms of thermodynamics. Some other recognized works on non-Fourier's heat equation are: [13,14,15,16 and 17].

Natural convection flow due to steady/periodic temperature is very essential because of its applications in different engineering and environmental problems. For instance, in automatic control systems, electrical and electronic components are frequently subjected to periodic heating and are cooled by natural convection processes. In an attempt to study the influence of periodic heat input on natural convective fluid flow, [18] considers the case in which the surface temperature varies slightly about a mean level, which is higher than the ambient temperature. They performed a perturbation analysis in terms of the amplitude of the surface temperature, but their results are restricted to small amplitudes of the surface temperature variation. Other works that considers flow of fluid within channel due to oscillatory surface temperature are [8] that studied the fully developed convection between two periodically heated parallel plates; a study that derived its relevance from miniaturization of electrical and electronic panels. With regard to the periodic boundary condition, most studies have dealt with conduction heat transfer using the parabolic (Fourier's law) heat equation or numerical schemes, whereas some relied on the hyperbolic heat equation. However, the hyperbolic model of the heat transfer cannot accurately predict the temperature in a medium.

Jha [19] extended the work of [9] to include suction and injection on the channel plates when the working fluid has the property of heat generation/absorption. Their work concluded that the introduction of suction/injection has distorted the symmetric nature of the flow considered by [9] as the thermal boundary layer is increasing towards the plate with injection and reducing towards the plate with suction.

Considering the volume of works done on the non-classical heat propagation and sinusoidal surface thermal conditions, it is interesting to investigate the influence of non-Fourier heat equation on the natural convection flow induced by periodic boundary condition, and hence the motivation for this work

2. Mathematical Analysis

We consider a fully developed free convection flow of viscous incompressible fluid in a vertical channel due to time/periodic temperature conditions on the plates. The channel plates are taken vertically, parallel in the *x*-direction at *2h* distance apart while the *y*-axis is taken normal to the plates (see Fig. 1). As the temperature increases on both plates, the density of the fluid adjacent to it reduces which makes it to embark on upward motions which signify the setting up of convection currents in the channel. Due to the viscosity of the working fluid, the velocity on the plates remains zero.

The governing momentum and energy equations for the present physical situation following [19] and taking into account finite speed of heat propagation are:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) \quad (1)$$

$$\tau_t \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{Q(T - T_0)}{\rho Cp} \quad (2)$$

