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MHD STAGNATION-POINT FLOW OF AN UPPER CONVECTED MAXWELLFLUID OVER A STRETCHING SURFACE BY MODIFIED ADOMIAN DECOMPOSITION METHOD (MADM1)

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Abstract

The Modified Adomian decomposition method (MADM1) is a very reliable method that has been developed in an attempt to improve the accuracy and at the same time provide faster computation of series solutions. It is a very efficient tool used to solve different types of nonlinear differential equations which includes application to MHD stagnation-point flow of an upper-convected Maxwell fluid problem. In this study, we have extended the application of MADM1 by showing its ability to solve the nonlinear differential equation of the MHD stagnation-point flow of an upper-convected Maxwell fluid problem. The results obtained show better agreement with existing work.

***Keywords*:** Modified Adomian decomposition method (MADM1), MHD, upper-convected Maxwell fluid

1.0 INTRODUCTION

Recently, so many reliable modifications of the Adomian decomposition method that provide faster computation and more improved accuracy of solution series for nonlinear differential equations have been developed. They include, first and second modified decomposition method by Wazwaz (MADM1 and MADM2), the two step Adomian decomposition method (TSADM), the restarted Adomian decomposition method (RADM), Laplace Adomian decomposition method (LADM), Kamal Adomian decomposition modified (KADM) and modified Laplace Adomian decomposition method (MLADM). (Al-mazmumy*et al.,* [1]; Alizadeh and Effati, [2]; Tomaizeh, [3]). The modified decomposition method in general requires only a slight variation from the standard decomposition method. It may also provide exact solutions by using few iterations and sometimes without theAdomian polynomials Xie, [4]. However, we will consider the first modification by Wazwaz (MADM1).

The first modification form was established based on the assumption that the function f can be divided into two parts namely, one assigned to the initial term of the series and the other to the second term. The remaining terms of the recursive relationship are obtained, thus resulting in a series that is different from the standard ADM being generated. The slight variation in reducing the number of terms of the initial component results in the reduction of the computational work and increases the accuracy of the series solution. It may also provide the exact series solution by using two iterations only. Hence, there may be no need to evaluate the Adomian polynomials required for the nonlinear equations. (Xie, [4]; Rach, [5];Alizadeh and Effati, [2]; Wazwaz, [6]). The MADM1 is therefore very efficient and saves time in computation while maintaining high accuracy of the series solution.

Hence, we will apply the MADM1 on MHD stagnation flow of an upper convected Maxwell fluid. The UCM fluid are highly viscous and elastic. The stress tensor in this fluid have a nonlinear relationship with the deformation rate tensor. These nonlinear relationships giverise to complicated higher order differential equations that are difficult to solve. As a result of this, so many researchers are involved in finding solutions to the nonlinear differential equations governing the motion of these fluids using different methods. Sadeghy*et al.,*[7] theoretically obtained the solution of a two dimensional stagnation point flow of an upper–convected Maxwell fluid using the Chebyshev pseudo-spectral collocation-point method. Kumari and Nath, [8] investigated a steady two-dimensional mixed convection MHD stagnation-point flow of upper-convected Maxwell fluid using the finite-difference method. Hayat, *et al.*, [9] studied a steady two dimensional MHD flow

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of an upper-convected Maxwell fluid near a stagnation point over a stretching surface using homotopy analysis method. The effects of magnetic field was examined. The result obtained showed that as the magnetic field parameter increases, the velocity of the flow decreased. Lok*et al.,* [10] numerically investigated the effects of elasticity, shrinking and suction parameters on a steady two-dimensional boundary layer stagnation-point flow and heat transfer of an upper-convected Maxwell fluid using the Keller-box method (an implicit finite-difference method with second-order accuracy) Mustaq*et al.,* [11] examined the effects of thermal radiation on a laminar two-dimensional stagnation point flow of an upper-convected Maxwell fluid using fourth order Runge-Kutta integration technique. Rahman, [12] investigated the effects of second order slip and magnetic field on a nonlinear mixed convection stagnation point flow of an upper-convected Maxwell fluid towards a vertical permeable stretching sheet using the bvp4c function from MATLAB. Mustafa *et al.*, [13] analyzed a non-aligned MHD stagnation-point flow of upper-convected Maxwell fluid in the presence of a linear radiative flux and heat source/sink using the fifth order Runge-kutta with shooting technique and also a collocation method based MATLAB package bvp4c. Hayat *et al.,* [14]studied the effects of mass transfer on a two-dimensional stagnation-point flow of an upper-convected Maxwell fluid using the homotopy analysis method. Hayat *et al.*, [15] studied a rheological model of a steady laminar boundary layer stagnation-point flow of an upper-convected Maxwell fluid with heat transfer past a stretching sheet using the homotopy analysis method. The effects of melting parameter was examined.

