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AN INVENTORY PRODUCTION MODEL OF DELAYED DETERIORATING ITEMS WITH CONSTANT PRODUCTION RATE.

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Abstract

In this study, an effort has been made to derive an optimal production model for delayed deteriorating items where both deterioration and production rates are regarded as constant. Shortages are not allowed and production is higher than the demand rate. The model is solved analytically to minimize the total variable cost of the system which leads to the determination of the best cycle length for the inventory. Numerical examples are provided to illustrate the application of the model developed.

Keywords**:** Delayed deterioration, Holding cost, Production rate, Demand rate

1.0 Introduction

Generally, inventory refers to any item of property held in stock by a firm or any organization for smooth and functional running of business. It is a balance between demand and supply. One of the most unrealistic assumptions in traditional inventory model was that items preserved their physical characteristics while in production or when kept in inventory. These assumptions are not generally true because items are subject to a continuous loss in their masses or utility throughout their life time due to decay, damage, dryness, spoilage, or other reasons. This phenomenon is refers to as deterioration. Owing to this fact, controlling and maintaining inventory of deteriorating items becomes a challenging problem for decision makers.

For several years, the control and maintenance of inventories of deteriorating items have attracted the attentions of many researchers. It all started with Ghare and Schrader [1] who developed a model of exponentially decaying inventory, with a constant rate of deterioration. After Ghare and Schrader [1], several reviews and models on deteriorating inventory has been developed. For example, Shah and Jaizwal [2] came up with an order level inventory model for a system with constant rate of deterioration. Aggarwal [3] extended the work of Shah and Jaiswal [2] by providing a note on an order level inventory model for a system with a constant rate of deterioration. Padmanabahna and Vrat provides an EOQ model for perishable items under stock dependent selling rate. The literatures mentioned above considered constant deterioration rates in their models. However, several researchers considered several scenarios of relationship between time and deteriorating rates. Some of these scenarios include: deteriorating rate as linear function of time, two-parameter Weibull distribution, three- parameter Weibull distribution and or deterioration rate as other function of time. Geol and Aggarwal [4] were the earlier researchers to consider Weibull deteriorating rate in their inventory model. Their research focused on maximization of profit rate function using the three parameter Weibull density function. Wu *et al.*[5] proposed an EOQ model for inventory of an item that deteriorates at a Weibull distribution rate where demand rate is a continuous function of time. In this model, the inventory started with an instant replenishment and ended with shortages. Wu [6] reconsidered the work of Wu *et al*. [5] by considering the model to start with zero inventories and ended without shortages.

In the above literature, the practitioners ignore or give little attention to items in the production centers. In the production circle, the production rate remains the primary key factor of production. Many production models considers constant production rate. A breakthrough to the production model of deteriorating items was that of Goyal and Giri [7] who developed a production inventory problem of a product with time varying demand, production and deterioration rate. Kaliraman*et a*l. [8] came up with an EPQ inventory model of deteriorating items with Weibull deterioration under stock dependent demand. Sakar [9] provided a production inventory model with probabilistic deterioration in two-echelon supply chain management. Sakar [10] came up with an economic manufacturing model with probabilistic deterioration in a production system. Palanivel and Uthayakumar [11] also proposed an EPQ model for deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation. Again Palanivel and Uthayakumar [12] reviewed their work in [11] by considering probabilistic deterioration. Karthikeyan and Viji [13] came up with Economic Production Quantity inventory model for three levels of production with deteriorating item. Uthayakumar and Tharani [14] proposed an Economic Production Model for deteriorating items and time dependent demand with rework and multiple production setups. Karthikeyan and Viji [15] reviewed their work in [13] by considering Weibull distribution and shortage.

Recent studieson non-instantaneous (delayed) deteriorating inventory models are that of Ahmad and Musa [16] who developed an EOQ model for delayed deteriorating items with time dependent exponential declining demand. Sani and Yakubu [17] presented an EOQ model for deteriorating items that exhibit delay in deterioration with discrete time. Musa and Adam [18] developed an ordering policies of delayed deteriorating items with unconstrained retailer's capital, linear trend in demand and shortages. A classical EPQ model of non-instantaneous

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deteriorating items by considering relationship between holding cost and ordering cycle length with backordering was presented by Ghasemi [19]. In this paper, an effort has been made to develop a production inventory model of delayed (Non-instantaneous) deteriorating items in which deteriorating rate, holding cost and demand are regarded as constant fraction of the production items. Shortages are not allowed.