Let the velocity and temperature on the channel plates be

$$u(\pm h, t) = 0, \qquad T(\pm h, t) = T_1 + T_2 \cos(\omega t) \quad (3)$$

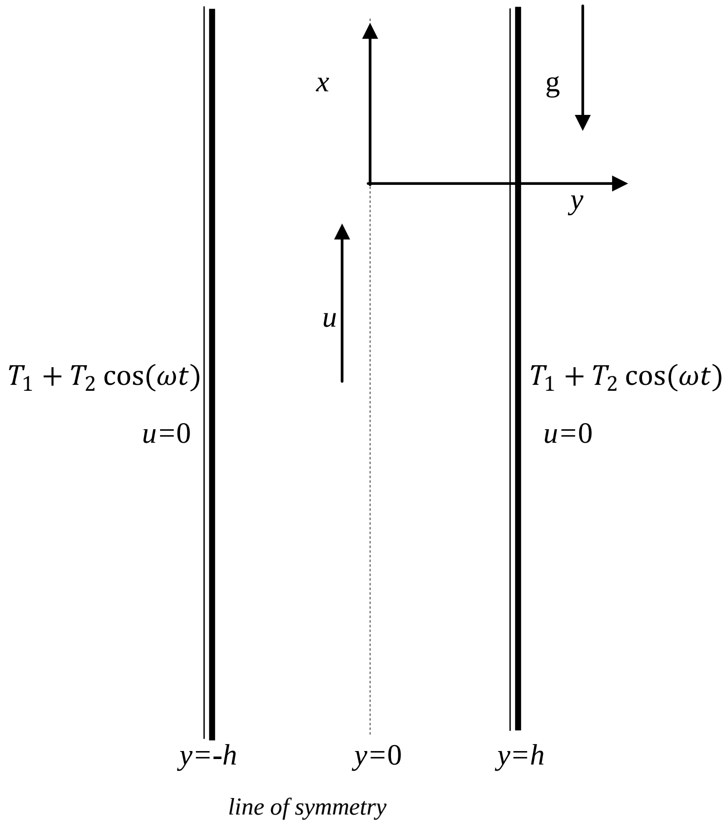

Fig. 1 Flow configuration and coordinates system

Since the flow is fully developed in the *x*-direction, then it is independent of *x*. Following [9]we separate the problem into steady and periodic parts using the following expressions

$$u = \frac{g\beta h^2}{\nu}\left[(T_1 - T_0)A(\eta) + T_2 B(\eta)e^{i\omega t}\right] \quad (4)$$

$$T = T_0 + (T_1 - T_0)F(\eta) + T_2 G(\eta)e^{i\omega t} \quad (5)$$

Where $A(\eta), F(\eta)$ represent the steady parts and $B(\eta), G(\eta)$ the periodic parts of velocity and temperature respectively.

Substituting equations (4) and (5) into equations (1) and (2) using the boundary conditions (3), we obtain the following second order ordinary differential equations in dimensionless form:

$$F''(\eta) - HF(\eta) = 0; \qquad F(\pm 1) = 1$$
$$A''(\eta) + F(\eta) = 0; \qquad A(\pm 1) = 0$$
$$G''(\eta) - [H + St\,\Pr(i - \lambda)]G(\eta) = 0; \qquad G(\pm 1) = 1$$
$$B''(\eta) - iStB(\eta) = -G(\eta); \qquad B(\pm 1) = 0 \quad (6)$$

Where

$$\eta = \frac{y}{h}, \quad St = \frac{h^2\omega}{\nu}, \quad \Pr = \frac{\nu}{\alpha}, \quad \lambda = \tau_t\omega, \quad H = \frac{Qh^2}{k} \quad (7)$$

St is the Strouhal number which varies directly as the frequency of the periodic heating, *Pr* is the Prandtl number which is the ratio of momentum to thermal diffusivity,λ is the dimensionless relaxation time and *H* is the dimensionless heat generation/absorption parameter, positive values of *H* means heat absorption and negative value is heat generation. The physical quantities used in the expressions (7) are defined in the nomenclature. Using the method of variation of parameters to obtain the particular solutions we solve the equations (6) to get the following solutions;

$$A(\eta) = -\frac{a_2}{H}\cosh\eta\sqrt{H} + b_4 \quad (8)$$

$$B(\eta) = a_3\cosh\eta\sqrt{iSt} + x_1\cosh\eta\sqrt{z} \quad (9)$$

$$F(\eta) = a_2\cosh\eta\sqrt{H} \quad (10$$

$$G(\eta) = a_1\cosh\eta\sqrt{z} \quad (11)$$

Where $a_1 = \frac{1}{\cosh\sqrt{z}}$ $\quad a_2 = \frac{1}{\cosh\sqrt{H}}$ $\quad a_3 = -\frac{x_1\cosh\sqrt{z}}{\cosh\sqrt{iSt}}$

$x_1 = -\frac{a_1}{z - iSt}$ $\quad z = H + St\,\Pr(i - \lambda)$ $\quad b_4 = \frac{a_2\cosh\sqrt{H}}{H}$

