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NON LINEAR TIME SERIES MODELLING AND CHANGE POINT DETECTION OF MUNICIPAL SOLID WASTE GENERATED IN LAGOS STATE, NIGERIA

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Abstract

In this study, we analyse municipal solid waste generated in Lagos State using non-linear time series model and change point detection technique. This is necessary because so much attention have been given to linear models when it comes to analysing municipal solid waste generated in Lagos State. Change point detection technique was used to measure the intervention of government on volume of municipal solid waste generated in Lagos State. We fitted non-linear time series model particularly bilinear time series model and compared it with linear model namely autoregressive integrated moving average model. The data used for this study exhibited non-stationary having conducted unit root test. Using Akaike information criterion the appropriate model fitted for nonlinear and linear models respectively were of the order (6, 1, 0) and (6, 1, 0, 1, 1).There was a single change point in the volume of solid waste generated between 1980 and 2014 and this occurred in 1987 with an upward shift. This change could be caused by increase in population and efficient service of the waste authorities. The non-linear model performed better than the linear model with a lower residual variance and root mean square error. We recommend non-linear times series model particularly bilinear model for predicting municipal solid waste that will be generated in Lagos State.

***Keywords*:** Solid waste, Change point, Pettitt approach, Intervention, Time series models

1.0 Introduction

Municipal solid waste generation has been a major concern to any government and the populace in any given society. Municipal solid waste is a daily activities which is natural. When it is not properly disposed there is tendency for it to increase indiscriminately and this can pose a threat to humans. The situation always become worse when there is no measure put in place when it comes to the disposal of these municipal solid waste. When these wastes are not properly disposed, it can result to outbreak of diseases in that society. The municipal solid waste generation has become serious problems which are more common in urban areas of many developing countries [1].The increase in the volume of municipal solid waste generated has been attributed to urbanization, high population growth, increase in per-capita incomes and technological development [2] and [3]. Municipal solid waste generation is defined as refuse from households, industrial, commercial and institutional establishments, market, yard and street sweepings and the financial and technical resources needed are not always enough to combat the volume of solid waste being generated that are always on the increase [4].

There has been large accumulation of municipal solid waste in Nigeria especially within the towns and cities which has led to blockage of roads/streets and drainages with associated environmental problems[5], [6] and diseases[7]. Factors that affect generation of municipal solid waste vary geographically and can be categorized into social, economic, and environmental [8].

Lagos being one of the populous city in Nigeria, has faced the problem of collection and disposal of municipal solid waste. The flooding that has been occurring in the recent time in Lagos is as a result of dumping of refuse at canals and drainages[9]. Lagos state particularly Lagos city has witnessed increase in her population in the recent time [10] and as a result if no proper measures are put in place in waste generation compare to its population that is on the high side, the result will be dangerous.

This study is set to detect change point in the volume of municipal solid waste generated which will enable us to know the impact of intervention by the government in waste generation. In addition, since less attention has been given to using non-linear time series model particularly bilinear time series models for predicting solid waste that will be generated in Lagos State, there is the

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need to suggest and build appropriate non-linear time series model. This will help policy makers to take right decision regarding the disposal and management of waste generated.

2.0 Theoretical Analysis

2.1 Data sources and Description

Data was collected from Lagos State Waste Management Authority (LAWMA). The scope of the data was 35 years (1980 to 2014). For the purpose of change point detection, the annual data was used. We employed an appropriate transformation technique, which was used to transform the data to quarterly data (140 data point) for time series modelling. The transformation is needed so as to obtain a better estimate.

2.1 Change Point Detection

2.2.1 Trend test

The Mann–Kendall (MK) test which has been found to be appropriate was used for trend detection and this is necessary before proceeding to change point detection. MK test statistic is given by:

$$U_{t,T} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \text{sgn}(X_j - X_i) \tag{2.1}$$

X_i and X_j represent the sequential data values, n is the number of the recorded data. We state the null and the alternative as follows:

H_0: There is no presence of trend in the data.

H_1: There is presence of trend in the data.

