Journal of the Nigerian Association of Mathematical Physics
***Volume 53* (November *2019 Issue*), pp89 – 102**
© J. of NAMP

COMPARATIVE ANALYSIS OF INITIAL FEASIBLE SOLUTIONS TO BALANCED TRANSPORTATION PROBLEMS

Akeremale O. C[1], AnthonyO. F.[2]and Olaiju A. O.[3]

[1,2]Department of Mathematics, Federal University Lafia, Nasarawa State, Nigeria
[3]Department of Mathematics, Federal Polytechnics, Ilaro Ogun State, Nigeria

Abstract

In this work, we compared three transportation approaches to find the most efficient transportation schedule and the best transportation route for the initial feasible solution in order to maximize profit. Excel solver as a tool was used to compute the optimal solution as a validation for the optimal test.

Keywords: transportation cost, transportation problem, initial basics feasible solution, excel solver, optimal solution

1. Introduction

One of the important applications of linear programming is in transportation problems. Transportation problem is a classical class optimization problem which involves moving goods from one location to another location depending on demand and supply of the goods at the various locations. Transportation problem was first formulated by [1] and [2]successfully developed efficient and robust method for the solution of the problem followed by [3]who later improved on the method of solution.

The simplified version of simplex method is transportation method and it can be used to solve linear programming problems. It is known as the transportation problem/model because in solving problems that involves several sources and destinations, transportation method is a major application. A transportation problem is said to be balanced if, total supply equals total demand, if there is a disparity between demand and supply, it is said to be unbalanced transportation problem. In this work, we will focus on balanced transportation problems

Many authors [4-12] hadworked on transportation problems with most of them laying emphasis on Vogel approximation method and northwest corner rule method for both the initial basic feasible solution and optimal solution. Though their results were found to be very good but, in this research, we intend to explore row minimal, column minimal and least square methods to obtain both the initial basic feasible solution and the optimal solution for possible comparison.

2. Problem Formulation

A firm produces goods at $m$locations, i.e. $i = 1,2.3, \dots, m$. The supply product at i^{th}location = si, the demand for the goods is spread out at n different demand locations i.e. $j = 1,2,3, \dots, n$. The demand at the j^{th} demand location is D_j. The problem of the firm is to get goods from supply locations to demand locations at minimum cost. Assume that the cost of shipping one unit from supply location$i$ to demand location$j$ is C_{ij} and that shipping cost is linear, which means that if you shipped X_{ij} unit from location$i$ to demand location j, then, the cost would be $C_{ij}X_{ij}$.

Where X_{ij} is the number of units shipped from supply location$i$ to demand location$j$ the problem is to identify the minimum or maximum shipping cost schedule. The constraints being that supply must meet demand at each demand location, and cannot exceed supply at each supply location.

By the assumption of linearity, the schedule cost is as below

$$max \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij} \qquad (1)$$

The total amount shipped out of supply location$i$ is

Corresponding Author**:** Akeremale O.C., Email: akere194@gmail.com, Tel: +2348062706593

$$\sum_{i=1}^{m} X_{ij} \tag{2}$$

i.e. X_{ij} is the unit shipped from $i$to j. Also, from i amount of unit can be shipped to any demand location$(j = 1,2,3, \ldots, n)$. The quantity cannot exceed the supply available, hence the constraint

$$\sum_{j=1}^{n} X_{ij} \le S_{i,} \quad i = 1,2,3, \ldots m \tag{3}$$

Similarly, the constraints that guarantee that the demand at each demand location is met is given by

$$\sum_{j=1}^{n} X_{ij} \ge D_j, j = 1,2,3, \ldots, n \tag{4}$$

Thus, we assume that the total supply is equal to total demand that is if

$$\sum_{j=1}^{n} D_j = \sum_{i=1}^{m} S_i \tag{5}$$

The constraints in the problem holds as equations

With the establishment that total supply is equal to total demand, the standard transportation model formulation is given below with the objective to find a transportation plan denoted by X_{ij} that satisfy (5) and solves

$$\left.\begin{array}{l} max \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij} \\ subject\ to \sum_{j=1}^{n} x_{ij} = S_i, \ \ i = 1,2,3, \ldots, m \\ \sum_{j=1}^{n} X_{ij} = D_i, \ \ j = 1,2,3, \ldots, n \end{array}\right\} \tag{6}$$

In the problem it is natural to assume that the variable X_{ij} takes on integer value (and non-negative ones). That is one can only ship items in whole number batches.

