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PRESSURE GRADIENT AND THERMAL RADIATION EFFECTS ON TRANSIENT BOUNDARY LAYER FLOW OVER A MOVING PERMEABLE VERTICAL PLATE

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Abstract

The transient laminar boundary flow past a moving vertical porous plate in the presence of pressure gradient and thermal radiation is examined. The nonlinear partial differential equations that govern the flow are transformed to ordinary differential equation using the relevant similarity transformations. The semi-analytic method of the Adomian decomposition is used to solve the resulting ordinary differential equations. The effects of the resulting parameters of the problem on velocity and temperature are analysed. The wall shear stress and wall heat transfer rates are also examined. A good agreement with previous works in literature is observed.

Keywords**:** Pressure gradient, transient, thermal radiation, boundary layer, Adomian decomposition.

Nomenclature:

k	=	thermal conductivity
K	=	absorption coefficient
q_r	=	radiative heat flux
g	=	gravitational acceleration
V_0	=	wall velocity
T_w	=	wall temperature
T_∞	=	ambienttemperature
t	=	time
c	=	suctionparameter
R	=	radiation parameter
Gr	=	local Grashof number
Pr	=	Prandtl number
u,v	=	velocity components

Greek Symbols:

β	=	volumetric expansion coefficient
α	=	thermal diffusivity
τ	=	Stefan-Boltzmann constant
ν	=	kinematic viscosity
η	=	transformed normal coordinate
λ	=	pressure gradient parameter

1.0 Introduction

Boundary layer flow over a moving plate has been found to play important role in many engineering and industrial processes, such as manufacture and extraction of polymer and rubber sheets, paper production, wire drawing and glass-fibre production, continuous casting and much more. Also the study of boundary layer control, gaseous diffusion, to mention a few, due to the heat transfer in fluid past a surface has attracted the interest of many researchers such as[1] and [2]. [3] first considered the study of boundary layer flows on a moving solid surface and he presented a theoretical study of boundary layer on a continuous semi-infinite sheet moving with a constant speed. The boundary layer solution of [3] which was carried out theoretically is found to be quite different from Blassius flow past a stationary flat plate. Later [4] confirmed the study of [3] experimentally. They also obtained the result analytically for the flow and heat transfer aspects developed by a continuously moving surface. [5] and [6] extended Sakiadis problem to include wall suction or blowing effects and determined its effect on heat and mass transfer in the boundary layer. The work of [5] was extended by [7] for the case of both variable surface temperature and heat flux on the heat transfer characteristics of a linearly stretched surface subject to wall suction and injection. Many investigators, which include [8], [9] and [10] considered the problems of stretched surface with variable temperature.
Transient boundary layers due to an impulsively started flat plate were also considered by [11] and [12] in a viscous fluid. Also [13]

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investigated the motion of viscous fluid past an impulsively started semi-infinite flat plate, while [14] examined the flow past a stretching sheet.

Radiation effects, which is also very important for industrial purposes, such as glass production, furnace design and in space technology applications, has been investigated by many researchers. Earlier in literature, laminar convection of a radiating gas in a vertical channel was considered by [15] and derived an exact solution to fully developed vertical channel flow for a radiative fluid. Also [16] considered thermal radiation and buoyancy effect on hydromagnetic flow over an accelerating permeable surface with heat source/sink and showed that the wall heat transfer increased due to the presence of the thermal buoyancy effect. Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer was also examined by [17].

Lately, [18] investigated the radiation effects on the thermal boundary layer flow over a moving plate with convective boundary condition and observed that dual solution exist when the plate and fluid move in the opposite direction. Also, [19] considered the effect of thermal radiation in laminar boundary layer flow over a permeable flat plate with Newtonian heating and showed that the presence of thermal radiation and blowing parameter increased the value of wall temperature, which is contrary to suction effect.

Radiation with pressure gradient effect on boundary layer flow has also been of interest. Earlier in literature, [20] examined the magnetohydrodynamic boundary layer in the presence of a pressure gradient and established that the relative effect of the magnetic field increases as the pressure gradient parameter increases. Lately, amongst other researchers, [21] investigated MHD boundary layer flow with low pressure gradient over a flat plate, and [22] examined it with convective heat transfer. Radiation and pressure gradient effects on incompressible laminar boundary layer flow was considered [23] who observed that the combination of adverse pressure gradient and radiation increased the temperature of the boundary layer in the case of a cooling plate and decreased the temperature in the case of a heating plate.