We assume that the data are independent and identically distributed with mean zero and the variance of S which is denoted by σ^2 is given as:

$$\sigma^2 = \frac{n(n-1)(2n+S)}{18} \tag{2.2}$$

The trend test statistic is given as:

$$Z = \begin{Bmatrix} \dfrac{S-1}{\sigma} & & S > 0 \\ 0 & if & S = 0 \\ \dfrac{S+1}{\sigma} & & S < 0 \end{Bmatrix} \tag{2.3}$$

Decision rule is that the null hypothesis will be rejected if the calculated Z statistic is greater than the critical value gotten from the standard normal distribution table corresponding to the pre-specified significance level. If the value Z is positive, it indicates an increasing trend and if is negative, it is a decreasing trend [11]. The trend magnitude is estimated using a non-parametric median based slope estimator proposed by [12] and extended by [13]. The slope estimation is given by:

$$\beta = Median\left|\frac{x_j - x_k}{j-k}\right| \forall k < j \tag{2.4}$$

2.2.2 Change point test

Pettitt change point test which is a rank-based and distribution-free test for detecting a significant change in the mean of a time series was used to detect occurrence of the abrupt change [11].The null and the alternative hypotheses which make use of non-parametric statistic

$K_T = \max|U_{t,T}| = \max(K_{T+}, K_{T-})$ are stated thus:

H_0: No change (or τ=T).

H_1: Change (or $1 \leq \tau < T$).

where

$$U_{t,T} = \sum_{i=1}^{t} \sum_{j=t+1}^{T} \text{sgn}(X_t - X_j) \tag{2.5}$$

$$Sgn(\theta) = \begin{Bmatrix} 1 & & \theta > 0 \\ 0 & if & \theta = 0 \\ -1 & & \theta < 0 \end{Bmatrix} \tag{2.6}$$

$K_{T+} = \max U_{t,T}$ for downward shift and $K_{T-} = \min U_{t,T}$ for upward shift. The confidence level associated with K_{T+}or K_{T-} is determined approximately by:

$$\rho = \exp\left(\frac{-6K_T^2}{T^3 - T^2}\right) \tag{2.7}$$

The probability for a change-point is given as $P = 1 - \rho$.It is clear that where a significant change point exists, the series is divided at the location of the change point into two subseries[11].

2.3 Time Series Modelling

In time series modelling, the first thing to be done is the plot of the series over time and by this we look at the outliers, troughs, presence of turning points that may be pronounced on the time plot. Another exploration is the trend which is the general direction the graph of a set of observation tends to be going over a period of time(usually at equal intervals).The nature of the study will dictate whether to observe the sets of observations daily, weekly, monthly, quarterly or annually. Details on trend are available in [14] and [15]. We tested the data for the presence of unit root using Augmented Dickey-Fuller statistic [16]. It was found out that the series is not stationary, that there is a presence of unit root which can result to system of equations explosion. Solving unit root problem, the data for this study was differenced and as a result the suggested model is Autoregressive Integrated Moving Average (ARIMA) model. Then, for ARIMA model, we fixed the order of integration (d) at one. The specification, estimation and order determination of ARIMA is available in [17]. Often time, since linearity is an assumption in which the real series under study may not exhibit linearity a linear model has always been fitted. Then, there is the need to consider a non- linear situation where a non-linear model is fitted. As a result, a type of nonlinear time series model namely autoregressive integrated moving average bilinear time series model was fitted to municipal solid waste generated. The specification and estimation of this model is available in [18]). There are quite some number of measures of accuracy in the modelling and forecasting literature [19]. For this study we used residual variance and root mean square error. Forecast using a non-linear time series model is scarce in literature and as a result in this study we used exponential smoothing but an autoregressive integrated moving average bilinear induced which gives the direction of future solid waste that would be generated.

3.0 Experimental Work

3.1 Change Point Detection

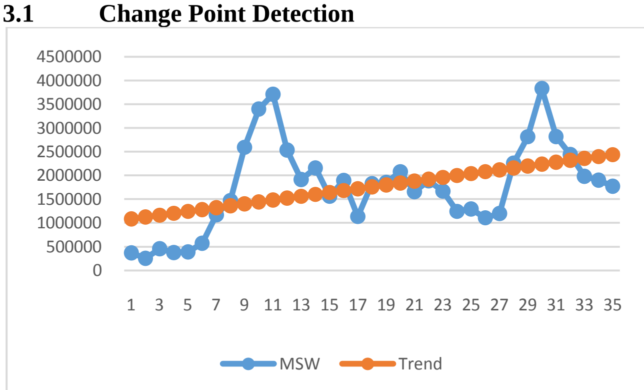

Figure 1: Plot of Annual Volume of Municipal Solid Waste (MSW) Generated in Lagos State and Associated Trend

From figure 1, we could see the behavior of municipal solid waste generated for a period of thirty five (35) years. We noticed that the highest municipal solid waste generated was recorded in 1990 and 2009. Years following 1990 and 2009 witnessed decrease in the municipal solid waste generated. Generally, an increasing trend was witnessed.

