Journal of the Nigerian Association of Mathematical Physics
Volume 53 **(November** *2019 Issue*), **pp39 – 50**
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ENHANCING THE MEAN RATIO ESTIMATORS FOR ESTIMATING THE POPULATION MEAN USING MIXED PARAMETERS

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Abstract

In this study we have incorporated a mixed parameter of the auxiliary variable to propose three new improved ratio estimators to enhance the mean ratio estimators for estimating the population mean. To enhance the efficiency of the proposed ratio estimators, a linear combination of the coefficient of kurtosis, first decile, quartile deviation, median and the population variance of the auxiliary variables have been harnessed. The properties related to the suggested estimators are assessed through constant, bias and mean square error. We have also provided empirical study for illustration and corroboration using two population data in evaluating the efficiencies of the proposed modified ratio estimators using percentage relative error (PRE). The suggested ratio estimators performed better than the other existing ratio estimators compared with in the study by emerging with a lesser mean squared error and should be preferred in application over the existing estimators, due to their smaller values of bias and Mean Square Error.

Keywords: Ratio, Estimation, Deciles, Variance, kurtosis, MSE

Nomenclature

N	-	Population size
n	-	Sample size
$f = n/N$	-	Sampling fraction
Y	-	Study/ main variable
X	-	Auxiliary/ supporting variable
$\bar{X}$	-	mean of X
$\bar{Y}$	-	mean of Y
$\bar{x}$	-	Sample mean of x
$\bar{y}$	-	Sample mean of y
x, y	-	Sample totals
S_x, S_y	-	standard deviations of X & Y
S^2_x, S^2_y	-	variances of X & Y
S_{xy}	-	covariance between variables X & Y
C_x, C_y	-	coefficient of variation X & Y
ρ	-	Correlation coefficient of X & Y
$\beta(.)$	-	Bias of the estimator
MSE (.)	-	Mean square error of the estimator
PRE	-	Percentage relative efficiency
$\tilde{Y}_i$	-	abid*et'al* (2016) existing modified ratio estimator of $\bar{Y}$
$\bar{Y}_{ps}$	-	Existing modified ratio estimator of $\bar{Y}$
$\bar{Y}_{pi}$	-	Proposed modified ratio estimator of $\bar{Y}$
Md	-	median of X
β_1	-	Population skewness
β_2	-	Population kurtosis
D	-	Deciles
DM	-	Decile mean
QD	-	Quartile Deviation
TM	-	Tri-mean
HL	-	Hodges-Lehmann estimator (Median)
MR	-	Mid-range
Kt	-	kurtosis
Sk	-	Skewness
Cv	-	Coefficient of variation

Subscript
i, j for existing estimators

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1. INTRODUCTION

Sampling is not just pure replacement of fractional coverage for a complete coverage. Sampling is the knowledge, art of governing and determining the consistency of useful statistical information through the concept of probability as given in [1]. The most common method of sampling is the simple random sampling, which is drawn fragment by fragment with the same chances of pool for each part at each draw. Ratio estimates makes use of well-known population totals of auxiliary variables to improve the weight of sample values for population estimations of interest. This is used to modify the sample estimate for the variable of concern. Most at times in sample surveys, information on auxiliary variable X , along with study variable Y is collected in which the auxiliary variable X is highly associated with the study variable Y . This information on auxiliary variable X , may well be exploited to acquire a more effective estimator of the population mean. In such cases the ratio technique of estimation is introduced to explore the information of the auxiliary variable X which is positively associated with the study variable Y, this is imperative to reduce bias and increase the precision estimate of the population mean. This can be attained by presenting a large number of modified ratio estimators which exploits the information by well-known values of coefficient of variation, coefficient of skewness, coefficient of kurtosis, median, standard deviation etc. Likewise, the ratio weights are specified by $\frac{X}{x}$ where X is the recognized population total for the auxiliary variable and x is the agreeing estimate of the total centered on all corresponding units in the sample. In [2] some modified ratio estimators were proposed using a linear mixture of coefficient of skewness, coefficient of kurtosis and standard deviation of the auxiliary variable X . This research work would propose some improved family of ratio estimators based on the linear mixture of the variance, conventional and non-conventional location parameters of the auxiliary variable X , adapted from existing methodology and to further compare its productivity with existing modified ratio estimators algebraically and empirically using a three population data. The population statistics of the data consist of the fixed capital which is denoted by X (supporting variable) and output of 80 factories which are represented by Y (study variable). The population statistics of the data can obtainable in [3].