All the works mentioned above are either steady or unsteady boundary layer flow past stretching or moving plate, with or without thermal radiation. Unsteady boundary layer problems are more difficult to determine an analytic or numerical solution that is uniformly valid. [13] applied the perturbation technique, while [24] utilized the transformation of [25] and the Keller-box numerical method. Recently [21] employed the homotopy perturbation method to obtain the solution of the boundary layer problem. An analytic solution of the unsteady boundary layer problem was determined by [26] and [27] using the homotopy analysis method.

Most recently, the adomian decomposition method has been found to be provide an accurate approximation of the solution, as it does not require discretization of the solution. [2] employed the method to determine the thermal radiation effect on the boundary layer flow past a moving vertical plate. [23] determined the solution for radiation with pressure gradient effects on laminar boundary layer flow using various semi-analytic methods. The adomian decomposition method for the solution of boundary layer convective heat transfer with low pressure gradient over a flat plate was used by [28], and showed that the ADM provide more efficient results for solving nonlinear models.

In this study, we consider the pressure gradient and thermal radiation effects on the unsteady boundary layer flow past a moving vertical porous plate. This is examined using the adomian decomposition method.

2.0 Mathematical Formulation

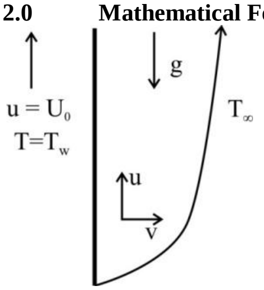


Figure 1: Physical Configuration and Coordinate System

An unsteady two-dimensional boundary layer flow of an incompressible fluid with thermal radiation and pressure gradient over a moving vertical plate is considered. The plate is assumed permeable to allow for possible wall fluid suction. The physical model and coordinate system of the problem are shown in figure (1). The variable pressure gradient is also taken into consideration. The physical variables are functions of y and t only, and all fluid properties are assumed to be constant except for density variations in the buoyancy force. Under these assumptions and neglecting viscous dissipation heat along with Boussinesq approximation, the equations that govern this flow are as follows:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{v\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{v\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + \frac{v\partial T}{\partial y} = \frac{\alpha\partial^2 T}{\partial y^2} - \frac{\alpha}{k}\frac{\partial q_r}{\partial y} \tag{3}$$

All the physical variables are defined in the nomenclature and the appropriate boundary conditions are;

$$\left.\begin{array}{ll} \text{At } y=0\,; & u=U_0\ ,\ T=T_w \\ \text{As } y\to\infty\,; & u\to 0\ ,\ T\to T_\infty \end{array}\right\} \qquad (4)$$

Now by making use of the Roseland approximation, the radiative heat flux term is simplified; that is

$$q_r = -\frac{4\sigma}{3\delta}\frac{\partial T^4}{\partial y} \qquad (5)$$

Assuming that the temperature differences within the flow are sufficiently small so that T^4 can be expanded in Taylor series about the free-stream temperature T_∞, with the higher order terms of the expansion being neglected, we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad (6)$$

Employing equations (5) and (6), equation (3) becomes

$$\frac{\partial T}{\partial t} + \frac{v\partial T}{\partial y} = \frac{\alpha\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\delta R}\frac{\partial^2 T}{\partial y^2} \qquad (7)$$

The similarity transformation of equations (1), (2) and (7) subjected to boundary conditions (4) can be expressed as follows [2];

$$\eta = \frac{y}{2\sqrt{vt}}\ ,\ u = U_0 f(\eta)\,,\ \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty} \qquad (8)$$

with the length scale $\delta = 2\sqrt{vt}$.