From literature, we observe that MHD stagnation flow of an upper-convected Maxwell fluid problem have been solved using homotopy analysis method. In this study therefore we will show the ability of theMADM1 to solve the nonlinear differential equations of the MHD stagnation-point flow of an upper convected Maxwell fluid.

2.0 BASIC IDEA OF THE FIRST MODIFIED ADOMIAN DECOMPOSITION METHOD (MADM1)

let us consider the nonlinear differential equation

$$Lu + Nu + Ru = g, \tag{1}$$

where,

L −is invertible and is taken as the highest order derivatives,

R −is the remainder of the linear operator.

$N(u)$ − represents the nonlinear terms,

g − is the specified inhomogeneous term.

To obtain Lu, we have,

$$Lu = g - Nu - Ru. \tag{2}$$

Applying the operator L^{-1}, the equation (3.1.2) becomes

$$L^{-1}Lu = L^{-1}g - NL^{-1}u - L^{-1}Ru, \tag{3}$$

where,

The solution series u is decomposed into a series of the form

$$u = \sum_{n=0}^{\infty} u_n \tag{4}$$

And the nonlinear term Nu is decomposed as,

$$N(u) = \sum_{n=0}^{\infty} A_n. \tag{5}$$

Substituting (3.1.4) and (3.1.5) into (3.1.3), we have

$$\sum_{n=0}^{\infty} u_n = \varphi + L^{-1}g - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n,$$

where

$$\varphi = \begin{cases} u(0) & if\ L = \frac{d}{dx} \\ u(0) + xu'(0), & if\ L = \frac{d^2}{dx^2} \\ u(0) + xu'(0) + \frac{x^2}{2!}u''(0)\,, & if\ L = \frac{d^3}{dx^3} \\ \vdots & \\ u(0) + xu'(0) + \frac{x^2}{2!}u''(0) + \cdots + \frac{x^n}{n!}u^{(n)}(0) & if\ L = \frac{d^{n+1}}{dx^{n+1}} \end{cases} \tag{6}$$

Recall that the Adomian decomposition method (ADM) suggests that the zeroth componentu_0 is usually defined by the function $u_0 = \varphi + L^{-1}g = f$.

Thus

$u_0 = f$,

$u_1 = -L^{-1}Ru_0 - L^{-1}A_0$,

$u_{n+1} = -L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \,, n \geq 0.$ (7)

Under this assumption, we set

$f = y_0 + y_1.$ (8)

The MADM1 recursive relation is formulated as follows;

$u_0 = y_0,$

$u_1 = y_1 - L^{-1}Ru_0 - L^{-1}A_0,$

$u_{n+1} = -L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \,, \quad n \geq 0.$ (9)

On comparing the recursive relation (7) of ADM with that of MADM1 (9), we observe that the zeroth component u_0 is defined by a part y_0 of f. The remaining part y_1 of f is added to the definition of the component u_1 in (9).

Hence ,we will use an example to illustrate that the reduction of terms of the zeroth component u_0 will result in a reduction of the computational work. Also the slight variation in the definition of the components u_0 and u_1 may provide the solution by using two iterations only. Furthermore, our calculation will show that there is no need sometimes to evaluate the Adomian polynomials required for the nonlinear differential equations.

3.0 APPLICATION

Consider a steady MHD stagnation-point flow of upper convected Maxwell fluid problem of Hayat *et al.*, [9]. The equations of motion of the flow are

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0,$ (10)

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_{1-}\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = a^2 x + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho},$ (11)

where $u, v -$ are the velocity components in the x and y directions respectively,

$\mu -$ is the kinematic viscosity of the fluid,

$\rho -$ is the density,

λ_{1-} the relaxation time,

$\sigma -$ the electrical conductivity of the fluid,

$B_0 -$ a constant magnetic field.