Table 1: Mann-Kendall Trend Test for the Annual Volume of Waste Generated in Lagos

Z	Sen Slope	P-Value	Trend
2.5279	41311.86	0.011<0.05	Increasing(sig)

Table 2: Pettitt Change Point Test for the Annual Volume of Waste Generated in Lagos

Year	K-Value	1-P	Confidence Level
1987	224	0.001	0.999

We could see in table 1 that the value of Z is positive indicating an increasing trend and this agreed with direction of rend in figure 1. Also, the Sen Slope is positive which support the increasing trend. The implication of this is that the volume of solid waste generated over the years is increasing meaning that the effort of waste authority is commendable since they are able to generate more waste as time progresses which resulted to a clean environment. Table2 revealed, Pettitt change point test where a single change point is recorded in the volume of waste generated between 1980 and 2014 and this occurred in 1987. When we observed the solid waste generated in 1987 from time plot in figure 1, we will notice an upward shift in the solid waste generated. This change in solid waste generated could be caused by increase in population, efficient/reliable service of the waste authorities (that is more public and private waste companies were available).

3.2 Time Series Modelling

Correlation between Solid Waste and Time

Correlation coefficient = 0.316, P-value = 0.000<0.05, N= 140

Trend Models

Linear $\hat{T}_t = 265377.742 + 2476.202t$

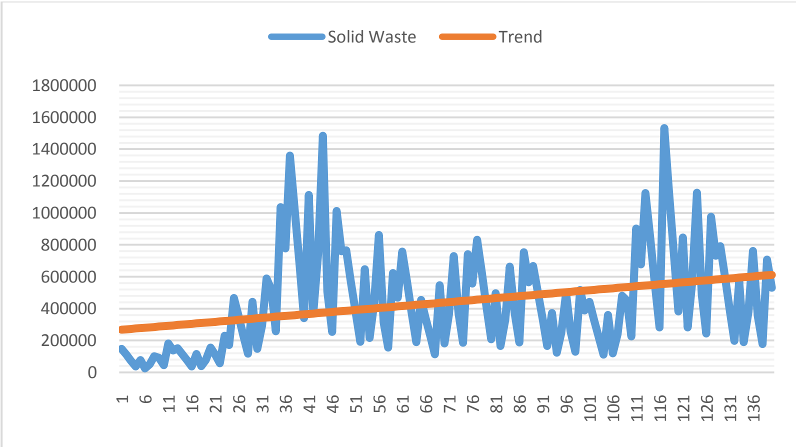

Figure 2: Time Plot and Linear Trend Model of Solid Waste Generated

There is a significant correlation between the solid waste generated and time since P <0.05. It means that solid waste generated increased over time. Likewise the trend equation of the quarterly data typified an increasing trend as we can see in figure 2. This support our result when annual data was used.

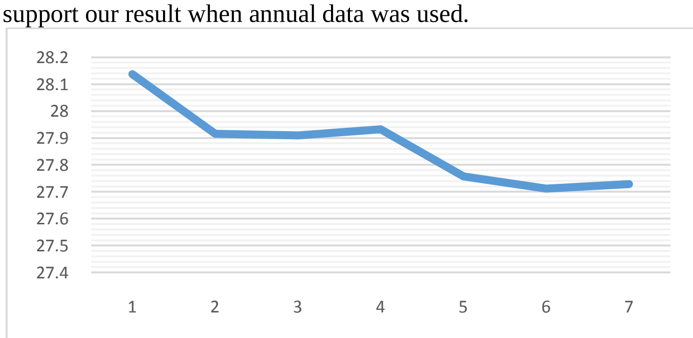

Figure 3: Plot of Akaike Information Criterion of Solid Waste Generated

In figure 3, the best model where we have minimum AIC is order 6. The estimated model is specified below and following is the non-linear model of order one.