Setting $\lambda = 0$ in the present problem, the solutions (8)-(11) coincide with the results obtained by [19].
The main concern of the present problem is to investigate the effect of periodic heating on the hydrodynamics and thermodynamics within the channel. We will therefore concentrate on the flow formations due to the periodic temperature $G(\eta)$ and the periodic velocity $B(\eta)$.
Taking C_R and C_i to denote respectively the real and imaginary parts of the complex number c, we obtain the phase (ψ) of temperature and that of velocity (χ) as follows;

$$\psi = \tan^{-1}\left(\frac{G_i}{G_R}\right) \qquad (12)$$

$$\chi = \tan^{-1}\left(\frac{B_i}{B_R}\right)$$

While the amplitudes of the periodic temperature and periodic velocity are given by

$$|G| = \left(G_R^2 + G_i^2\right)^{1/2} \qquad (13)$$

$$|B| = \left(B_R^2 + B_i^2\right)^{1/2}$$

Where G_R and B_R are the periodic temperature and velocity profile in phase, and G_i and B_i are the periodic temperature and velocity profile 90^0 out of phase. Positive values of B_R depicts velocity in the positive direction while a negative value is an indication of reverse flow. The phase χ of velocity is the angle by which the flow deviates from the upward (in phase) direction, while the amplitude $|B|$ denotes the absolute velocity at any given point in the channel.
We obtain the rate of heat transfer (Nu) on the channel plates by taking $\left.\frac{dG}{d\eta}\right|_{\eta=\pm1}$ to get

$$Nu = a_1\sqrt{z}\sinh\eta\sqrt{z} \qquad (14)$$

While the skin-friction (τ) is obtained by considering $\left.\frac{dB}{d\eta}\right|_{\eta=\pm1}$ to get

$$\tau = a_3\sqrt{iSt}\sinh\eta\sqrt{iSt} + x_1\sqrt{z}\sinh\eta\sqrt{z} \qquad (15)$$

3. Results and Discussion

A free convective flow of viscous incompressible fluid is considered between two infinite vertical parallel plates. The convection current is set up due to a steady-periodic heating of the channel plates. The flow is governed by four basic parameters namely; Heat source/sink (H), positive values of H represents heat sink while negative values represents heat generation in the channel, Strouhal number (St) which represents the frequency of the periodic heat input on the plate, Dimensionless relaxation time (λ), and Prandtl number (Pr) which is inversely proportional to the thermal diffusivity of the working fluid. The values of H are chosen between -1 and +1 to accommodate for cases of heat source/sink. The values of Pr are chosen between the non-dimensional values of 0.001 and 7.0 to accommodate fluids like mercury (0.008–0.041), H_2O vapor (0.882–0.994), oxygen (0.729–0.759), air (0.703-0.784), water (5.18–8.91) etc, [20]. The values of St are chosen over the range $0 \le St \le 10$. The values of dimensionless relaxation time (λ) are chosen between 0.01 and 0.5. The problem is presented in graphical form in Figs. 2–25 so as to clearly reveal the influence of each governing parameters on flow behaviour.

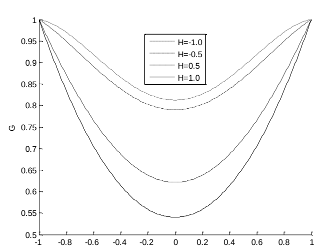

Fig. 2 Temperature profiles for different H [Pr=0.71 λ=0.1 St=2.0]

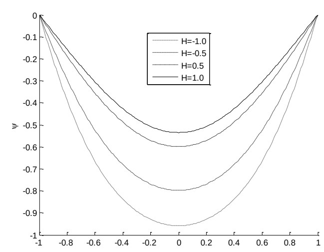

Fig. 3 Phase of Periodic temperature for different H [Pr=0.71 λ=0.1 St=2.0]

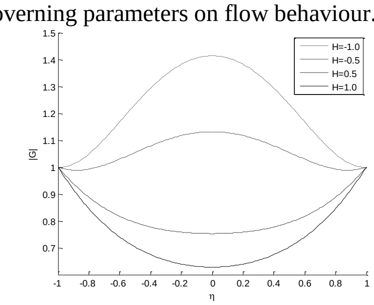

Fig. 4 Amplitude of periodic temperature for different H [Pr=0.71 λ=0.1 St=2.0]