A colossal literature in the modification of ratio estimators are available in previous studies. Some of these modified ratio estimators are given in [1-16], which all made use of the same methodology in deriving the constants, bias and mean square error of their proposed modified ratio estimators.

Consider a finite population $Z=\{Z_1, Z_2, Z_3, ..., Z_n\}$ of N different and recognizable bounds. Let Y be the study variable with value Y_i of Z_i , $i=1,2,3,...,N$ giving a vector $Y=\{Y_1, Y_2, Y_3, ..., Y_n\}$. The objective is to evaluate the population mean on the basis of a random sample.

A new linear combinations of ratio type estimators in simple random sampling by using non-conventional measures like Hodges Lehman estimator, population mid-range and population tri-mean as supporting information was introduced in [17]. Another new linear combinations of ratio type estimators in simple random sampling by using non-conventional dispersion parameters like Hodges Downton's method, Gini's mean difference and probability weighted moment as supporting information which outperformed previous modified ratio estimators was given in [18]. In [19]a new linear combinations of ratio type estimators in simple random sampling by using deciles of the auxiliary variable which performed better than the compared estimators in the literature.

In [5] a classical ratio type estimator for the estimation of finite population mean using one supporting variable under simple random sampling scheme was proposed. In [3] a product type estimator to calculate the population mean or total of the study variable by using supporting information when coefficient of correlation is negative was suggested. The historical development of the ratio method of estimation is given in [20]. These improved ratio estimators, although biased, have low mean squared errors as compared with the classical ratio estimator. In [8] some ratio type estimators with the use of coefficient of variation and coefficient of kurtosis of the auxiliary variable was proposed. In [10] a ratio type estimators for the population mean in the simple random sampling with the use of some known auxiliary information on coefficient of kurtosis and coefficient of variation was Proposed. They showed that their proposed estimators are more efficient than traditional ratio estimator in the estimation of the population mean. Also in [12] some modified ratio estimators using known values of coefficient of correlation, Kurtosis and coefficient of Variation was Proposed. The use of coefficient of Skewness and coefficient of Kurtosis, respectively and showed that it provides better estimates for the population mean in comparison to the usual ratio estimator and different existing estimators was studied in [14]. In [1] an estimator with the use of coefficient of skewness and quartile deviation of the auxiliary information in the simple random sampling for the estimation of the population mean was studied as well. An improved ratio estimator based on linear mixture of median, coefficients of skewness and kurtosis of the auxiliary variable, which outshone some of the ratio estimators it was compared with using a population data was suggested in [15]. Another ratio type estimator using the linear mixture of kurtosis, median and quartile deviation. They evaluated its performance using a population data and it performed better than

prevailing estimators was put forward in [16]. A new linear combination of ratio type estimators in simple random sampling by using non-conventional measures like Hodges Lehman estimator, population mid-range and population tri-mean, Downton's, Gini's mean difference, Probability weighted moment and deciles as supporting information which also outperformed previous existing estimators was introduced in [17-19].
Another new linear combinations of ratio type estimators in simple random sampling by using the standard deviation, coefficient of skewness, coefficient of kurtosis and coefficient of variation of the auxiliary variable in proposing new modified ratio estimators which outperformed the estimators in the study is given in [21].
In [22]a new linear combination of ratio type estimators in simple random sampling by using the deciles and quartile deviation of the concomitant variable as supporting information which also outperformed the estimators considered in the study was proposed.
In [23]the variance of the auxiliary variable was incorporated to propose three improved ratio estimators of the population mean using a linear combination of the population coefficient of variation, kurtosis, skewness and the population variance of the auxiliary variable. The suggested improved ratio estimators performed better than other ratio estimators in the literature when compared using bias and mean square error.
Many modified ratio estimators have been proposed in the past in order to reduce the mean square error as given in the literature above. However, the results obtained still shows some level of mean squared error. This research utilizes a linear combination of the coefficient of kurtosis, variance, quartile deviation, median and first decile of the auxiliary variable X to propose a new class of the modified ratio estimators to enhance the mean ratio estimators which is hoped to have reduced mean square error when compared to the existing ones.