2.0 ASSUMPTIONS AND NOTATIONS

The model is developed based on the following assumptions and notations

Assumptions

(i)Shortages not allowed
(ii) Lead time is zero
(iii) There is no repair or replacement of items that deteriorate during a cycle.
(iv)The deterioration occurs only when the item are effectively in stock.

Notations

(i) α Constant production rate
(ii) d Constantdemand rate
$(iii)\vartheta$ Constant deterioration rate
(iv) T_1The time the production stops and deterioration begins.
(v) TThe length of time for the cycle
$(vi)A_0$The set up cost
$(vii)c$The unit production cost of item
$(viii)I_1(t)$Inventory level at any time t during the production period.
$(ix)I_2(t)$Inventory level at any time t during the deterioration period.
$(x)TVC(T)$ The total variable cost per unit time.
$(xi)h$ Inventory carrying charge.

3.0 Formulation and Solution of the model

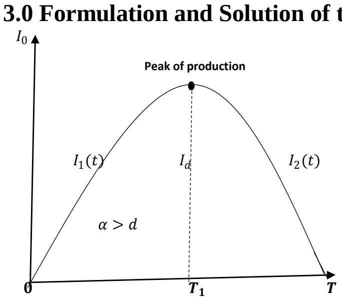

Figure 1: Inventory movement in a delayed deteriorating production system

The production level during the production period $[0, T_1]$ is a function of production rate $\alpha$and the demand rate d. the depletion of the inventory during the period$[T_1, T]$is function of deteriorating rate$\vartheta$and the constant demand rate d. The differential equations governing the inventory system are given below:

$$\frac{dI_1(t)}{dt} = \alpha - d, \qquad 0 \le t \le T_1 \tag{1}$$

$At\ t = 0,\ I_1(t) = I_0,\ at\ t = T_1,\ I_1(t) = I_d$

$$\frac{dI_2}{dt} + \vartheta I_2 = -d, T_1 \le t \le T \tag{2}$$

$At\ t = T_1,\ I_2(t) = I_d. \quad at\ t = T,\ I_2(t) = 0$

From (1)

$$\frac{dI_1(t)}{dt} = \alpha - d$$

We solve (1) using separation of variables

$$\int dI_1(t)dt = (\alpha - d)\int dt$$

$$\Rightarrow I_1(t) = (\alpha - d)t + C_1 \tag{3}$$

Where C_1 is an arbitrary constant

$at\ t = 0, I_1(t) = I_0.$

$$I_0 = C_1 \tag{4}$$

Substituting for $I_0 = C_1$ in (3), yields:

$$I_1(t) = (\alpha - d)t + I_0 \tag{5}$$

$at\ t = T_1,\ I_1(t) = I_d$

$$\therefore I_d = (\alpha - d)T_1 + I_0 \tag{6}$$

$$\Rightarrow I_0 = I_d - (\alpha - d)T \tag{7}$$

From (2)

$$\frac{dI_2(t)}{dt} + \vartheta I_2(t) = -d$$

The integrating factor to the given first order linear equation is $I = e^{\int \vartheta dt}$. Solving the equation using the integrating factor gives

$$I_2(t) = \frac{-d}{\vartheta} e^{2\vartheta t} + C_2 \quad (8)$$

$at\ t = T_1$, $I_2(t) = I_d$

$$\therefore I_d = -\frac{d}{\vartheta} e^{2\vartheta T_1} + C_2$$

$$\Longrightarrow C_2 = I_d + \frac{d}{\vartheta} e^{2\vartheta T_1} \quad (9)$$

Substituting C_2 into equation (7), we get

$$I_2(t) = -\frac{d}{\vartheta} e^{2\vartheta t} + I_d + \frac{d}{\vartheta} e^{2\vartheta T_1} \quad (10)$$

$at\ t = T,\ I_2(t) = 0.$

From (10),

$$-\frac{d}{\vartheta} e^{2\vartheta T} + I_d + \frac{d}{\vartheta} e^{2\vartheta T_1} = 0$$

$$\Longrightarrow I_d = \frac{d}{\vartheta}(e^{2\vartheta T} - e^{2\vartheta T_1}) \quad (11)$$

Substituting (11) in (7), we get

$$I_0 = \frac{d}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (\alpha - d)T \quad (12)$$

Substituting (12) into (5) we have,

$$I_1(t) = (\alpha - d)(t - T) + \frac{d}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) \quad (13)$$

Substitute (10) into (9) we get final expression for I_2 as,

$$I_2(t) = \frac{d}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta t}\right) \quad (14)$$