Then the standard mathematical model for this problem is:

$$\left.\begin{array}{l} \max Z = \sum_{i}^{m}\sum_{j}^{n} X_{ij}C_{ij} \\ subject\ to \sum_{j}^{n} X_{ij} \le a_i, \ \ i = 1,2,3, \ldots m \\ \sum_{j}^{n} X_{ij} \ge b_j, \ \ j = 1,2,3, \ldots, n \end{array}\right\} \tag{7}$$

where

$m =$ number of sources

$n =$ number of destinations

$a_i =$ capacity of i^{th}source

$b_j =$ capacity of j^{th}destination

$C_{ij} =$ cost coefficient for materials shiped from i^{th}source to j^{th} destination

$X_{ij} =$ amount of materials shiped between i^{th}source to j^{th} destination

3.0 Methodology

In this section, we consider column minima, row minima and least square methods for the initial feasible solution of balanced transportation problems while we improve on each of the method for the optimality solution. The basic steps involved in solving a transportation problem are; (i) find an initial basic feasible solution, (ii) test the solution for optimality, (iii) improve the solution when it is not optimal at the initial feasible point. In a case where the solution is not optimal at the initial feasible solution, steps (ii) and (iii) are repeated until an optimal solution is arrived at. Excel solver was also used for the optimality test.

4.0 Data Collection and Analysis

Data for this research work were collected from Pleasure Travels Limited, Peace Mass Transit, Blue Whales Transport Limited and Young Shall Grow Transport Limited and used for comparison of transportation algorithm(s) with the view of finding the most efficient transportation algorithm and the best transportation route for the benefit of the companies involved in order to maximise profit. Three methods of finding initial feasible solution (Column minima method, Row minima method and Least cost method) where employed

Excel solver was used to compute the optimal solution as a validation tool for the transportation problem.

4.1 Construction of Pleasure Travels Transportation Tableau

Consider a transportation company Pleasure Travels that allocates buses to different route on a daily bases and the total income generated on each per trip (to-fro) as well as the total passengers transported per trip (to-fro) as shown in table 4.1 below:

Table 1DataCollectedfromPleasureTravels Limited

ROUTE	PASSENGERS PER BUS	NO OF VEHICLE		NO OF PASSENGERS CARRIED	INCOME PER TRIP
		TO	FRO		
ABUJA-CALABAR	14	2	2	56	₦380,800
	13	1	1	26	₦171,600
ABUJA-OWERRI	36	2	2	144	₦1,022,400
	13	1	1	26	₦182,000
ABUJA-LAGOS	42	1	1	84	₦546,000
	36	1	1	72	₦460, 800
ABUJA-IBADAN	42	1	1	84	₦487,200
	13	1	1	26	₦140,400
TOTAL		10	10	518	₦ 3,391,200

Table1 is transformed into transportation tableau and the initial feasible solution of the transportation problem is computed as follows:

Table 2: Pleasure Travels Transportation Tableau

	A	B	C	D	Supply
Abuja- Calabar	**6800** **28**	**6800** **28**	**6600** 26	**6800**	**82**
Abuja -Owerri	**7100** 72	**7100** **72**	**7000** **26**	**7100**	**170**
Abuja - Lagos	**6500** **42**	**6500** 42	**6400** 72	**6500**	**156**
Abuja - Ibadan	**5800** **42**	**5800** 42	**5400** 26	**5800**	**110**
Demand	**184**	**184**	**150**		**518**

Now, we compute the total number of passengers that uses bus A to all the routes from Abuja