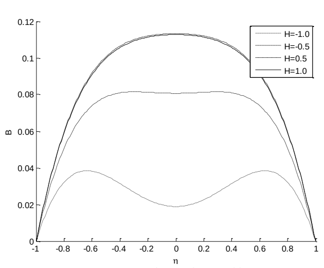

Fig. 5 Velocity profiles for different H [Pr=0.71 λ=0.1 St=2.0]

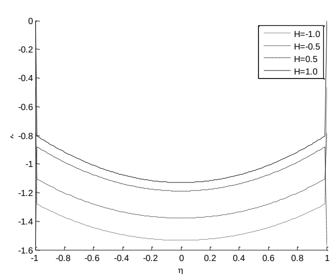

Fig. 6 Phase of periodic velocity for different H [Pr=0.71 λ=0.1 St=2.0]

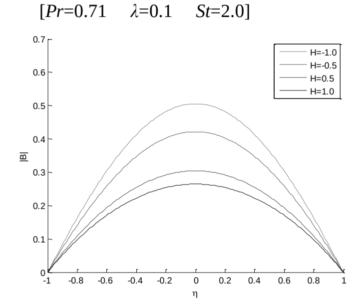

Fig. 7 Amplitude of Periodic velocity for different H [Pr=0.71 λ=0.1 St=2.0]

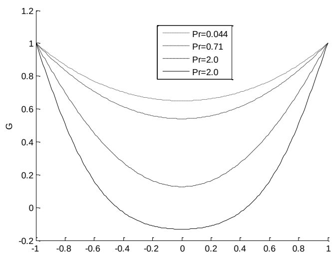

Fig. 8 Temperature profiles for different *Pr*
[H=1.0 λ=0.1 St=2.0]

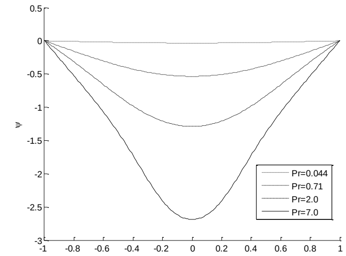

Fig. 9 Phase of periodic temperature for different *Pr*
[H=1.0 λ=0.1 St=2.0]

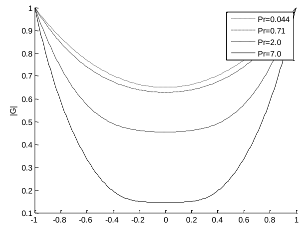

Fig. 10 Amplitude of periodic temperature for different *Pr*
[H=1.0 λ=0.1 St=2.0]

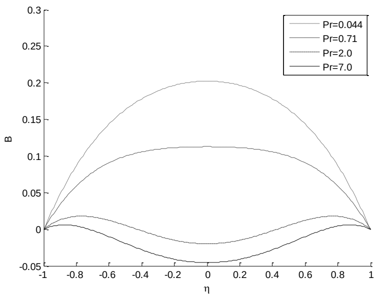

Fig. 11 Velocity profiles for different *Pr*
[H=1.0 λ=0.1 St=2.0]

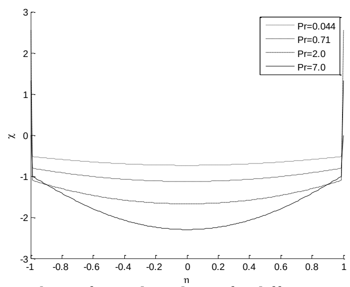

Fig. 12 Phase of periodic velocity for different *Pr*
[H=1.0 λ=0.1 St=2.0]

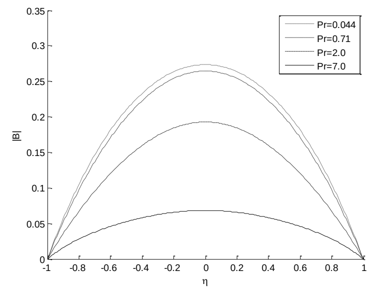

Fig. 13 Amplitude of periodic velocity for different *Pr*
[H=1.0 λ=0.1 St=2.0]

Figure 2 presents fluid temperature within the channel for variation in the heat generation/absorption parameter. The figure reveals that fluid temperature increases with growing heat source (H<0) while heat sink causes a decrease in the temperature. However, it is observed in Fig. 3 that the phase of periodic temperature (deviation from in-phase direction) increases with increase in heat generation, which decreases heat flux within the channel. Due to the increase in heat generation in the channel, the amplitude of periodic temperature increases as shown in Fig. 4, so that fluid temperature increases within the channel. This shows that the effect of amplitude increase overshadows the negative influence of phase increase caused by growing heat generation.