2. METHODOLOGY

The classical mean ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is given in [5] as

$$\hat{Y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} \quad (1)$$

The bias, constant and the mean square error of the ratio estimator are given by

$$B(\hat{Y}_r) = \frac{(1-f)}{n}\frac{1}{\bar{Y}}(RS_x^2 - \rho S_x S_y) \; R = \frac{\bar{Y}}{\bar{X}}; \qquad MSE(\hat{Y}_r) = \frac{(1-f)}{n}(S_y^2 + R^2S_x^2 - 2R\rho S_x S_y) \quad (2)$$

The mean ratio estimator above is used to enhance the precision of the estimate of the population mean in comparison with the sample mean estimator whenever a positive correlation exists between the study variable and the auxiliary variable.

Estimators proposed in [17]are given as:

$$\hat{Y}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + TM)}(\bar{X} + TM) \quad (3)$$

$$\hat{Y}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + TM)}(\bar{X}C_x + TM) \quad (4)$$

$$\hat{Y}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + TM)}(\bar{X}\rho + TM) \quad (5)$$

$$\hat{Y}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + MR)}(\bar{X} + MR) \quad (6)$$

$$\hat{Y}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}c_x + MR)}(\bar{X}c_x + MR) \quad (7)$$

$$\hat{Y}_6 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + MR)}(\bar{X}\rho + MR) \quad (8)$$

$$\hat{Y}_7 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + HL)}(\bar{X} + HL) \quad (9)$$

$$\hat{Y}_8 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + HL)}(\bar{X}C_x + HL) \quad (10)$$

$$\hat{Y}_9 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + HL)}(\bar{X}\rho + HL) \quad (11)$$

The biases, related constants and mean square (MSE) of the estimators given in [17] are respectively given by:

$$B(\hat{\bar{Y}}_1)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2,\ R_1=\frac{\bar{Y}}{(\bar{X}+TM)};\ MSE(\hat{\bar{Y}}_1)=\frac{(1-f)}{n}(R_1^2S_x^2+S_y^2(1-\rho^2)) \quad (12)$$

$$B(\hat{\bar{Y}}_2)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2,\ R_2=\frac{\bar{Y}C_x}{(\bar{X}C_x+TM)};\ MSE(\hat{\bar{Y}}_2)=\frac{(1-f)}{n}(R_2^2S_x^2+S_y^2(1-\rho^2)) \quad (13)$$

$$B(\hat{\bar{Y}}_3)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2,\ R_3=\frac{\bar{Y}p}{(\bar{X}\rho+TM)};\ MSE(\hat{\bar{Y}}_3)=\frac{(1-f)}{n}(R_3^2S_x^2+S_y^2(1-\rho^2)) \quad (14)$$

$$B(\hat{\bar{Y}}_4)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2,\ R_4=\frac{\bar{Y}}{(\bar{X}+MR)};\ MSE(\hat{\bar{Y}}_4)=\frac{(1-f)}{n}(R_4^2S_x^2+S_y^2(1-\rho^2)) \quad (15)$$

$$B(\hat{\bar{Y}}_5)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2,\ R_5=\frac{\bar{Y}C_x}{(\bar{X}C_x+MR)};\ MSE(\hat{\bar{Y}}_5)=\frac{(1-f)}{n}(R_5^2S_x^2+S_y^2(1-\rho^2)) \quad (16)$$

$$B(\hat{\bar{Y}}_6)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2,\ R_6=\frac{\bar{Y}\rho}{(\bar{X}\rho+MR)};\ MSE(\hat{\bar{Y}}_6)=\frac{(1-f)}{n}(R_6^2S_x^2+S_y^2(1-\rho^2)) \quad (17)$$

$$B(\hat{\bar{Y}}_7)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2,\ R_7=\frac{\bar{Y}}{(\bar{X}+HL)};\ MSE(\hat{\bar{Y}}_7)=\frac{(1-f)}{n}(R_7^2S_x^2+S_y^2(1-\rho^2)) \quad (18)$$

$$B(\hat{\bar{Y}}_8)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2,\ R_8=\frac{\bar{Y}C_x}{(\bar{X}C_x+HL)};\ MSE(\hat{\bar{Y}}_8)=\frac{(1-f)}{n}(R_8^2S_x^2+S_y^2(1-\rho^2)) \quad (19)$$

$$B(\hat{\bar{Y}}_9)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2,\ R_9=\frac{\bar{Y}\rho}{(\bar{X}\rho+HL)};\ MSE(\hat{\bar{Y}}_9)=\frac{(1-f)}{n}(R_9^2S_x^2+S_y^2(1-\rho^2)) \quad (20)$$