Demand = 28+72+42+42 = 184

Total number of passengers that uses bus B to all the routes

Demand = 28+72+42+42 =184

Total number of passengers that uses bus C to all the routes from Abuja

Demand = 26+26+72+26 = 150

Total Demand= 518

Number of passengers that moved from Abuja to Calabar using bus A to D= 82
Number of passengers that moved from Abuja to Owerri using bus A to D= 170
Number of passengers that moved from Abuja to Lagos using bus A to D= 156
Number of passengers that moved from Abuja to Ibadan using bus A to D= 110
Total supply = 82+170+156+110= 518. This gives a balanced transportation problem hence; it satisfies the condition Total Demand = Total Supply

Total income generated is calculated below
6800 * 56 = ₦ 380,800
6600 * 26 = ₦ 171,600
7100 * 144 = ₦ 1,022,400
7000 * 26 = ₦ 182,000
6500 * 84 = ₦ 546,000
6400 * 72 = ₦ 460,800
5800 * 84 = ₦ 487,200
5400 * 26 = ₦ 140,400
Total income =₦ 3,391,200

4.2 Finding Initial Feasible Solution

4.2.1 Column Minima Method

The initial basic feasible solution is obtained using column minima method. Following the algorithm presented and starting from the least cell in the first column until all demand and supply are met as shown in table 3.

Table 3: Column Minima Method

	A	B	C	D	Supply
A	**6800** **82**	**6800**	**6600**	**6800**	**82**
B	**7100**	**7100** **20**	**7000** 150	**7100**	**170**
C	**6500** 102	**6500** **54**	**6400**	**6500**	**156**
D	**5800**	**5800** 110	**5400**	**5800**	**110**
Demand	**184**	**184**	**150**		**518**

Total income generated by the column minima method
6800 * 82 = ₦ 557,600
7100 * 20 = ₦ 142,000
7000 * 150 = ₦ 1,050,000
6500 * 102 = ₦ 663,000
6500 * 54 = ₦ 351,000
5800*110= ₦ 638,000
Total income = ₦ 3,401,600

4.2.2 Row Minima Method

The Row Minima Method is applied starting allocation to the cell with the minimum cost in the first row following the algorithm mentioned earlier as shown in Table 4

Table 4: Row Minima Method

	A	B	C	D	Supply
A	**6800**	**6800**	**6600** 82	**6800**	**82**
B	**7100**	**7100** **102**	**7000** 68	**7100**	**170**
C	**6500** 156	**6500**	**6400**	**6500**	**156**
D	**5800** 28	**5800** 82	**5400**	**5800**	**110**
Demand	**184**	**184**	**150**		**518**

Total income generated using the Row Minima Method
6600 * 82 = ₦ 541,200
7000 * 68 = ₦ 476,000
7100 * 102 = ₦ 724,200
6500 * 156 = ₦ 1,014,000
5800 * 28 = ₦ 162,400
5800* 82 = ₦ 475,600
TOTAL INCOME = ₦ 3,393,400

4.2.3 Least Cost Method

The initial basic feasible solution is obtained using the least cost method following the algorithm presented earlier starting from the least cell until all demand and supply are met as shown in table 5

TABLE 5: Least Cost Method

	A	B	C	D	Supply
A	**6800** 82	**6800**	**6600**	**6800**	**82**
B	**7100** 20	**7100**	* **7000** 150	**7100**	**170**
C	**6500** 82	**6500** 74	**6400**	**6500**	**156**
D	**5800** 28	**5800** 110	**5400**	**5800**	**110**
Demand	**184**	**184**	**150**		**518**

Total income generated using the least cost method is calculated below
6800 * 82 = ₦ 557,600
7100 * 20 = ₦ 142,000
7000 * 150 = ₦ 1,050,000
6500 * 82 = ₦ 533,000
6500 * 74 = ₦ 481,000
5800 * 110 = ₦ 638,000
Total income =3,401,600