Along with boundary conditions

$u(x,0) = cx\,;\quad v(x,0) = 0;\quad$ at$y = 0,$

$u(x,\infty) \to ax;\quad$ as$\eta \to \infty,$

Where c -is the stretching rate.

Using the following similarity variables as given by Hayat *et al.*, [9],

$$\eta = \sqrt{\frac{c}{\mu}}y \quad ; \quad u = cxf'(\eta) \quad ; \quad v = -\sqrt{\mu c}\, f(\eta),$$

the above governing equation is transformed as

$f''' - M^2 f' - f'^2 + ff'' + \lambda^2 + \beta(2ff'f'' - f^2 f''') = 0,$ (12)

Subject to

$f(0) = 0,\ f'(0) = 1 \qquad at\ \eta = 0\,,$

$f'(\infty) = \lambda \qquad as\ \ \eta \to \infty.$ (13)

with $M^2 = \frac{\sigma B_0^2 x}{\rho \cup_\infty},\ \beta = \lambda_1 c$ and $\lambda = \frac{a}{c.}$

4.0 METHOD OF SOLUTION

Recall from ADM

$L_f^{-1}f = M^2 L_f^{-1}f' + L_f^{-1}f'^2 + L_f^{-1}\,ff'' - L_f^{-1}\lambda^2 - \beta L_f^{-1}(2ff'f'' - f^2 f'''),$

where

$$L_f^{-1} = \int_0^\eta\int_0^\eta\int_0^\eta d\eta d\eta d\eta, \quad :$$

hence,

$$f = f(0) + \eta f'(0) + \frac{\eta^2}{2}f''(0) - L_f^{-1}\lambda^2 + M^2 L_f^{-1}\sum_{n=0}^{\infty} f_n' + L_f^{-1}\sum_{n=0}^{\infty} f_n'^2 + L_f^{-1}\sum_{n=0}^{\infty} f_n f_n'' - \beta L_f^{-1}\sum_{n=0}^{\infty}(2f_n f_n' f_n'' - f_n^2 f_n''').$$

On substituting the boundary conditions (13), we have

$$f = \eta + \frac{A}{2}\eta^2 - \frac{\lambda^2}{6}\eta^3 + M^2 L_f^{-1}\sum_{n=0}^{\infty} f_n' + L_f^{-1}\sum_{n=0}^{\infty} f_n'^2 + L_f^{-1}\sum_{n=0}^{\infty} f_n f_n'' - \beta L_f^{-1}\sum_{n=0}^{\infty}(2 f_n f_n' f_n'' - f_n^2 f_n'''). \quad (14)$$

By MADM1, $f = y_0 + y_1$,where $y_0 = \eta - \frac{\lambda^2}{6}\eta^3, y_1 = \frac{A}{2}\eta^2$

Thus,

$$f_0 = \eta - \frac{\lambda^2}{6}\eta^3,$$

$$f_1 = \frac{A}{2}\eta^2 + M^2 L_f^{-1}\sum_{n=0}^{\infty} f_n' + L_f^{-1}\sum_{n=0}^{\infty} G_n + L_f^{-1}\sum_{n=0}^{\infty} C_n - \beta L_f^{-1}\sum_{n=0}^{\infty}(2D_n - E_n), \quad (15)$$

$$f_{n+1} = M^2 L_f^{-1} f_n' + L_f^{-1}\sum_{n=0}^{\infty} G_n + L_f^{-1}\sum_{n=0}^{\infty} C_n - \beta L_f^{-1}\sum_{n=0}^{\infty}(2D_n - E_n), \quad (16)$$

where G_n, C_n, D_n, and E_n are the Adomian polynomials and may be computed using the Adomian general formula

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, 3 \ldots \quad (17)$$

Thus,

$$f_1 = \frac{1}{2}A\,\eta^2 + \frac{M^2}{6}\eta^3 - \frac{M^2\lambda^2}{240}\eta^5 + \frac{1}{240}(1+\lambda^2)\eta^5 - \frac{\lambda^2}{630}\eta^7 - \frac{\lambda^4}{2016}\eta^8 + \frac{\lambda^4}{18144}\eta^9 + \frac{\lambda^2\beta}{30}\eta^5 - \frac{\lambda^4\beta}{210}\eta^7 - \frac{\lambda^4\beta}{630}\eta^7 + \frac{\lambda^6\beta}{3024}\eta^9 + \frac{\lambda^2\beta}{60}\eta^5 - \frac{\lambda^4\beta}{630}\eta^7 + \frac{\lambda^4}{18144}\eta^9. \quad (18)$$