Estimators proposed in [18] are given as:

$$\hat{\bar{Y}}_{10}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+G)}(\bar{X}+G) \quad (21)$$

$$\hat{\bar{Y}}_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+G)}(\bar{X}\rho+G) \quad (22)$$

$$\hat{\bar{Y}}_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+G)}(\bar{X}C_x+G) \quad (23)$$

$$\hat{\bar{Y}}_{13}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+D)}(\bar{X}+D) \quad (24)$$

$$\hat{\bar{Y}}_{14}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D)}(\bar{X}\rho+D) \quad (25)$$

$$\hat{\bar{Y}}_{15}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D)}(\bar{X}C_x+D) \quad (26)$$

$$\hat{\bar{Y}}_{16}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+S_{pw})}(\bar{X}+S_{pw}) \quad (27)$$

$$\hat{\bar{Y}}_{17}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+S_{pw})}(\bar{X}\rho+S_{pw}) \quad (28)$$

$$\hat{\bar{Y}}_{18}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+S_{pw})}(\bar{X}C_x+S_{pw}) \quad (29)$$

The biases, related constants and mean square (MSE) of the estimators given in [18] are respectively given by:

$$B(\hat{\bar{Y}}_{10})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2,\ R_{10}=\frac{\bar{Y}}{(\bar{X}+G)};\qquad MSE(\hat{\bar{Y}}_{10})=\frac{(1-f)}{n}(R_{10}^2S_x^2+S_y^2(1-\rho^2)) \quad (30)$$

$$B(\hat{\bar{Y}}_{11})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2,\ R_{11}=\frac{\bar{Y}\rho}{(\bar{X}\rho+G)};\ MSE(\hat{\bar{Y}}_{11})=\frac{(1-f)}{n}(R_{11}^2S_x^2+S_y^2(1-\rho^2)) \quad (31)$$

$$B(\hat{\bar{Y}}_{12})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2,\ R_{12}=\frac{\bar{Y}C_x}{(\bar{X}C_x+G)};\ MSE(\hat{\bar{Y}}_{12})=\frac{(1-f)}{n}(R_{12}^2S_x^2+S_y^2(1-\rho^2)) \quad (32)$$

$$B(\hat{\bar{Y}}_{13})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{13}^2,\ R_{13}=\frac{\bar{Y}}{(\bar{X}+D)};\ MSE(\hat{\bar{Y}}_{13})=\frac{(1-f)}{n}(R_{13}^2S_x^2+S_y^2(1-\rho^2)) \quad (33)$$

$$B(\hat{\bar{Y}}_{14})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2,\ R_{14}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D)};\ MSE(\hat{\bar{Y}}_{14})=\frac{(1-f)}{n}(R_{14}^2S_x^2+S_y^2(1-\rho^2)) \quad (34)$$

$$B(\hat{\bar{Y}}_{15})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{15}^2,\ R_{15}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D)};\ MSE(\hat{\bar{Y}}_{15})=\frac{(1-f)}{n}(R_{15}^2S_x^2+S_y^2(1-\rho^2)) \quad (35)$$

$$B(\hat{\bar{Y}}_{16})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{16}^2,\ R_{16}=\frac{\bar{Y}}{(\bar{X}+S_{pw})};\ MSE(\hat{\bar{Y}}_{16})=\frac{(1-f)}{n}(R_{16}^2S_x^2+S_y^2(1-\rho^2)) \quad (36)$$

$$B(\hat{\bar{Y}}_{17})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{17}^2,\ R_{17}=\frac{\bar{Y}\rho}{(\bar{X}\rho+S_{pw})};\ MSE(\hat{\bar{Y}}_{17})=\frac{(1-f)}{n}(R_{17}^2S_x^2+S_y^2(1-\rho^2)) \quad (37)$$

$$B(\hat{\bar{Y}}_{18})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{18}^2,\ R_{18}=\frac{\bar{Y}C_x}{(\bar{X}C_x+S_{pw})};\ MSE(\hat{\bar{Y}}_{18})=\frac{(1-f)}{n}(R_{18}^2S_x^2+S_y^2(1-\rho^2)) \quad (38)$$

Estimators proposed in [19] are given as:

Case I

$$\hat{\bar{Y}}_{pk}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D_k)}(\bar{X}\rho+D_k) \text{ Where } k=1,2,...,10 \quad (39)$$