The total demand over deteriorating period $[T_1, T]$ is denoted by D_T

$$D_T = d(T - T_1). \quad (15)$$

The amount of item that deteriorate over the period $[T_1, T]$ is denoted by A_d

$$A_d = I_d - D_T$$

$$\Longrightarrow A_d = d\left(\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right) \quad (16)$$

The total cost of items that deteriorate over the period $[T_1, T]$ is given by

$$D_c = CA_d$$

$$\Longrightarrow D_c = Cd\left(\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right) \quad (17)$$

The inventory holding cost is given by:

$$H_c = \int_o^{T_1} hI_1(t)dt + \int_{T_1}^{T} hI_2(t)dt$$

$$= h\left(\left(\frac{Td}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(e^{2\vartheta T}\left(T - T_1 - \frac{1}{2\vartheta}\right) + \frac{1}{2\vartheta} e^{2\vartheta T_1}\right)\right) \quad (18)$$

The Total variable cost TVC is given by the expression,

$$TVC = A_0 + H_c + D_c$$

Thus;

$$TVC = A_0 + h\left(\left(\frac{Td}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(e^{2\vartheta T}\left(T - T_1 - \frac{1}{2\vartheta}\right) + \frac{1}{2\vartheta} e^{2\vartheta T_1}\right)\right)$$

$$+ \ Cd\left(\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right) \quad (19)$$

The total inventory cost per unit time is,

$$TVC(T) = \frac{A_0}{T} + \frac{h}{T}\left[\left(\frac{Td}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(e^{2\vartheta T}\left(T - T_1 - \frac{1}{2\vartheta}\right) + \frac{1}{2\vartheta} e^{2\vartheta T_1}\right)\right]$$

$$+ \ \frac{cd}{T}\left[\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right] \quad (20)$$

Differentiating (20) with respect to T we get,

$$\frac{dTVC\,(T)}{dT} = \frac{A_0}{T^2} + \frac{h}{\vartheta T^2}\left(dT_1 + \frac{d}{2\vartheta^2} - 2e^{-2\vartheta T_1}\right) + \frac{hd}{\vartheta^3} e^{2\vartheta T} - \frac{h(\alpha - d)}{2}$$

$$+ \frac{1}{\vartheta T^2}\left(e^{2\vartheta T}(cd\vartheta - Tcd) + e^{2\vartheta T_1}(cd) - cT_1 d\vartheta\right) \quad (21)$$

Multiplying (21) through by T^2 and equate to zero yields;

$$A_0 + \frac{h}{\vartheta}\left(dT_1 + \frac{d}{2\vartheta^2} - 2e^{-2\vartheta T_1}\right) + \left(\frac{hd}{\vartheta^3} e^{2\vartheta T} - \frac{h(\alpha - d)}{2}\right)T^2 + \frac{1}{\vartheta}\left(e^{2\vartheta T}(cd\vartheta - Tcd) + cde^{2\vartheta T_1} - cdT_1\vartheta\right) = 0 \quad (22)$$

The solution of equation (21) gives us the optimal cycle length $T = T^*$.

The optimal total variable cost per unit time for the inventory system is obtained from equation (20) with $T = T^*$.

EPQ = Total demand before deterioration + total demand after deterioration + total items that deteriorated.

$$= dT + d(T - T_1) + d\left[\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right]$$

$$= d\left[T + \frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right)\right]. \quad (23)$$

4.0 Numerical Examples

The following are 5 numerical examples with different parameter values. The output obtained (using Maple 2017 Mathematical software) gives the optimal cycle length $T = T^*$, Optimal total variable cost per unit time of the inventory system, $TVC(T)$ and the Optimal production quantity EPQ.

Table 1: EPQ, Total variable cost and Optimal cycle length for production model with constant production rate.

S/N	A_0	C	ϑ	d	h	α	T_1	$T(days)$	TVC	EPQ
1	100	500	0.30	250	0.06	100	0.0023	324	20,510	808
2	500	100	0.50	250	0.02	200	0.005	331	56,935	965
3	300	500	0.40	150	0.02	150	0.002	325	14,401	523
4	1000	500	0.15	300	0.01	500	0.003	333	19,553	902
5	700	100	0.75	500	0.01	500	0.003	358	17,892	2,731

5.0 Conclusion

In this paper we derived an inventory production model for items that exhibit delay in deterioration. Both the production rate and demand rate were taken as constant, shortages were not allowed in the model and the optimal solution of total variable cost per unit time and EPQ were obtained. The outcome of the result reveals that the higher the demand rate the higher the EPQ. This is reasonable as the production must rise with rise in the demand to avoid under stocking which will result in the shortage cost. The model can be extended by considering shortages and backordering.

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