In general, we have our series solution for two iterations as

$$f = \eta - \frac{\lambda^2}{6}\eta^3 + \frac{1}{2}A\,\eta^2 + \frac{M^2}{6}\eta^3 - \frac{M^2\lambda^2}{240}\eta^5 + \frac{\eta^3}{6} + \frac{\lambda^4}{840}\eta^7 - \frac{\lambda^4}{1470}\eta^7 + \frac{\lambda^2\beta}{30}\eta^5 - \frac{\lambda^4\beta}{210}\eta^7 - \frac{\lambda^4\beta}{630}\eta^7 + \frac{\lambda^6\beta}{3024}\eta^9 + \frac{\lambda^2\beta}{60}\eta^5 - \frac{\lambda^4\beta}{630}\eta^7 + \frac{\lambda^4}{18144}\eta^9 \quad (19)$$

and

$$f' = 1 - \frac{\lambda^2}{2}\eta^2 + A\,\eta + \frac{M^2}{2}\eta^2 - \frac{M^2\lambda^2}{48}\eta^4 + \frac{1}{2}\eta^2 + \frac{\lambda^4}{120}\eta^6 - \frac{\lambda^4}{210}\eta^6 + \frac{\lambda^2\beta}{6}\eta^4 - \frac{\lambda^4\beta}{30}\eta^6 - \frac{\lambda^4\beta}{90}\eta^6 + \frac{\lambda^6\beta}{336}\eta^8 + \frac{\lambda^2\beta}{12}\eta^4 - \frac{\lambda^4\beta}{90}\eta^6 + \frac{\lambda^6\beta}{2016}\eta^8 \quad (20)$$

5.0 RESULTS AND DISCUSSION

The results obtained from the variation of the resulting parameters such as magnetic field parameter(M), Deborah number (β), (that is the dimensionless elastic parameter) and the stretching parameter (λ) on the velocity of the flow are presented graphically using Mathematical 8 to illustrate their effects on the flow. Results for the effects of β, λ *and* M variations on the skin friction coefficient are also shown .