This yield the transformed equations:

$$f^{"} = (c-2\eta)f' - Gr\,\theta + \lambda \qquad (9)$$

$$\theta'' = \frac{\Pr}{1+R}(c-2\eta)\theta' \qquad (10)$$

with the transformed boundary conditions:

$$\left.\begin{array}{l} f(0) = \theta(0) = 1 \\ f(\infty) = \theta(\infty) = 1 \end{array}\right\} \qquad (11)$$

where

$$R = \frac{16\alpha\sigma T_\infty^3}{3\delta R}\ ,\ Gr = \frac{g\beta(T_w - T_\infty)\delta^2}{vU_0}\ ,\ \Pr = \frac{v}{\alpha}\ ,\ \lambda = \frac{\delta^2}{vUo}\frac{\partial P}{\partial x} \qquad (12)$$

The physical quantities of interest are the skin friction coefficient,

$$Cf = \frac{\tau_w\delta}{\mu U_0} = f'(0) \qquad (13)$$

and the Nusselt number,

$$Nu = \frac{q_w\delta}{K(T_w - T_\infty)(1+R)} = -\theta'(0) \qquad (14)$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial\eta}\right)_{\eta=0}\ ,\ q_w = -K\left[\frac{\partial T}{\partial\eta}\right]_{\eta=0} - \left[\frac{4\sigma}{3K}\frac{\partial T^4}{\partial\eta}\right]_{\eta=0}$$

3.0 Adomian Decomposition method of solution

The Adomian decomposition method was introduced by Adomian [29] is very useful in obtaining the analytic and numerical solution of linear and nonlinear differential equations. The technique has many advantages over other classical techniques, as it avoids perturbation in order to determine solutions of given nonlinear equations. The main advantage of this method is that it provides an accurate approximation of the solution over traditional numerical methods. It does not require discretization of the solution. The method does not result in any large system of linear or nonlinear equation and so does not require the computation round off errors. Solutions are found more precisely. To apply this method, we consider the general differential equation,

$$Lu(x) + Ru(x) + N(u(x)) = g(x)$$

where L is the highest order derivative which is assumed to be invertible, R is a linear differential operator of less order than L, Nu represents the nonlinear terms, and g(x) is known analytic function.
The method is based on applying the inverse operator L^{-1} formally to the expression

$$Lu(x) = g(x) - Ru(x) - N(u(x)) \quad (15)$$

So, by using the given conditions we obtain

$$u(x) = f(x) - L^{-1}Ru(x) - L^{-1}N(u(x)) \quad (16)$$

where the fn f(x) represents the terms arising from integrating the source term g(x). The standard Adomian decomposition method defines the solution u(x) by the series

$$Lu(x) = \sum_{i=0}^{\infty} u_i(x)$$

where the components $u_0, u_1, u_2, \ldots$ are usually determined recursively by using the relation:

$$u_{k+1}(x) = -L^{-1}Ru_k - L^{-1}N(u_k), \quad k \geq 0$$

The decomposition method suggests that the zeroth component u_0 is usually identified by the function f in equation [16]. For nonlinear equations, the nonlinear operator Nu = F(u) is usually represented by an infinite series of the so-called Adomian polynomials, defined as:

$$F(u) = \sum_{k=0}^{\infty} A_k \quad (17)$$

Where theAdomian polynomial A_n may be computed by the formula

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{n} \lambda^i u_i(x)\right)\right]_{\lambda=0} \quad (18)$$

Applying the Adomian decomposition method to equations (9) and (10) subject to equation (11):
In operator form we have:

$$L\eta\, f = (C-2\eta)f' - Gr\theta + \lambda \quad (19)$$

and $$L\eta\,\theta = \frac{\Pr}{1+R}(C-2\eta)\theta' \quad (20)$$

where the operator, $L\eta = \frac{d^2}{d\eta^2}$

so that the inverse operator $L_\eta^{-1} = \iint(\bullet)\,ds\,ds$

Operating L_η^{-1} on both sides of eqns (19) and (20) and using the initial conditions gives:

$$f(\eta) = f(0) + \eta f'(0) + L_\eta^{-1}((C-2\eta)f') - Gr\,L_\eta^{-1}(\theta) + L_\eta^{-1}(\lambda) \quad (21)$$

and $$\theta(\eta) = \theta(0) + \eta\theta'(0) + \frac{\Pr}{1+R}\,L^{-1}((C-2\eta)\theta') \quad (22)$$

Let $$f = \sum_{n=0}^{\infty} f_n \quad ; \quad \theta = \sum_{n=0}^{\infty} \theta_n \quad (23)$$