4.3Construction of Peace Mass Transit Transportation Problem

Table 6: Data Collected from Peace Mass Transit

ROUTE	PASSENGER PER BUS	NO OF VEHICLE		TOTAL PASSENGER	INCOME
		TO	FRO		
ABUJA-CALABAR	14	2	2	56	375,200
	13	2	2	52	343,200
ABUJA-PORTHHARCOURT	13	3	3	78	468,000
	14	1	1	28	170,800
ABUJA – OWERRI	36	1	1	72	489,600
	13	1	1	26	182,000
ABUJA-IBADAN	42	1	1	84	478,800
	13	2	2	52	291,200
TOTAL		13	13	448	2,798,800

Table 6 is transformed into a transportation tableau and the initial feasible solution of the transportation problem is computed as follows:

Table 7: Peace Mass Transit Transportation Tableau

	A	B	C	D	Supply
A	28 **6700**	28 **6700**	26 **6600**	26 **6600**	108
B	26 **6000**	26 **6000**	26 **6000**	**28** **6100**	106
C	72 **6800**	26 **7000**	**6800**	**6800**	**98**
D	84 **5700**	**26** **5600**	26 **5600**	**5600**	**136**
Demand	**210**	**106**	**78**	54	**448**

4.3.1 Column Minima Method

Table8: Column Minima Method

	A	B	C	D	Supply
A	104 **6700**	**6700**	4 **6600**	**6600**	108
B	106 **6000**	**6000**	**6000**	**6100**	106
C	**6800**	98 **7000**	**6800**	**6800**	**98**
D	**5700**	**8** **5600**	74 **5600**	*54* **5600**	**136**
Demand	**210**	**106**	**78**	54	**448**

Total income generated by the column minima method
6700 * 104 = ₦ 698,800
6600 * 4 = ₦ 26,400
6000 * 106 = ₦ 636,000
7000 * 98 = ₦ 686,000
5600 * 8 = ₦ 44,800
5600 * 74 = ₦ 414,400
5600*54 = ₦ 302,400
Total income = ₦ 2,806,800

4.3.2 The Row Minima Method

Table9: Row Minima Method

	A	B	C	D	Supply
A	**6700**	**6700**	78 **6600**	30 **6600**	108
B	106 **6000**	**6000**	**6000**	**6100**	106
C	**6800**	98 **7000**	**6800**	**6800**	**98**
D	104 **5700**	**8** **5600**	**5600**	*24* **5600**	**136**
Demand	**210**	**106**	**78**	54	**448**

Total income generated using the row minima method
6600 * 78 = ₦ 514,800
6600 * 30 = ₦ 198,000

6000 * 106 = ₦ 636,000
7000 * 98 = ₦ 686,000
5700 * 104 = ₦ 592,800
5600 * 8 = ₦ 44,800
Total income = ₦ 2,806,800

4.3.3 The Least Cost Method

Table10: Least Cost Method

	A	B	C	D	Supply
A	**6700** 104	**6700** 4	**6600**	**6600**	108
B	* **6000** 106	**6000**	**6000**	**6100**	106
C	**6800**	**7000** 98	**6800**	**6800**	**98**
D	**5700** 104	**5600** **4**	**5600** 78	**5600** *54*	**136**
Demand	**210**	**106**	**78**	54	**448**

Total income generated using the least cost method
6700 * 104 = ₦ 696,800
6700 * 4 = ₦ 26,800
6000 * 106 = ₦ 636,000
7000 * 98 = ₦ 686,000
5600 * 4 = ₦ 22,400
5600 * 78 = ₦ 436,800
5600 * 54 = ₦ 302,400
Total income= ₦ 2,807,200

4.4 Construction of Bluewhales Transport Company Limited Transportation Problem

Table 11: Data Collected from BlueWhales Travels

ROUTE	VEHICLE MAKE	NO OF VEHICLE		NO OF PASSENGERS	INCOME
		TO	FRO		
Abuja – Lagos	22	2	2	88	506,000
	22	1	1	44	253,000
Abuja – Delta	22	1	1	44	233,200
	36	1	1	76	395,200
Abuja Yenagoa	42	1	1	84	638,400
	22	1	1	44	334,400
Abuja – Edo	22	3	3	132	871,200
	36	1	1	72	475,200
TOTAL		11	11	584	₦3,706,600

Table 11 above can be transformed into transportation tableau as shown in the next table.

