

NUMERICAL SIMULATION OF LOGISTIC MODEL EQUATIONS

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Abstract

The dynamical system that defines the interaction between two competing legume, for a limit resource is defined. By using O.D.E 45 numerical simulation we have studies a differential effect of the intrinsic growth rate on the biodiversity scenario. The full results of this study are presented and discussed quantitatively.

Keywords: Numerical simulation, logistic, model equation biodiversity, intrinsic growth rate.

Introduction

One of the processes of understanding the interaction between two legume such as cowpea and groundnut depend on the construction of a deterministic mathematical model that has the structure of a logistic model formation this model is defined by two intrinsic growth rate parameter values, two intra-species coefficient; initial condition and the length of the growing season. Following Ekaka-a [1] and Ekaka-a etal [2], Atsu and Ekaka-a, [3], De Mazancourt etal [4], Ekaka-a et al [5], Troost, Kori and Koo, Sam [6] Ford, Lumb and Ekaka-a [7]. The study of the effect of a model parameter variation on biodiversity is an ongoing active area of research.

In this study, we are interested to find out the differential effects of the intrinsic growth rate parameter value on the extent of a biodiversity gain and a biodiversity loss by using the method of ordinary differential equations of order 45 (ODE 45).

Mathematical Formations

Following Ekaka-a etal [8], we shall consider a continuous dynamical system of non-linear first order ordinary differential equation having the following logistic model structure.

$$\frac{dc(t)}{dt} = \alpha_1 c(t) - B_1 c^2(t) \tag{1}$$

$$\frac{dg(t)}{dt} = \alpha_2 g(t) - B_2 g^2(t) \tag{2}$$

with the initial condition

$$c(0) = c_0 > 0 \tag{3}$$

$$g(0) = g_0 > 0 \tag{4}$$

where:

c(t) defines the biomass of the cowpea legume at time (t) in the unit of days.

g(t) defines the biomass of groundnut at legume at time (t) in the unit of days.

α_1 defines the intrinsic growth rate of the cowpea legume

α_2 defines the intrinsic growth rate of groundnut

B_1, B_2 defines the intra-competition coefficients.

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Following Ekaka-a and Atsu [9] we have utilized the following parameter values

$$\alpha_1 = 0.0225$$

$$\alpha_2 = 0.0446$$

$$B_1 = 0.006902$$

$$B_2 = 0.0133$$

The length of the growing season is in the unit of days.

Method of Analysis

For the purpose of this study, we have utilize the application of ODE 45 which is computational more efficient than other numerical methods such as ODE 23, ODE 23tb and ODE 155.

Results

The full results of this present study are presented as displayed on Table 1-Table 3. The first column data represent the cowpea biomass when all the model parameter values are fixed.

The second column data represent the cowpea biomass when only the intrinsic growth rate model parameter values are varied.

The third column data represent the effect of the second column data otherwise called the perturbed data on the first column data otherwise called the original data.

The fifth column data represent the groundnut biomass when only the intrinsic growth rate model parameter values are varied.

The sixth column data represent the effect of the fifth column data otherwise called the perturbed data on the fourth column data otherwise called the original data.

Table 1: Quantifying the effect of decreasing the intrinsic growth rates of two interacting cowpea and groundnut by 10 percent on biodiversity loss: length of growing season is ten (10) months.

Example	$C_0(t)$	$C_n(t)$	BL(%)	$G_0(t)$	$G_n(t)$	BL(%)
1	0.4800	0.4800	0	0.4800	0.4800	0
2	0.4893	0.4795	2.0013	0.4986	0.4791	3.9221
3	0.4987	0.4790	3.9561	0.5179	0.4782	7.6659
4	0.5083	0.4785	5.8653	0.5377	0.4773	11.2398
5	0.5180	0.4780	7.7301	0.5581	0.4764	14.6515
6	0.5279	0.4775	9.5516	0.5792	0.4755	17.9084
7	0.5379	0.4770	11.3307	0.6009	0.4746	21.0176
8	0.5481	0.4765	13.0684	0.6232	0.4737	23.9859
9	0.5584	0.4760	14.7658	0.6461	0.4729	26.8199
10	0.5689	0.4755	16.4237	0.6697	0.4720	29.5255

Table 2: Quantifying the effect of decreasing the intrinsic growth rates of two interacting cowpea and groundnut by 99.8 percent on biodiversity loss: length of growing season is ten (10) months.

Example	$C_0(t)$	$C_n(t)$	BL(%)	$G_0(t)$	$G_n(t)$	BL(%)
1	0.4800	0.4800	0	0.4800	0.4800	0
2	0.4893	0.4893	0.0045	0.4986	0.4986	0.0089
3	0.4987	0.4987	0.0090	0.5179	0.5178	0.0177
4	0.5083	0.5082	0.0134	0.5377	0.5376	0.0265
5	0.5180	0.5179	0.0179	0.5581	0.5579	0.0352
6	0.5279	0.5278	0.0223	0.5792	0.5789	0.0438
7	0.5379	0.5378	0.0267	0.6009	0.6006	0.0523
8	0.5481	0.5479	0.0311	0.6232	0.6228	0.0608
9	0.5584	0.5582	0.0355	0.6461	0.6457	0.0692
10	0.5689	0.5687	0.0398	0.6697	0.6692	0.0774

Table 3: Quantifying the effect of increasing the intrinsic growth rates of two interacting cowpea and groundnut by 150 percent on biodiversity loss: length of growing season is ten (10) months.

Example	$C_0(t)$	$C_n(t)$	BL(%)	$G_0(t)$	$G_n(t)$	BL(%)
1	0.4800	0.4800	0	0.4800	0.4800	0
2	0.4893	0.4948	1.1294	0.4986	0.5098	2.2475
3	0.4987	0.5100	2.2676	0.5179	0.5413	4.5293
4	0.5083	0.5256	3.4144	0.5377	0.5745	6.8438
5	0.5180	0.5417	4.5696	0.5581	0.6094	9.1893
6	0.5279	0.5582	5.7330	0.5792	0.6462	11.5640
7	0.5379	0.5751	6.9044	0.6009	0.6848	13.9656
8	0.5481	0.5924	8.0835	0.6232	0.7254	16.3916
9	0.5584	0.6102	9.2701	0.6461	0.7679	18.8394
10	0.5689	0.6285	10.4639	0.6697	0.8124	21.3060

Discussion of Results

Since the length of the growing season is consistent with ten (10) months, the decreased variations of the intrinsic growth rates of the cowpea and groundnut have predicted biodiversity loss whereas the increased variation of the intrinsic growth rates of the cowpea and groundnut has predicted biodiversity gain.

Conclusion

In this study, we have applied the method of a numerical method (ODE 45 simulation modeling) to predict biodiversity loss due to a decreased variation of the intrinsic growth rates together and also predict biodiversity gain due to an increased variation of the intrinsic growth rates together. The effective calculation of the interval of the growth rates where biodiversity loss changes to a biodiversity gain remains to be an open problem that we would consider in our next research investigation.

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