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FIVE-STEP SIMPSON'S MULTI OFF-STEP POINTS FOR THE SOLUTION OF STIFF ORDINARY DIFFERENTIAL EQUATIONS

A.I. Bakari[1], S. Babuba[2], P. Tumba[3] and A. Danladi[4]

[1,2,4]**Department of Mathematics, Federal University, Dutse, Nigeria.**
[3]**Department of Mathematics, Federal University, Gashua, Nigeria.**

Abstract

In this paper, we construct five-step Simpson's multi off-step points for the solution of stiff ordinary differential equations. The approach was based on interpolation and collocation procedures. In addition, the properties of the new methods such as order, error constant, zero stability, consistency, convergence and region of absolute stability were also investigated. The developed methods were then applied to solve linear and nonlinear initial value problems of stiff first order ordinary differential equations. We used MAPLE 18 in developing schemes and, well-known ODE solver (ODE 23) for generating numerical solutions.

KEYWORDS: Conventional, Block Method, Backward Differentiation Formula, Second Derivative, Stiff ODEs

1. INTRODUCTION

We consider the first order differential equation

$y' = f(x, y),\ y(\alpha) = y_0 \quad , x \in [a, b]$ (1)

Where $f: R \times R^N \to R^N,\ y, y_0 \in R^N f$ Satisfies Lipschitz conditions [1].

Many methods have been derived by researchers to overcome Dahlquist barrier theorem [2], the order of an A-stable implicit linear multistep method cannot exceed two.

Among them are [3] which derived a class of multistep methods based on super-future method for solving initial value problems. Also, [4] constructed efficient A-stable numerical methods for the solutions of stiff differential equations.[5]modified the extended backward differentiation formulae for the integration of stiff initial value problems of ODEs. [6] developed a new class of one-step hybrid methods for the numerical solutions of ordinary differential equations. [7] developed an L (α)-Stable second derivative block linear multistep method for the solutions of stiff initial value problems of ordinary differential equations. [8] modified extended backward differentiation formula. Some implicit schemes were developed based on the linear multi-step method. [9] proposed efficient A-stable numerical methods for the solutions of stiff differential equations. In this paper, we proposed five-step Simpson's multi off-step points for the solution of stiff ordinary differential equations.

2. DEVELOPMENT OF THE METHOD

In this section, derivation of the continuous formulation of the constructed block method five-step Simpson's multi off-step points for the solution of stiff ordinary differential equations.

$y(x) = \sum_{j=0}^{r-1} \alpha_j(x) y_{n+j} + h \sum_{j=0}^{s-1} \beta_j(x) f_{n+j}$ (2)

Where $\alpha_j(x)$ and $h\beta_j(x)$ are the continuous coefficient of the method

$\alpha_j(x) = \sum_{i=0}^{r+s-1} \alpha_{j.i+1} x^j;\ j \in \{0,1,2, \ldots \ldots . r-1\}$ (3)

$h\beta_j(x) = \sum_{i=0}^{r+s-1} h\beta_{j.i+1} x^j;\ j \in \{0,1,2, \ldots \ldots . s-1\}$ (4)

2.1 FIVE-STEP SIMPSON'S MULTI OFF-STEP POINTS

From equation (2) we obtained the continuous formulation five-step Simpson's with multi off-step points formula of the form

Corresponding Author**:** Bakari A.I., Email: bakaribrahimabba@gmail.com, Tel: +2348134910855

$$y(x)=\sum_{j=0}^{k}\alpha_0(x)y_n+h\beta_0(x)f_n+h\beta_{\frac{1}{2}}f_{n+\frac{1}{2}}+h\beta_1f_{n+1}+h\beta_{\frac{3}{2}}f_{n+\frac{3}{2}}+h\beta_2f_{n+2}+h\beta_{\frac{5}{2}}f_{n+\frac{5}{2}}+h\beta_3f_{n+3}+h\beta_{\frac{7}{2}}f_{n+\frac{7}{2}}+h\beta_4f_{n+4}+h\beta_{\frac{9}{2}}f_{n+\frac{9}{2}}+h\beta_5f_{n+5} \quad (5)$$

Where:

$\alpha_0(x)=1$

$$\beta_0(x)=t-\frac{7381t^2}{2520h}+\frac{177133t^3}{37800h^2}-\frac{84095t^4}{18144h^3}+\frac{341693t^5}{113400h^4}-\frac{8591t^6}{6480h^5}+\frac{7513t^7}{18900h^6}-\frac{121t^8}{1512h^7}+\frac{88t^9}{8505h^8}-\frac{11t^{10}}{14175h^9}+\frac{4t^3}{155925h^{10}}$$

$$\beta_{\frac{1}{2}}(x)=\frac{10t^2}{h}-\frac{4861t^3}{189h^2}+\frac{79913t^4}{2520h^3}-\frac{663941t^5}{28350h^4}+\frac{6041t^6}{540h^5}-\frac{6709t^7}{1890h^6}+\frac{67t^8}{90h^7}-\frac{844t^9}{8505h^8}+\frac{4t^{10}}{525h^9}-\frac{8t^{11}}{31185h^{10}}$$

$$\beta_1(x)=-\frac{45t^2}{2h}+\frac{6121t^3}{84h^2}-\frac{115923t^4}{1120h^3}+\frac{26476t^5}{3150h^4}-\frac{92771t^6}{2160h^5}+\frac{18047t^7}{1260h^6}-\frac{1123t^8}{360h^7}+\frac{1214t^9}{2835h^8}-\frac{53t^{10}}{1575h^9}+\frac{4t^{11}}{3465h^{10}}$$

$$\beta_{\frac{3}{2}}(x)=\frac{40t^2}{h}-\frac{26164t^3}{189h^2}+\frac{400579t^4}{1890h^3}-\frac{867478t^5}{4725h^4}+\frac{13349t^6}{135h^5}-\frac{10838t^7}{315h^6}+\frac{349t^8}{45h^7}-\frac{3104t^9}{2835h^8}+\frac{416t^{10}}{4725h^9}-\frac{32t^{11}}{10395h^{10}}$$

$$\beta_2(x)=-\frac{105t^2}{2h}+\frac{6751t^3}{36h^2}-\frac{71689t^4}{240h^3}+\frac{728587t^5}{2700h^4}-\frac{163313t^6}{1080h^5}+\frac{34343t^7}{630h^6}-\frac{2281t^8}{180h^7}+\frac{248t^9}{135h^8}-\frac{34t^{10}}{225h^9}+\frac{8t^{11}}{1485h^{10}}$$

$$\beta_{\frac{5}{2}}(x)=\frac{252t^2}{5h}-\frac{13754t^3}{75h^2}+\frac{1197t^4}{4h^3}-\frac{62549t^5}{225h^4}+\frac{43319t^6}{270h^5}-\frac{93773t^7}{1575h^6}+\frac{128t^8}{9h^7}-\frac{856t^9}{405h^8}+\frac{8t^{10}}{45h^9}-\frac{16t^{11}}{2475h^{10}}$$

$$\beta_3(x)=-\frac{35t^2}{h}+\frac{6961t^3}{54h^2}-\frac{461789t^4}{2160h^3}+\frac{273431t^5}{1350h^4}-\frac{129067t^6}{1080h^5}+\frac{28603t^7}{630h^6}-\frac{1999t^8}{180h^7}+\frac{76t^9}{45h^8}-\frac{98t^{10}}{675h^9}+\frac{8t^{11}}{1485h^{10}}$$

$$\beta_{\frac{7}{2}}(x)=\frac{120t^2}{7h}-\frac{4012t^3}{63h^2}+\frac{22439t^4}{210h^3}-\frac{485278t^5}{4725h^4}+\frac{8321t^6}{135h^5}-\frac{358t^7}{15h^6}+\frac{1877t^8}{315h^7}-\frac{2624t^9}{2835h^8}+\frac{128t^{10}}{1575h^9}-\frac{32t^{11}}{10395h^{10}}$$

$$\beta_4(x)=-\frac{45t^2}{8h}+\frac{3533t^3}{168h^2}-\frac{3986t^4}{1120h^3}+\frac{435893t^5}{12600h^4}-\frac{45449t^6}{2160h^5}+\frac{10427t^7}{1260h^6}-\frac{757t^8}{360h^7}+\frac{944t^9}{2835h^8}-\frac{47t^{10}}{1575h^9}+\frac{4t^{11}}{3465h^{10}}$$