Table12: Blue Whales Travels Transportation Tableau

	A	B	C	D	Supply
A	**5750** 44	**5750** 44	**5750** 44	**5750**	132
B	**5300** **44**	**5200** 76	**5300**	**5300**	**120**
C	**7600** 84	**7600** 44	**7600**	**7600**	**128**
D	**6600** 44	**6600** 44	**6600** 44	**6600** *72*	**204**
Demand	**216**	**208**	**88**	72	**584**

4.4.1 Column Minima Method

Table 13: Column Minima Method

	A	B	C	D	Supply
A	**5750** 132	**5750**	**5750**	**5750**	132
B	**5300** **84**	**5200** 36	**5300**	**5300**	**120**
C	**7600**	**7600** 128	**7600**	**7600**	**128**
D	**6600**	**6600** 44	**6600** 88	**6600** *72*	**204**
Demand	**216**	**208**	**88**	72	**584**

Total income generated using the column minima method

5750 * 132 = ₦ 759,000
5300 * 84 = ₦ 445,200
7600 * 128 = ₦ 972,800
6600 * 80 = ₦ 528,000
6600 * 88 = ₦ 580,800
6600 * 36 = ₦ 237,600
5300 * 36 = ₦ 190,800
Total income = ₦ 3,714,200

4.4.2 Row Minima Method

Table 14: Row Minima Method

	A	B	C	D	Supply
A	**5750** 132	**5750**	**5750**	**5750**	132
B	**5300** 84	**5200** 36	**5300**	**5300**	**120**
C	**7600**	**7600** 128	**7600** 44	**7600**	**128**
D	**6600** 132	**6600** 44	**6600** 88	**6600** *72*	**204**
Demand	**216**	**208**	**88**	72	**584**

Total income generated using the row minima method

5750 * 132 = ₦ 759,000
5300 * 84= ₦ 445,200

5200 * 36 = ₦ 187,200
7600 * 128 = ₦ 972,800
6600 * 44 = ₦ 290,400
6600 * 88 = ₦ 580,800
6600 * 72 = ₦ 475,200
Total income = ₦ 3,701,600

4.4.3The Least Cost Method

Table15: Least Cost Method

	A	B	C	D	Supply
A	**5750**	**5750**	**5750** 88	**5750** 44	132
B	* **5300** 120	**5200**	**5300**	**5300**	**120**
C	**7600** 96	**7600** 32	**7600**	**7600**	**128**
D	**6600**	**6600** 176	**6600**	**6600** *28*	**204**
Demand	**216**	**208**	**88**	72	**584**

Total income generated using the least cost method
5750 * 88 = ₦ 506,000
5750 * 44 = ₦ 253,000
5300 * 120 = ₦ 636,000
7600 * 96 = ₦ 72,960
7600 * 32 = ₦ 243,200
6600 * 176 = ₦ 1,161,600
6600 * 28 = ₦ 184,800
Total income = ₦ 3,714,200

4.5 Construction of Young Shall Grow Transportation Tableau

Table 16: Data Collected from Young Shall Grow Transport

ROUTE	PASSENGER PER BUS	NO OF VEHICLE		TOTAL PASSENGER	INCOME
		TO	FRO		
ABUJA-ENUGU	22	3	3	132	607,200
	50	1	1	100	450,000
ABUJA-	22	2	2	88	369,600
ONITSHA	50	2	2	200	820,000
ABUJA – ABA	22	1	1	44	233,200
	50	1	1	100	530,000
ABUJA-	22	2	2	88	440,000
UMUAHIA	50	1	1	100	500,000
TOTAL		13	13	852	3,950,000

Table16 above can be transformed into transportation tableau as shown in the next table