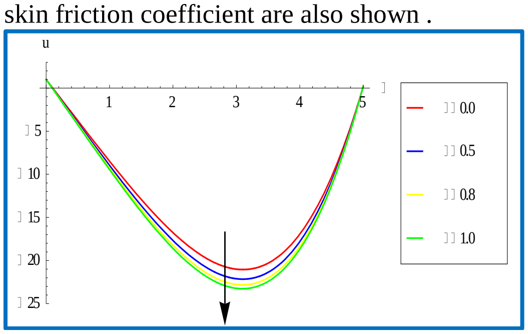

Figure 1: Effect of β on Velocity 1

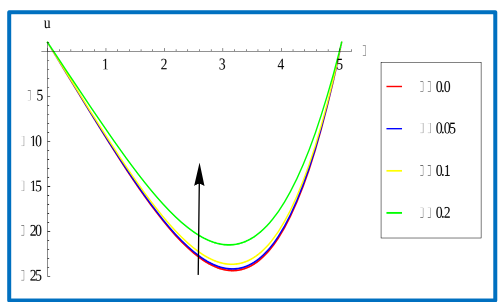

Figure 3: Effect of λ on Velocity 1

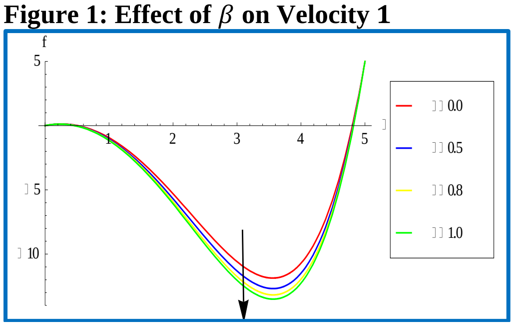

Figure 2: Effect of β on Velocity 2

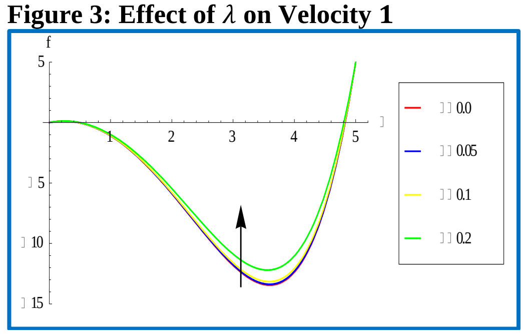

Figure 4: Effect of λ on Velocity 2

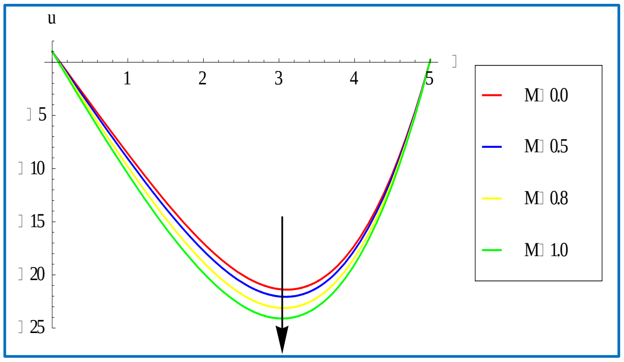

Figure 5: Effect of M on Velocity 1

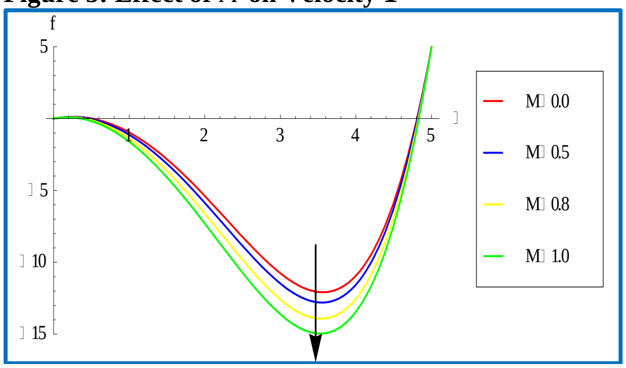

Figure 6: Effect of Mon Velocity 2

Table 1: The effect of (M) variation on Skin Friction(Sk) coefficient

M	λ	β	Sk
0.0	0.2	0.2	-9.66516
0.5	0.2	0.2	-10.2641
1.0	0.2	0.2	-12.061
1.5	0.2	0.2	-15.0558

Table 2: The effect of (β) variation on Skin Friction(Sk) coefficient

β	M	λ	Sk
0.0	0.5	0.5	-2.88432
0.5	0.5	0.5	-3.00435
1.0	0.5	0.5	-3.12439
1.5	0.5	0.5	-3.24442

Table 3: The effect of (λ) variation on Skin Friction(Sk) coefficient

λ	M	β	Sk
0.005	0.5	0.2	-11.1783
0.15	0.5	0.2	-10.7058
0.25	0.5	0.2	-9.65304
0.35	0.5	0.2	-7.80362

Figures (1) and (2) represent the effects for varying values of$\beta = 0.0, 0.5, 0.8, 1.0$,at $M = 0.2$ *and* $\lambda = 0.2$ on the velocity. We notice that as the Deborah number increases, the velocity of the flow decreases. Figures (3) and (4) represent the effects for varying values of$\lambda = 0.0, 0.05, 0.1, 0.2$, at$M = 0.2$ *and* $\beta = 0.2$on the velocity profile. It is observed that increase in the stretching parameter results to a corresponding increase in the flow velocity.The effects of magnetic field parameter is shown in figures (5) and (6) for $\lambda = 0.2$ *and* $\beta = 0.2$with decrease in the flow velocity as M increases that is($M =$ *0.0, 0.5, 0.8, 1.0*.These agrees with the results obtained by Hayat, *et al.*, [9]. As M *and* β increases from tables (1) and (2), the skin friction coefficient deceases whereas, λ increases as the skin friction coefficient decreases.

6.0Conclusion

In this study, we have discussed the first Modified Adomiandecompositioin method (MADM1) and then use it to effectively obtain the series solution of nonlinear differential equation of MHD stagnation-point flow of an upper-convected Maxwell fluid problem. The slight variation in reducing the number of terms of f_0 resulted in the reduction of the computational work and increased the accuracy of the series solution. It also provided the exact series solution by using two iterations only. Hence, there was no need to evaluate the Adomian polynomials required for the nonlinear equations. Results show that the method is reliable, efficient and requires fewer computations. Comparison of the MADM1 with other methods such as HAM shows excellent agreement.

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