$$\beta_{\frac{9}{2}}(x)=\frac{10t^2}{9h}-\frac{263t^3}{63h^2}+\frac{161353t^4}{22680h^3}-\frac{197741t^5}{28350h^4}+\frac{6947t^6}{1620h^5}-\frac{3229t^7}{1890h^6}+\frac{119t^8}{270h^7}-\frac{604t^9}{8505h^8}+\frac{92t^{10}}{14175h^9}-\frac{8t^{11}}{31185h^{10}}$$

$$\beta_5(x)=-\frac{t^2}{10h}+\frac{7129t^3}{18900h^2}-\frac{1303t^4}{2016h^3}+\frac{9046t^5}{14175h^4}-\frac{19t^6}{48h^5}+\frac{3013t^7}{18900h^6}-\frac{t^8}{24h^7}+\frac{58t^9}{8505h^8}-\frac{t^{10}}{1575h^9}+\frac{4t^{11}}{155925h^{10}}$$

Evaluating (5) at $x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}, x_{n+\frac{5}{2}}, x_{n+\frac{7}{2}}, x_{n+4}, x_{n+\frac{9}{2}}$ and x_{n+5} yields the following discrete methods which constitute the new five-step Simpson's multi off-step points-block method.

$$y_{n+\frac{1}{2}}-y_n=\frac{26842253}{191600640}hf_n+\frac{164046413}{239500800}hf_{n+\frac{1}{2}}-\frac{296725183}{31933440}hf_{n+1}+\frac{12051709}{7983360}hf_{n+\frac{3}{2}}-\frac{33765029}{17740800}hf_{n+2}+\frac{2227571}{1247400}hf_{n+\frac{5}{2}}-\frac{21677723}{17740800}hf_{n+3}+\frac{23643791}{39916800}hf_{n+\frac{7}{2}}-\frac{12318413}{63866880}hf_{n+4}+\frac{9071219}{239500800}hf_{n+\frac{9}{2}}-\frac{3250433}{958003200}hf_{n+5}$$

$$y_{n+2}-y_n=\frac{2046263}{14968800}hf_n+\frac{645431}{748440}hf_{n+\frac{1}{2}}-\frac{2149811}{14968800}hf_{n+1}+\frac{355583}{311850}hf_{n+\frac{3}{2}}-\frac{1258463}{831600}hf_{n+2}+\frac{904403}{623700}hf_{n+\frac{5}{2}}-\frac{166931}{166320}hf_{n+3}+\frac{21833}{44550}hf_{n+\frac{7}{2}}-\frac{800243}{4989600}hf_{n+4}+\frac{118291}{3742200}hf_{n+\frac{9}{2}}-\frac{8501}{2993760}hf_{n+5}$$

$$y_{n+\frac{3}{2}}-y_n=\frac{108223}{788480}hf_n+\frac{840607}{985600}hf_{n+\frac{1}{2}}-\frac{879183}{3942400}hf_{n+1}+\frac{762497}{492800}hf_{n+\frac{3}{2}}-\frac{670233}{394240}hf_{n+2}+\frac{24399}{15400}hf_{n+\frac{5}{2}}-\frac{2136259}{1971200}hf_{n+3}+\frac{259071}{492800}hf_{n+\frac{7}{2}}-\frac{675441}{3942400}hf_{n+4}+\frac{6637}{197120}hf_{n+\frac{9}{2}}-\frac{11899}{3942400}hf_{n+5}$$

$$y_{n+2}-y_n=\frac{64121}{467775}hf_n+\frac{400138}{467775}hf_{n+\frac{1}{2}}-\frac{2159}{8910}hf_{n+1}+\frac{39752}{22275}hf_{n+\frac{3}{2}}-\frac{23423}{17325}hf_{n+2}+\frac{230788}{155925}hf_{n+\frac{5}{2}}-\frac{17879}{17325}hf_{n+3}+\frac{2248}{4455}hf_{n+\frac{7}{2}}-\frac{25754}{155925}hf_{n+4}+\frac{15226}{467775}hf_{n+\frac{9}{2}}-\frac{547}{187110}hf_{n+5}$$