Table 17: Young Shall Grow Transportation Tableau

	A	B	C	D	Supply
A	4600 44	**4600** 44	4600 44	**4500** 100	123
B	4200 44	4200 44	4100 100	*4100* **100**	288
C	**5300** 44	5300 100	**5300**	*5300*	**144**
D	**5000** 44	**5000** 44	**5000** 100	**5000**	**188**
Demand	**176**	**232**	**244**	200	**852**

4.5.1 Column Minima Method

Table18: Column Minima Method

	A	B	C	D	Supply
A	4600 176	**4600** 56	4600	**4500**	232
B	4200	4200 176	4100 112	*4100*	288
C	**5300**	5300	**5300**	*5300* *144*	**144**
D	**5000**	**5000**	**5000** 132	**5000** *56*	**188**
Demand	**176**	**232**	**244**	200	**852**

Total income generated using the column minima method

4600 * 176 = ₦ 809,600
4600 * 56 = ₦ 257,600
4200 * 176= ₦ 739,200
4100 * 112 = ₦ 459,200
5300 * 144 = ₦ 763,200
5000 * 132 = ₦ 660,000
5000 * 56 = ₦ 280,000
Total income = ₦ 3,968,800

4.5.2 The Row Minima Method

Table19: Row Minima Method

	A	B	C	D	Supply
A	4600 176	**4600** 56	4600	**4500**	232
B	4200	4200 176	4100 112	*4100*	288
C	**5300**	5300	**5300** 132	*5300* *12*	**144**
D	**5000**	**5000**	**5000**	**5000** *188*	**188**
Demand	**176**	**232**	**244**	200	**852**

Total income generated using row minima method

4600 * 176 = ₦ 809,600
4600 * 56 = ₦ 257,600
4200 * 176= ₦ 739,200
4100 * 112 = ₦ 459,200
5300 * 132 = ₦ 699,600
5300 * 12 = ₦ 63,600
5000 * 188 = ₦ 940,000
Total income = ₦ 3,968,800

4.5.3 The Least Cost Method

Table 20: Least Cost Method

	A	B	C	D	Supply
A	4600	4600 120	4600 112	4500	232
B	4200 * 176	4200 112	4100	*4100*	288
C	5300	5300	5300 132	*5300* *12*	144
D	5000	5000	5000	5000 *188*	188
Demand	176	232	244	200	852

Total income generated using least cost method

4600 * 120= ₦ 552,000
4600 * 112 = ₦ 515,200
4200 * 176= ₦ 739,200
4200 * 112 = ₦ 470,400
5300 * 132 = ₦ 699,600
5300 * 12 = ₦ 63,600
5000 * 188 = ₦ 940,000

Total income = ₦ 3,980,000

5 Result, Conclusion and Recommendation

5.1 Result

Using the excel solver application, the optimal solution for the four companies were obtained as follows:Pleasure Travels Limited **= ₦ 3,401,600**, Peace Mass Transit**= ₦2,823,600**, Blue Whales Transport Company Limited**= ₦3,714,200**, Young Shall Grow Motors Limited**= ₦3,980,000**. The table below shows the companies name, initial income generated by the companies, the initial basic feasible solution using the three (3) different methods of solving transportation problems, the optimal solution obtained using the excel solver, the difference between the optimal solution and the initial basic feasible solution methods and the percentage (%) difference.

Table 21: Analysis of Results

COMPANY NAME	INITIAL SOLUTION	METHODS			OPTIMAL SOLUTION	DIFFERENCE			% DIFFERENCE		
COMPANY	INITIAL	CMM	RMM	LCM	OPTIMAL	OPT-CMM	OPT-RMM	OPT-LCM	CMM	RMM	LCM
PT	**3,391,200**	3,401,600	3,393,400	3,401,600	3,401,600	0	8,200	0	0	0.24%	0
PMT	**2,798,800**	2,806,800	2,806,800	2,807,200	2,823,600	16,800	16,800	16,400	0.59%	0.59%	0.58%
BTL	**3,706,600**	3,714,200	3,710,600	3,714,200	3,714,200	0	3,600	0	0	0.09%	0
YSGML	**3,950,000**	3,968,800	3,968,800	3,980,000	3,980,000	11,200	11,200	0	0.28%	0.28%	0