Fig. 5 depicts velocity profiles for different heat source/sink parameter. It is observed from the figure that velocity increases with decreasing heat source and an increase in the heat sink brings about a slight decrease in the velocity. The deviation of periodic velocity from in-phase direction increases as heat source is noticed to be decreased. This is clearly seen in figure 6, and also the amplitude of periodic velocity increase is caused by an increase in the heat source and a decrease in the heat sink which is shown in figure 7. The combined effect of phase change an amplitude of velocity reveals that the response of phase change to variations in heat source/sink is dominant over the effect of the amplitude and this is evident in the fluid velocity that decreases with increasing heat generation.

An observation from Fig. 8 which shows Temperature profiles for different values of *Pr* is seen that temperature of the system is reduced as the value of *Pr* increases.This can be attributed to the fact that increase in *Pr* is synonymous to a decrease in thermal diffusivity of the working fluid which translates to a decrease in thefluid temperature. This is an evidence of the physical fact that the thermal diffusivity of the fluid decreases with growing *Pr* which consequently reduces the heat flux into the system from the applied heat on the boundary. Figures 9 and 10 presents the phase and amplitude of periodic temperature for different values of *Pr*. ψ and |G| are seen as dropping within the system with increasing *Pr*.This is due to the fact that, as the Prandtl number increases, the thermaldiffusivity of the fluid reduces which results in a corresponding decrease in the fluid temperature within the channel.

In figure 11 it is observed that velocity is decreasing with increasing values of*Pr*. This is the consequence of temperature decrease caused by growing *Pr*, which acts to weaken the convection current within the channel so that the fluid velocity decrease. Similarly, increasing *Pr* is an evidence of growing fluid viscosity and eventually decreases the fluid buoyancy. Phase and amplitude of periodic velocity for different values of *Pr* is the subject of the figures 12 and 13 and it is clearly seen that the velocity decreases with increasing values of *Pr* and the periodic velocity is shown as a decreasing function of the Prandtl number. This translates to the fact that thermal diffusivity decreases as the Prandtl number increases thereby reducing heatpenetration. Consequently convection currents becomes weak and hence result in a decrease in velocity.

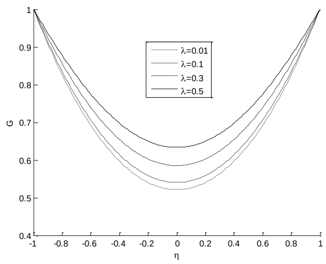

Fig. 14 Temperature profiles for different λ
[H=1.0 St=2.0 Pr=0.71]

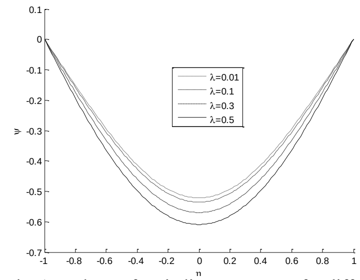

Fig. 15 Phase of periodic temperature for different λ
[H=1.0 St=2.0 Pr=0.71]

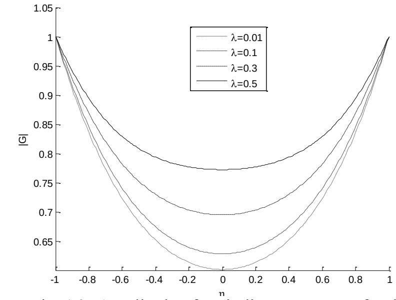

Fig. 16 Amplitude of periodic temperature for different λ
[H=1.0 St=2.0 Pr=0.71]