$$y_{n+\frac{5}{2}}-y_n=\frac{5256425}{38320128}hf_n+\frac{8183125}{9580032}hf_{n+\frac{1}{2}}-\frac{2996375}{12773376}hf_{n+1}+\frac{2793625}{1596672}hf_{n+\frac{3}{2}}-\frac{2306375}{2128896}hf_{n+2}+\frac{89035}{49896}hf_{n+\frac{5}{2}}-\frac{2325625}{2128896}hf_{n+3}+\frac{838375}{1596672}hf_{n+\frac{7}{2}}-\frac{311375}{1824768}hf_{n+4}+\frac{320875}{9580032}hf_{n+\frac{9}{2}}-\frac{114985}{38320128}hf_{n+5}$$

$$y_{n+3}-y_n=\frac{1689}{12320}hf_n+\frac{13169}{15400}hf_{n+\frac{1}{2}}-\frac{14787}{61600}hf_{n+1}+\frac{1363}{770}hf_{n+\frac{3}{2}}-\frac{35229}{30800}hf_{n+2}+\frac{16083}{7700}hf_{n+\frac{5}{2}}-\frac{25373}{30800}hf_{n+3}+\frac{1887}{3850}hf_{n+\frac{7}{2}}-\frac{2007}{12320}hf_{n+4}+\frac{71}{2200}hf_{n+\frac{9}{2}}-\frac{179}{61600}hf_{n+5}$$

$$y_{n+\frac{7}{2}}-y_n=\frac{18775351}{136857600}hf_n+\frac{5843887}{6842880}hf_{n+\frac{1}{2}}-\frac{10669897}{45619200}hf_{n+1}+\frac{9973607}{5702400}hf_{n+\frac{3}{2}}-\frac{2767667}{2534400}hf_{n+2}+\frac{353633}{178200}hf_{n+\frac{5}{2}}-\frac{241129}{506880}hf_{n+3}+\frac{4148249}{5702400}hf_{n+\frac{7}{2}}-\frac{8312311}{45619200}hf_{n+4}+\frac{1190357}{34214400}hf_{n+\frac{9}{2}}-\frac{84427}{27371520}hf_{n+5}$$

$$y_{n+4}-y_n=\frac{12818}{93555}hf_n+\frac{400448}{467775}hf_{n+\frac{1}{2}}-\frac{38176}{155925}hf_{n+1}+\frac{278272}{155925}hf_{n+\frac{3}{2}}-\frac{121184}{10395}hf_{n+2}+\frac{330368}{155925}hf_{n+\frac{5}{2}}-\frac{34432}{51975}hf_{n+3}+\frac{176896}{155925}hf_{n+\frac{7}{2}}-\frac{2998}{155925}hf_{n+4}+\frac{2368}{93555}hf_{n+\frac{9}{2}}-\frac{1184}{467775}hf_{n+5}$$

$$y_{n+\frac{9}{2}}-y_n=\frac{542331}{3942400}hf_n+\frac{837567}{985600}hf_{n+\frac{1}{2}}-\frac{167427}{788480}hf_{n+1}+\frac{829089}{492800}hf_{n+\frac{3}{2}}-\frac{1880253}{1971200}hf_{n+2}+\frac{27459}{15400}hf_{n+\frac{5}{2}}-\frac{537219}{1971200}hf_{n+3}+\frac{10773}{14080}hf_{n+\frac{7}{2}}-\frac{2065743}{3942400}hf_{n+4}+\frac{199809}{985600}hf_{n+\frac{9}{2}}-\frac{4671}{788480}hf_{n+5}$$

$$y_{n+5}-y_n=\frac{80335}{598752}hf_n+\frac{132875}{149688}hf_{n+\frac{1}{2}}-\frac{80875}{199584}hf_{n+1}+\frac{28375}{12474}hf_{n+\frac{3}{2}}-\frac{24125}{11088}hf_{n+2}+\frac{89035}{24948}hf_{n+\frac{5}{2}}-\frac{24125}{11088}hf_{n+3}+\frac{28375}{12474}hf_{n+\frac{7}{2}}-\frac{80875}{199584}hf_{n+4}+\frac{132875}{149688}hf_{n+\frac{9}{2}}-\frac{80335}{598752}hf_{n+5} \quad (6)$$