Model Estimation

Autoregressive Integrated Moving Average (ARIMA) (6, 1, 0)

$$\hat{X}_t = -0.576929X_{t-1} - 0.570078X_{t-2} - 0.304501X_{t-3} - 0.136482X_{t-4} - 0.251257X_{t-5} + 0.261168X_{t-6}$$

Bilinear(BL) (6, 1, 0, 1, 1)

$$\hat{X}_t = -0.576929X_{t-1} - 0.570078X_{t-2} - 0.304501X_{t-3} - 0.136482X_{t-4} - 0.251257X_{t-5} + 0.261168X_{t-6} - 1.79E-07X_{t-1}e_{t-1}$$

Table 3: Measures of Performance between ARIMA and BL

	Residual Variance		**Root Mean Square Error for Forecast**		**Akaike Information Criterion**	
Models	**ARIMA**	**BL**	**ARIMA**	**BL**	**ARIMA**	**BL**
Solid Waste	$(241588.8)^2$	$(234043.4)^2$	517595.3	233990.3	27.71189	27.57823

We could see from the above table the performance of non-linear and linear model considered in this study. When we looked at the two models at model level, bilinear model performed better than autoregressive integrated moving average model-the linear type with a smaller residual variance. The same thing occurred at forecasting level with a smaller root mean square error.

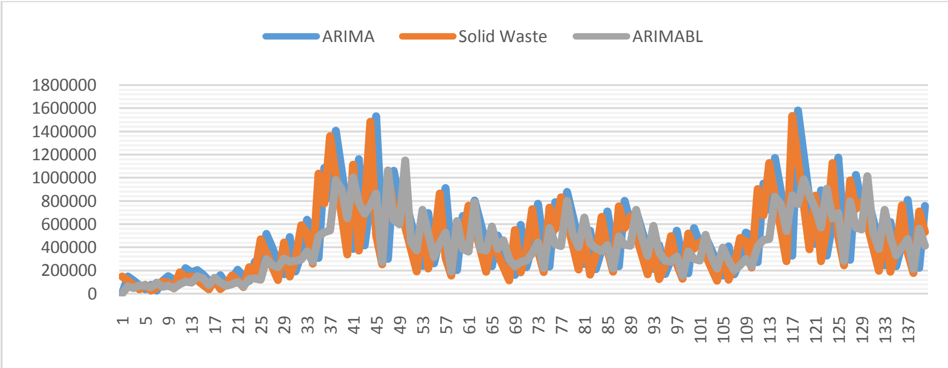

Figure 3: Plot of In-sample Forecasts and Solid Waste Generated

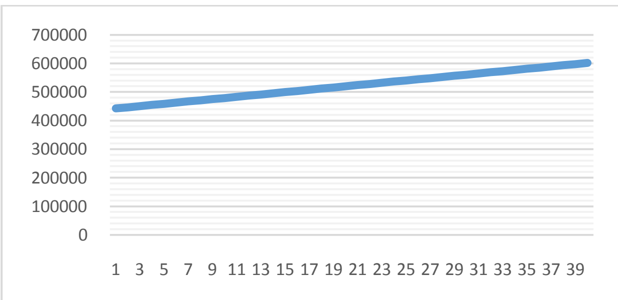

Figure 4: Plot of Out-sample Forecast (2015-2024)

4.0 Conclusion

This study employs change point detection using Pettitt approach to measure the intervention of government on volume of solid waste generated. The study also provides a suitable non-linear model that can be used to predict the municipal solid waste generated. There was a single change point in the volume of waste generated between 1980 and 2014 and this occurred in 1987with an upward shift. This change could be caused by increase in population and efficient service of the waste authorities. The data exhibited non-stationary having conducted unit root test. The appropriate models fitted were autoregressive integrated moving average and bilinear models of order (6, 1, 0) and (6, 1, 0, 1, 1), respectively. The bilinear model performed better than linear model with a lower residual variance and root mean square error. We recommend bilinear model for predicting future municipal solid waste that will be generated in Lagos State. The out-sample forecast revealed that the solid waste that would be generated in future will increase over time.

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