Case II

$$\hat{\bar{Y}}_{pl}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_l)}(\bar{X}C_x+D_l) \text{ Where } l=1,2,...,10 \quad (40)$$

The biases, related constants and mean square (MSE) of the estimators given in [19] are respectively given by:

$$B(\hat{\bar{Y}}_{pk})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{pk}^2,\ R_{pk}=\frac{\bar{Y}\rho}{(\bar{X}\rho+D_k)};\ MSE(\hat{\bar{Y}}_{pk})=\frac{(1-f)}{n}(R_{pk}^2S_x^2+S_y^2(1-\rho^2)) \quad (41)$$

$$B(\hat{\bar{Y}}_{pl})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{pl}^2,\ R_{pl}=\frac{\bar{Y}C_x}{(\bar{X}C_x+D_l)};\ MSE(\hat{\bar{Y}}_{pl})=\frac{(1-f)}{n}(R_{pl}^2S_x^2+S_y^2(1-\rho^2)) \quad (42)$$

Estimators proposed in [21]are given as:

$$\hat{\bar{Y}}_{19}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+S_x)}(\bar{X}\rho+S_x) \quad (43)$$

$$\hat{\bar{Y}}_{20}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+S_x)}(\bar{X}\beta_2+S_x) \quad (44)$$

$$\hat{\bar{Y}}_{21}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+S_x)}(\bar{X}\beta_1+S_x) \quad (45)$$

$$\hat{\bar{Y}}_{21}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+S_x)}(\bar{X}C_x+S_x) \quad (46)$$

The biases, related constants and mean square error (MSE) of the estimators given in [21] are respectively given by:

$$B(\hat{\bar{Y}}_{19})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{19}^2,\ R_{19}=\frac{\bar{Y}\rho}{(\bar{X}\rho+S_x)};\ MSE(\hat{\bar{Y}}_{19})=\frac{(1-f)}{n}(R_{19}^2S_x^2+S_y^2(1-\rho^2)) \quad (47)$$

$$B(\hat{\bar{Y}}_{20})=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{20}^2,\ R_{20}=\frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+S_x)};\ MSE(\hat{\bar{Y}}_{20})=\frac{(1-f)}{n}(R_{20}^2S_x^2+S_y^2(1-\rho^2)) \quad (48)$$

$$B(\hat{Y}_{21}) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{21}^2,\ R_{21} = \frac{\bar{Y}\beta_1}{(\bar{X}\beta_1 + S_x)};\ MSE(\hat{Y}_{21}) = \frac{(1-f)}{n}(R_{21}^2 S_x^2 + S_y^2(1-\rho^2)) \quad (49)$$

$$B(\hat{Y}_{22}) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{22}^2,\ R_{22} = \frac{\bar{Y}C_x}{(\bar{X}C_x + S_x)};\ MSE(\hat{Y}_{22}) = \frac{(1-f)}{n}(R_{22}^2 S_x^2 + S_y^2(1-\rho^2)) \quad (50)$$

Also [22] Proposed 20 estimators with the most performing estimator amongst all given by;

$$\hat{Y}_{23} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}Q_d + D_{10})}(\bar{X}Q_d + D_{10}) \quad (51)$$

The bias, related constant and mean square error (MSE) of the estimator given in [22] is respectively given as:

$$B(\hat{Y}_{23}) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{23}^2,\ R_{23} = \frac{\bar{Y}Q_d}{(\bar{X}Q_d + D_{10})};\ MSE(\hat{Y}_{23}) = \frac{(1-f)}{n}(R_{23}^2 S_x^2 + S_y^2(1-\rho^2)) \quad (52)$$

Estimators proposed in [23]are given as:

$$\hat{Y}_{24} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + S_x^2)}(\bar{X}C_x + S_x^2) \quad (53)$$

$$\hat{Y}_{25} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + S_x^2)}(\bar{X}\beta_2 + S_x^2) \quad (54)$$

$$\hat{Y}_{26} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + S_x^2)}(\bar{X}\beta_1 + S_x^2) \quad (55)$$