From equation (23) we have that

$$\sum_{n=0}^{\infty} f_n(\eta) = \varphi_{01} + L_\eta^{-1}\left((C-2\eta)\sum_{n=0}^{\infty} f_n^1\right) - Gr\,L_\eta^{-1}\sum_{n=0}^{\infty}(\theta_n) \quad (24)$$

and $$\sum_{n=0}^{\infty} \theta_n(\eta) = \varphi_{02} + \frac{\Pr}{1+R}\,L_\eta^{-1}\left((C-2\eta)\sum_{n=0}^{\infty}\theta_n^1\right) \quad (25)$$

where $\varphi_{01} = f(0) + \eta f'(0) + L_\eta^{-1}(\lambda)$

$$= 1 + A\eta + \frac{\lambda\eta^2}{2}$$

and $\varphi_{02} = \theta(0) + \eta\theta'(0)$

$$= 1 + B\eta$$

It follows that

$$f_0 = \varphi_{01} = 1 + A\eta + \frac{\lambda\eta^2}{2} \tag{26}$$

and $\theta_0 = \varphi_{02} = 1 + B\eta$ (27)

Hence, from equations (24) and (25) we have

$$f_{n+1} = L_\eta^{-1}(C-2\eta)f_n^1 - Gr L^{-1}(\theta_n);\ n \geq 0 \tag{28}$$

$$\theta_{n+1} = \frac{\Pr}{1+R}(C-2\eta)\theta_n^1,\quad n \geq 0 \tag{29}$$

Where $f'(0) = A$ and $\theta'(0) = B$ are constants are to be determined using the boundary conditions at infinity.
Equation (28) and (29) on iteration yield respectively.

$$f_0 = 1 + A\eta + \frac{\lambda\eta^2}{2} \tag{30}$$

$$f_1 = \frac{AC\eta^2}{2} + \frac{\lambda C\eta^3}{6} - \frac{A\eta^3}{3} - \frac{\lambda\eta^4}{6} - Gr\left(\frac{\eta^2}{2} + \frac{B\eta^3}{6}\right) \tag{31}$$

$$\begin{aligned} f_1 = {} & \frac{AC^2\eta^3}{6} + \frac{\lambda C^2\eta^4}{24} - \frac{3AC\eta^4}{12} - \frac{2\lambda C\eta^5}{60} \\ & - GrC\left(\frac{\eta^3}{6} + \frac{B\eta^4}{24}\right) - \frac{2\lambda C\eta^5}{40} \\ & + \frac{2A\eta^5}{20} + \frac{4\lambda\eta^6}{90} + 2Gr\left(\frac{\eta^4}{12} + \frac{B\eta^5}{40}\right) \\ & - \frac{Gr\Pr B}{1+R}\left(\frac{C\eta^4}{24} - \frac{\eta^5}{60}\right) \end{aligned} \tag{32}$$

and $\theta_0 = 1 + B\eta$ (33)

$$\theta_1 = \frac{\Pr B}{1+R}\left(\frac{C\eta^2}{2} - \frac{\eta^3}{3}\right) \tag{34}$$

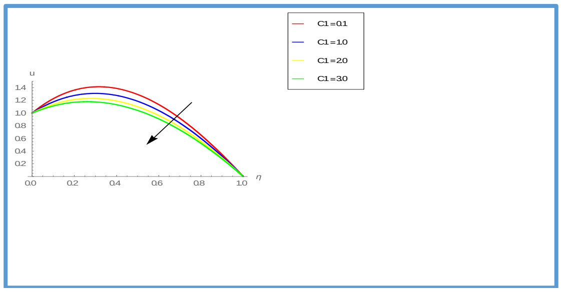
$$\theta_2 = \left(\frac{\Pr}{1+R}\right)^2\left(\frac{BC^2\eta^3}{6} - \frac{BC\eta^4}{4} + \frac{B\eta^5}{10}\right) \tag{35}$$

4.0 Results and Discussion

Using the results in equations (30) to (35) we analyse the effects of the various parameters on thevelocity and temperature using the MATHEMATICA program. Throughout the analysis we have taken some standard values of the Prandtl number, to 0.72, the Grashof's number to be 5.0, the radiation parameter to be 1.0, the pressure gradient parameter to be 3.0. The results are presented graphically in figures 2 to 8.

Figures 2 and 4 show the effects of suction radiation parameters on the fluid velocity respectively. The velocity attains it maximum a short distance away from the plate. An increase in the fluid suction gives rise to a decrease in the fluid velocity, while the fluid velocity increases with increase in radiation parameter.