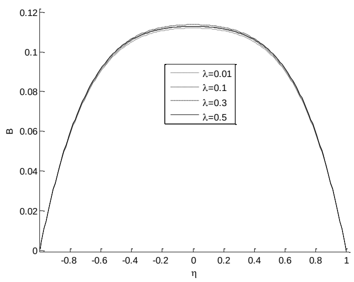

Fig. 17 Velocity profiles for different
[H=1.0 St=2.0 Pr=0.71]

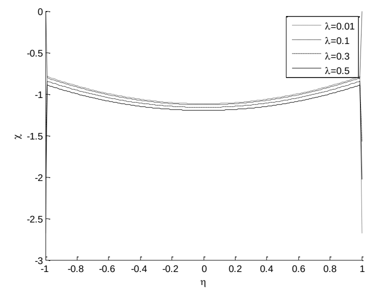

Fig. 18 Phase of periodic velocity for different λ
[H=1.0 St=2.0 Pr=0.71]

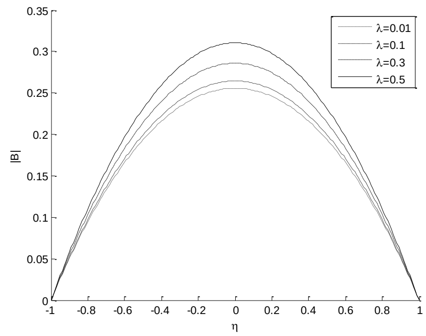

Fig. 19 Amplitude of periodic velocity for different λ
[H=1.0 St=2.0 Pr=0.71]

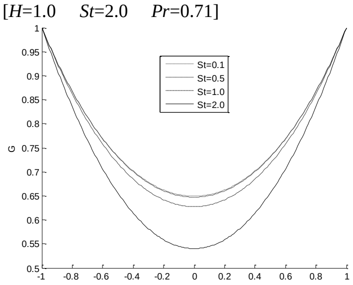

Fig. 20 Temperature profiles for different St
[H =1.0 Pr=0.71 λ=0.1]

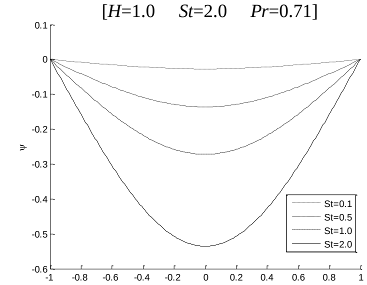

Fig. 21 Phase of periodic temperature for different St
[H =1.0 Pr=0.71 λ=0.1]

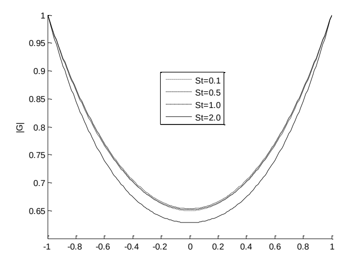

Fig. 22 Amplitude of periodic temperature for different St
[H =1.0 Pr=0.71 λ=0.1]

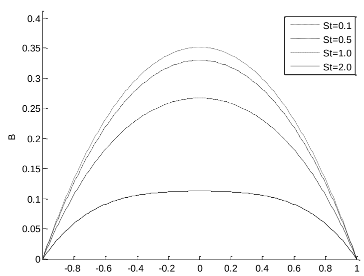

Fig. 23 Velocity profiles for different St
[H =1.0 Pr=0.71 λ=0.1]

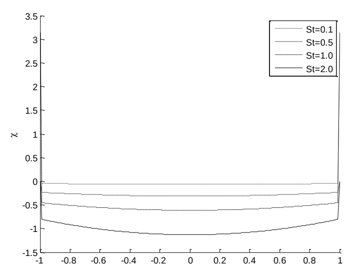

Fig. 24 Phase of periodic velocity for different St
[H=1.0 Pr=0.71 λ=0.1]