Difference = Optimal Solution – Initial Basic Feasible Solution

$$\% \, difference = \frac{optimal\ solution - initial\ basic\ feasible\ solution}{optimal\ solution} \times 100$$

From the table above it is clear that the column minima method generated the same income as that of the optimal solution for two companies (Pleasure Travels and Bluewhales Travels), and generated income less than that of the optimal solution for two companies(Peace Mass Transit and Young Shall Grow Motors Limited). However, the column minima method generates a higher income than the initial income for all the companies

The row minima method generated an income the same as the optimal solution for none of the companies. However, the row minima method also generates a higher income than the initial income for all the companies.

The least cost method yields the best initial basic feasible solution for all the companies as its income generated were found to be the same as that of optimal solution for three companies (Pleasure Travels, Blue whales Transport Company and Young Shall Grow Motors Limited) and generates an income less than that of the optimal solution for one company (Peace Mass Transit).

This could be achieved because it takes into consideration the least cost associated with each route alternatives. Although it takes a longer time to compute, this is something the Column Minima Method and RowMinima Method could not have achieved.

The above data is shown graphically below

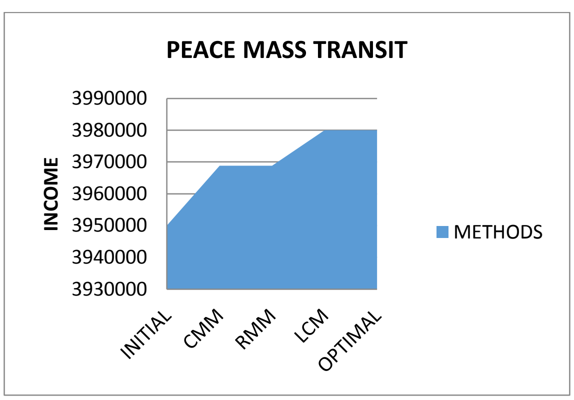

Fig 1: income for peace mass transit

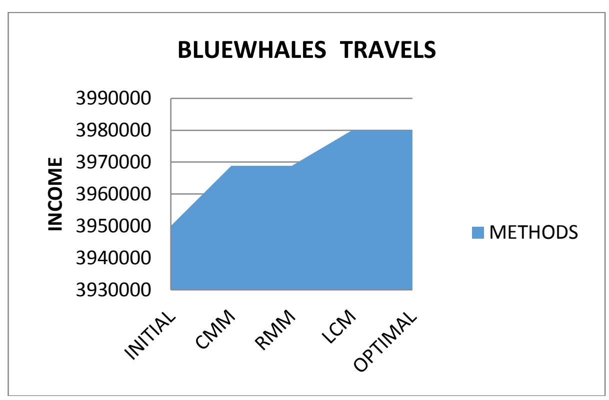

Fig 2: income for bluewhales travels

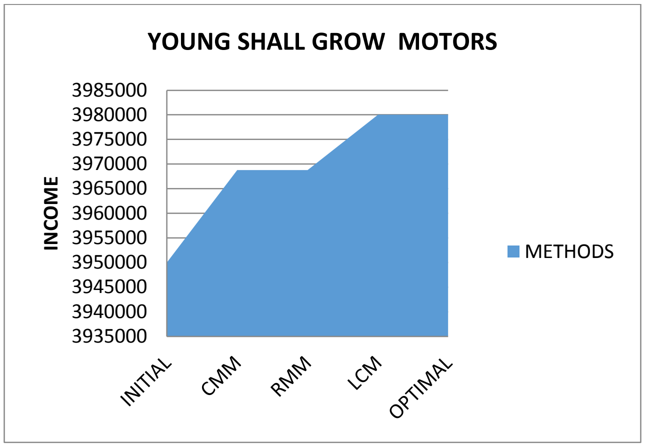

Fig 3: income for young shall grow motors

5.2 Conclusion

An advantage of the transportation problem algorithm is that, the solution process involves only main variables, artificial variables are not required as in the case of simplex method. Large transportation problem could relatively be solved using the transportation algorithm. It was shown that solving balanced transportation problem was easier using the column minima method and row minima method using the transportation problem formulated for Pleasure Travels, Bluewhales Transport Co. Ltd, Peace Mass Transit and Young Shall Grow Motors Ltd. The least cost method was found to be a bit difficult to calculate, yet it always yields a better result near optimal if not optimal. It can therefore be concluded that the least cost method although not quite as easy as the other methods, facilitates a better initial basic feasible solution than the column minima method and the row minima method. The optimal solution obtained using the excel solver application for the four (4) companies were found to be **₦3,401,600.00, ₦3,714,000.00, ₦2,823,600.00** and **₦3,980,000.00** respectively.
It should be noted that although the Least cost method facilitates an optimal or near optimal result, it is not too reliable since it did not yield an optimal result for all the companies.