The fluid velocity attains its maximum at the wall as shown in figures 3 and 5.Increase in both radiation and suction parameters yield increase in the fluid temperature.The effects of fluid suction on skin friction and heat transfer rate are shown in figures 6 and 7. The skin friction decreases with increase in fluid suction, this helps to reduce the skin friction at the plate. There is also a decrease in heat transfer rate as the fluid suction increases.
The pressure gradient causes a drastic decrease in the fluid velocity as it increases, such that fluid velocity attains its maximum at the wall
velocity



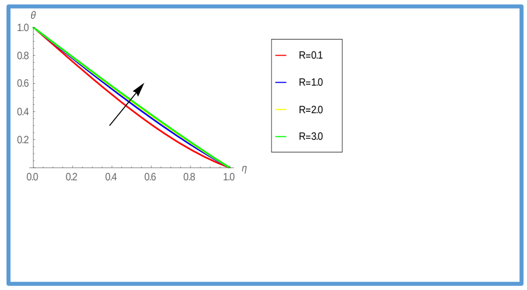
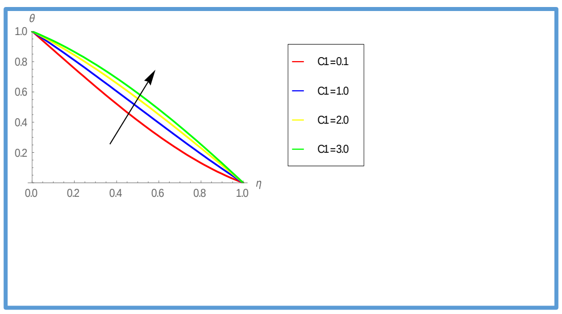


Figure 2: Effect of suction parameter on velocity

Figure 5: Effect of radiation parameter on temperature



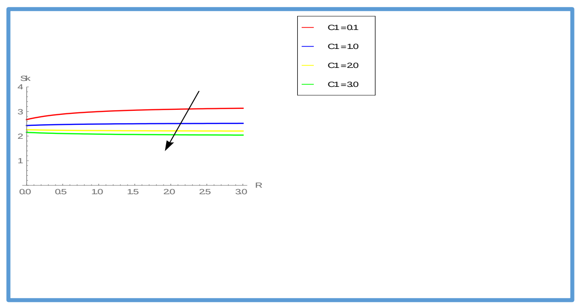
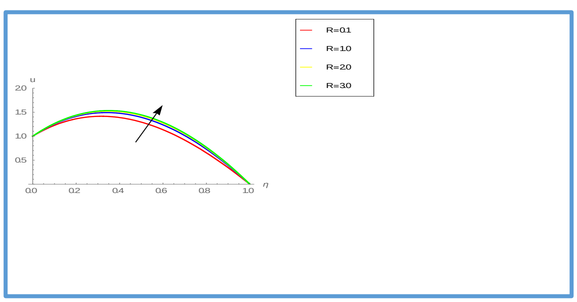


Figure 3:Effect of suction parameter on temperature

Figure 6:Effect of suction on wall shear stress



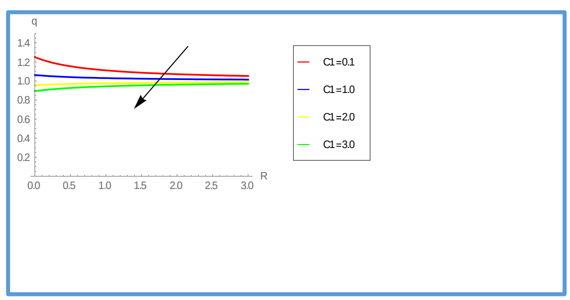


Figure 4: Effect of radiation parameter on velocity

Figure 7: Effect of suction on wall heat transfer

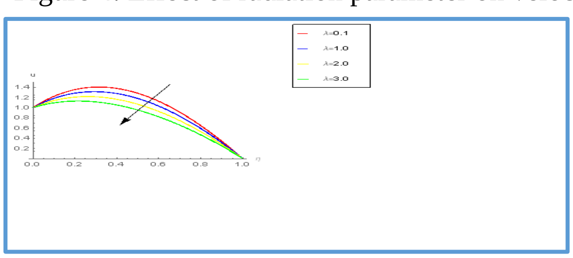


Figure 8: Effect of pressure gradient parameter on velocity

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