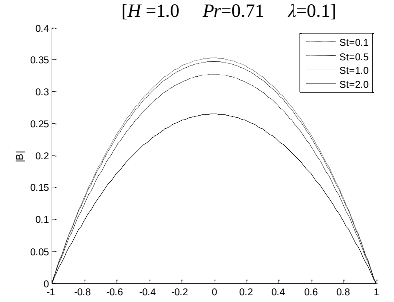

Fig. 25 Amplitude of periodic velocity for different St
[H =1.0 Pr=0.71 λ=0.1]

It is observed from Fig. 14 that temperature is increasing with growing dimensionless relaxation time (λ) and the deviation from in-phase direction tend to increase with increasing dimensionless relaxation time (λ) which is noticed from figure 15, and thereby causing thermal accumulation within the channel leading to an increase in the fluid temperature. A similar phenomenon is seen in figure 16 where it is seen that amplitude of periodic temperature is increasing with increasing dimensionless relaxation time (λ). In Fig. 17 it is noticed that the effect of dimensionless relaxation time (λ) is not much felt on the velocity while the phase of periodic velocity is seen to be increasing when dimensionless relaxation time (λ) is being increased as shown in Fig. 18. However Fig. 19 illustrates the amplitude of periodic velocity to be an increasing function of dimensionless relaxation time (λ). This is due to the fact that as λ increases, convection current is stronger and hence causes an increase in the periodic velocity amplitude. Amplitude increase overshadows negative effect of phase increase caused by increasing dimensionless relaxation time (λ).

A very similar case is seen in figures 20, 21 and 22 where temperature, phase and amplitude of periodic temperature are observed to be decreasing as the value of Strouhal number (*St*) is being increased. This physical fact is hinged upon the fact that kinematic viscosity of the fluid is reduced due to the decrease in temperature caused by growing *St*. Also, increasing *St*results to a decrease in temperature as shown in fig. 20. This is due to the fact that intensity of the heating on the plates reduces as *St*increases.

In figures 23, 24 and 25, it is noticed that the velocity, phase and amplitude of periodic velocity are reducing as the value of *St* is being increased. The temperature reductionbrought about by increasing *St* has definitely weakened the convection current which reduced the buoyancy of the fluid and this leads to a reduction in velocity.

Table 1: Comparison between Jha and Ajibade [46] and the present work (λ=0)

	Jha and Ajibade (2009)		Present work ($\lambda = 0$)	
H	ψ	$\lvert G \rvert$	ψ	$\lvert G \rvert$
-1.0	-0.9069405380	1.3265320396	-0.9069405380	1.3265320396
-0.5	-0.7613285054	1.0638804951	-0.7613285054	1.0638804951
0.0	-0.6566171584	0.8646168358	-0.6566171584	0.8646168358
0.5	-0.5790585352	0.7139002026	-0.5790585352	0.7139002026
1.0	-0.5198160755	0.5983678650	-0.5198160755	0.5983678650

From the table above, it is clearly seen that when λ=0 is considered in the present work, there is an excellent agreement with the work of [19].

4. CONCLUSION

The Heat source/sink, Strouhal number, Prandtl number and dimensionless relaxation time have been the main parameters studied and their effects on the heat and fluid flow phenomena have been determined. The effects of natural convection, periodic temperature and velocity have also been investigated. The main conclusion of this work is that an increase in the flow parameters of the fluid either increase or decrease the values of heat source/sink parameter, Prandtl number, Strouhal number and dimensionless relaxation time within the channel. It is also concluded that the amplitudes and phases of temperature and velocity exerts strong influence on the temperature and velocity throughout the system. Also part of the conclusion of this work are;

i- Phase increase contributes to decrease of temperature as well as velocity within the channel.
ii- Fluid temperature and velocity decrease with growing Strouhal number.
iii- Heat generation increases the thermodynamics and hydrodynamics within the channel.

Also, when *λ* was set to be zero in this work, there is an excellent agreement with the results and conclusions of[19]**,** as observed from the numerical values in table 1.

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