5.2.1 Recommendation

The use of a scientific approach gives a systematic and transparent solution as compared with a haphazard method. Using the more scientific assignment problem model for a given transportation problem gives a better result and management may benefit from the proposed approach to guarantee optimal profits from them. We therefore recommend that the assignment problem model should be adopted by managements of transportation companies for maximum profits.
In general, the passenger population (demand) is always greater than the vehicle availability (supply). But this can only be verified (for an unbalanced transportation problem), if the actual data management and records are always available for researches to be carried out on them. Further research needs to be carried out in the area of bus services and maintenance looking at the number of times the buses could breakdown.

References

[1] Hitchcock, F.L. (1941) 'The Distribution of a Product from Several Sources to Numerous Localities' *Journal of Mathematics and Physics*, **20**, 224-230. http://dx.doi.org/10.1002/sapm1941201224

[2] Dantzig, G.B. (1951) Application of the Simplex Method to a Transportation Problem, Activity Analysis of Production and Allocation. In: Koopmans, T.C., Ed., John Wiley and Sons, New York, 359-373.

[3] Charnes, A., Cooper, W.W. and Henderson, A. (1953) An Introduction to Linear Programming. John Wiley & Sons,New York.

[4] Taghrid, I., Gaber, E., Mohamed, G., Iman, S. (2009) 'Solving transportation problems using object-oriented model' *International Journal of Computer Science and Network Security,* Vol. 9, No. 2, 353-361.

[5] Abdul S.S, Gurudeo A.T, Ghulam, M.B., (2014) 'A comparative study of initial feasible solution methods for transportation problems' *Mathematical Theory and Modelling Vol.4, No.1 2014, ISSN 2225-0522, 134-136*

[6] Abdul., Q., Shakeel., J., and Khalid., M.M., (2012) 'A new method for finding an optimal solution for transportation problems' *International Journal of Computer Science and Engineering,* 1271-1274

[7] Veena, A., and Krysztof, K. (2009) 'Alternate solutions analysis for transportation problem' *Journal of Business and Economic Research, 7(6)*, 6-9.

[8] Anthony, C., Zhong, Z., Piya, C., Seungkyu, R., Chao, Y. and Wong, S.C., (2011)'Transport network design problem under uncertainty: A review and new developments' *Transport Reviews: A Transnational Transdisciplinary Journal*, 31(6), 743-768.

[9] Odior, A.O., Charles-Owaba, O.E., and Oyawale, F.A., (2010) 'Determining feasible solutions of a multi-criteria assignment problem' *Journal of Applied Science and nvironmental Management, 14(1)*, 35-38.

[10] Ashayeri, J and Gelders, L.F. (1985) 'Warehouse design optimization' *Journal of Operation Research*, 21(1985), 285-294.

[11] Mollah., M.A.A., Aminur, R.K., Md. Sharif U., Faruque A. (2016) 'A new approach to solve transportation problems' *Open Journal of Optimization* 5(2016), 22-30.