3. THE BASIC PROPERTIES OF THE SCHEME

3.1 Order, Error Constant Zero Stable and Consistency of the method

Following the scheme derived above are discrete schemes from the class of Linear Multistep Method (LMM) of the form

$$y(x)=\sum_{j=0}^{r-1}\alpha_j(x)y_{n+j}+h\sum_{j=0}^{s-1}\beta_j(x)f_{n+j} \quad (7)$$

to be linear difference operator

$$[y(x);h]=\sum_{j=0}^{k}\alpha_j y_{n+j}+h\beta_k f_{n+k}\ . \quad (8)$$

Where $y(x)$ is an arbitrary sufficiently differentiable on the interval $[a,b]$. we can we expand the terms in (8) as a Taylor series and comparing the coefficients of h gives

$$L[y(x);h]=c_0y(x)+c_1hy^{'(x)}+c_2h^2y^{''(x)}+\cdots+c_ph^py^p(x)+\cdots \quad (9)$$

Where the constants $C_p, p=0,1,2\ \ldots,, j=1,2,\ldots k$ are given as follows:

$$C_0=\alpha_0+\alpha_1+\alpha_2+\cdots+\alpha_k$$
$$C_1=(\alpha_0+\alpha_1+\alpha_2+\cdots+\alpha_k)-(\beta_0+\beta_1+\beta_2+\cdots+\beta_k)$$

. . . .

. . . .

$$C_q=\frac{1}{q!}(\alpha_0+\alpha_1+\alpha_2+\cdots+\alpha_k)-\frac{1}{(q-1)!}(\beta_1+2^{q-1}\beta_2+\cdots+k^{q-1}\beta_k)$$

Where $C_0=C_1=C_2\ldots C_p=0$ and$C_{p+1}\neq 0$. Therefore,C_{p+1} is the error constant and $C_{p+1}h^{p+1}y^{(p+1)}(x_n)$ is the principal local truncation error at appoint x_n. It was established from the evaluation of hybrid block methods have order and error constants.

Using the concept above, the hybrid block methods are obtained with the help of MAPLE 18 SOFTWARE to have the following uniform order and error constants.

3.2 THE ORDER AND ERROR CONSTANTS OF THE METHO

Below is the table for the step number, scheme number, order and error constant and it was determined from the evaluation of derived methods.

TABLE 1: THE ORDER AND ERROR CONSTANTS OF THE CONSTRUCTED METHO

Step number	Scheme number	Order	Error constant
5	1	11	$\frac{4671}{3229614080}$
5	2	11	$\frac{37}{29937600}$
5	3	11	$\frac{2989}{2306867200}$
5	4	11	$\frac{1}{788480}$
5	5	11	$\frac{202025}{156959244288}$
5	6	11	$\frac{1}{788480}$
5	7	11	$\frac{2989}{2306867200}$
5	8	11	$\frac{37}{29937600}$
5	9	11	$\frac{4671}{3229614080}$
5	10	11	0

3.3 ZERO STABILITY OF THE METHOD

The Five-step of new constructed method is normalized and write them as block from which we obtain the stability polynomial of the method as

$\rho(r) = \det(rA - B)$

$$= det\left[r\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix} - \begin{bmatrix} 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix}\right]$$

$$= det\begin{bmatrix} r&0&0&0&0&0&0&0&0&0\\ 0&r&0&0&0&0&0&0&0&0\\ 0&0&r&0&0&0&0&0&0&0\\ 0&0&0&r&0&0&0&0&0&0\\ 0&0&0&0&r&0&0&0&0&0\\ 0&0&0&0&0&r&0&0&0&0\\ 0&0&0&0&0&0&r&0&0&0\\ 0&0&0&0&0&0&0&r&0&0\\ 0&0&0&0&0&0&0&0&r&0\\ 0&0&0&0&0&0&0&0&0&r-1 \end{bmatrix}$$

$= r^{10} - r^9 = 0$
$r_1 = 1, r_2 = r_3 = r_4 = r_5 = r_6 = r_7 = r_8 = r_9 = r_{10} = 0 < 1$
The block method (6) is zero stable and is of order $P = 11$. Hence by [1] it is convergent.

4. NUMERICAL EXPERIMENTS

We present the numerical experiments of the newly constructed five-step Simpson's multi off-step points for the solution of stiff ordinary differential equations and we conducted using MATLAB_R2010A

PROBLEM 1

$y_1' = -1002y_1 + 1000y_2^2$
$y_2' = y_1 - y_2 - y_2^2$
Exact $y_{1(x)} = e^{-2x} y_1(0) = 1$
$y_2(x) = e^{-x} y_2(0) = 1$
$with\ h = 0.1$

TABLE 2: NUMERICAL RESULTS OF THE CONSTRUCED FIVE-STEP BLOCK METHOD FOR PROBLEM 1

x	New Method $y_{1(x)}$	New Method $y_2(x)$	Theoretical $y_{1(x)}$	Theoretical $y_2(x)$
9.1	0.000123128294470	0.011109006178534	0.000000015229980	0.000123409804087
9.2	0.000111411087894	0.010567213545812	0.000000012469253	0.000111665808490
9.3	0.000100808920971	0.010051844452367	0.000000010208961	0.000101039401837
9.4	0.000091215683647	0.009561610206908	0.000000008358390	0.000091424231478
9.5	0.000082535363573	0.009095284968365	0.000000006843271	0.000082724065557
9.6	0.000074681085183	0.008651702680652	0.000000005602796	0.000074851829888
9.7	0.000067574240217	0.008229754156913	0.000000004587182	0.000067728736491
9.8	0.000061143700978	0.007828384305979	0.000000003755667	0.000061283495053
9.9	0.000055325108466	0.007446589494090	0.000000003074880	0.000055451599432
10	0.000050060228248	0.007083415035285	0.000000002517499	0.000050174682056

TABLE 3: ABSOLUTE ERRORS FOR PROBLEM 1

X	Absolute errors $y_{1(x)}$	Absolute errors $y_2(x)$
9.1	0.000123113064490	0.010985596374447
9.2	0.000111398618641	0.010455547737322
9.3	0.000100798712011	0.009950805050530
9.4	0.000091207325257	0.009470185975429
9.5	0.000082528520302	0.009012560902808
9.6	0.000074675482387	0.008576850850764
9.7	0.000067569653035	0.008162025420422
9.8	0.000061139945311	0.007767100810926
9.9	0.000055322033586	0.007391137894658
10	0.000050057710749	0.007033240353229

PROBLEM 2

$y_1' = -50y_1$
$y_2' = 70y_1 - 120y_2$
Exact $y_{1(x)} = e^{-50x} y_1(0) = 1$
$y_2(x) = e^{-50x} + e^{-120x} y_2(0) = 2$
$with\ h = 0.1$

TABLE 4:NUMERICAL RESULTS OF THE CONSTRUCED FIVE-STEP BLOCK METHOD FOR PROBLEM 2

x	New Method $y_{1(x)}$	New Method $y_2(x)$	Theoretical $y_{1(x)}$	Theoretical $y_2(x)$
0.1	1.000000000000000	2.000000000000000	1.000000000000000	2.000000000000000
0.2	0.075771656591913	0.029751869539527	0.006737946999085	0.006744091211439
0.3	0.005741343942683	0.007859164743028	0.000045399929762	0.000045399967514
0.4	0.000435031141601	0.000337569479354	0.000000305902321	0.000000305902321
0.5	0.000032963030268	0.000037448195211	0.000000002061154	0.000000002061154
0.6	0.000002497663410	0.000002291257074	0.000000000013888	0.000000000013888
0.7	0.000000189252094	0.000000198750870	0.000000000000094	0.000000000000094
0.8	0.000000014339945	0.000000013902813	0.000000000000001	0.000000000000001
0.9	0.000000001086561	0.000000001106678	0.000000000000000	0.000000000000000
1	0.000000000082331	0.000000000081405	0.000000000000000	0.000000000000000

TABLE5: ABSOLUTE ERRORS FOR PROBLEM 2

x	Absolute errors $y_{1(x)}$	Absolute errors $y_2(x)$
0.1	0.000000000000000	0.000000000000000
0.2	0.069033709592827	0.023007778328088
0.3	0.005695944012920	0.007813764775514
0.4	0.000434725239281	0.000337263577033
0.5	0.000032960969115	0.000037446134057
0.6	0.000002497649522	0.000002291243186
0.7	0.000000189252001	0.000000198750776
0.8	0.000000014339944	0.000000013902812
0.9	0.000000001086561	0.000000001106678
1	0.000000000082331	0.000000000081405

5. CONCLUSION

The new block-scheme of five-step Simpson's multi off-step points for the solution of stiff ordinary differential equations of uniform order eleven was constructed based on multistep collocation approach. The stability analysis of the constructed method showed that the method is zero-stable and convergence. The result of this study shows that newly derived method for five-step approximate well the solutions of the linear and non- linear stiff systems of ordinary differential equations as compared with the exact solutions.

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