The biases, related constants and mean square error (MSE) of the estimators given in [23] are respectively given by:

$$B(\hat{Y}_{24}) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{24}^2,\ R_{24} = \frac{\bar{Y}C_x}{(\bar{X}C_x + S_x^2)};\ MSE(\hat{Y}_{24}) = \frac{(1-f)}{n}(R_{24}^2 S_x^2 + S_y^2(1-\rho^2)) \quad (56)$$

$$B(\hat{Y}_{25}) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{25}^2,\ R_{25} = \frac{\bar{Y}\beta_2}{(\bar{X}\beta_2 + S_x^2)};\ MSE(\hat{Y}_{25}) = \frac{(1-f)}{n}(R_{25}^2 S_x^2 + S_y^2(1-\rho^2)) \quad (57)$$

$$B(\hat{Y}_{26}) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{26}^2,\ R_{26} = \frac{\bar{Y}\beta_1}{(\bar{X}\beta_1 + S_x^2)};\ MSE(\hat{Y}_{26}) = \frac{(1-f)}{n}(R_{26}^2 S_x^2 + S_y^2(1-\rho^2)) \quad (58)$$

The Proposed Ratio Estimators

Driven by the modified ratio estimators, proposed in [17-19] and [21-23] We suggest three new modified ratio estimators by adapting the methodology of the stated estimators, using the linear mixture of the coefficient of Kurtosis, quartile deviation, first decile and population variance of the auxiliary variable, on the assumption that the auxiliary variable is related to the study variable and any known information of the auxiliary variable can be utilized in modifying new ratio estimators that can be used to estimate the population mean with the aim of reducing MSE, the suggested modified ratio estimators given as follows:

$$\hat{Y}_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\omega_1 + S_x^2)}(\bar{X}\omega_1 + S_x^2) \quad (59)$$

$$\hat{Y}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\omega_2 + S_x^2)}(\bar{X}\omega_2 + S_x^2) \quad (60)$$

$$\hat{Y}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\omega_3 + S_x^2)}(\bar{X}\omega_3 + S_x^2) \quad (61)$$

Where;

$\omega_1 = D_1 \times \beta_2, \omega_2 = QD \times \beta_2$ and $\omega_3 = Md \times \beta_2$

Where ω_1 is the combination of the first decile and the coefficient of kurtosis while

ω_2 is the combination of quartile deviation and kurtosis and

ω_3 Is the combination of median and kurtosis all from the auxiliary variable

The properties of the proposed estimators can be obtained as follows:
MSE of these estimators can be gotten using Taylor's series technique defined as

$$h(\bar{x},\bar{y}) \cong h(\bar{X},\bar{Y}) + \frac{\partial h(C,d)}{\partial c} \mid \bar{X},\bar{Y}(\bar{x}-\bar{X}) + \frac{\partial h(C,d)}{\partial d} \mid \bar{X},\bar{Y}(\bar{y}-\bar{Y}) \quad (62)$$

where $h(\bar{x},\bar{y}) = \hat{R}_{pi}$ and. $h(\bar{X},\bar{Y}) = R$

As shown in [24], equation (62) can be applied to the proposed estimators in order to obtain the bias, relative constants and MSEs of the proposed class of modified ratio estimators given above.

For the mixture of first decile, coefficient of Kurtosis and Variance of the auxiliary variable.

$$\hat{R}_{p1} - R \cong \frac{\partial \frac{[(\bar{y}+b(\bar{X}-\bar{x}))]}{(\bar{x}\omega_1+S_x^2)}}{\partial \bar{x}} \mid \bar{X},\bar{Y}(\bar{x}-\bar{X}) + \frac{\partial \frac{[(\bar{y}+b(\bar{X}-\bar{x}))]}{(\bar{x}\omega_1+S_x^2)}}{\partial \bar{y}} \mid \bar{X},\bar{Y}(\bar{y}-\bar{Y}) \quad (63)$$

$$\cong -\left(\frac{\bar{y}\omega_1}{(\bar{x}\omega_1+S_x^2)^2} + \frac{b(\bar{X}\omega_1+S_x^2)}{(\bar{x}\omega_1+S_x^2)^2}\right) \mid \bar{X},\bar{Y}(\bar{x}-\bar{X}) + \frac{1}{(\bar{x}\omega_1+S_x^2)} \mid \bar{X},\bar{Y}(\bar{y}-\bar{Y}) \quad (64)$$

$$E(\hat{R}_{p1}-R)^2 \cong \frac{(\bar{Y}\omega_1+\beta(\bar{X}\omega_1+S_x^2))^2}{(\bar{X}\omega_1+S_x^2)^4}V(\bar{x}) - 2\frac{(\bar{Y}\omega_1+B(\bar{X}\omega_1+S_x^2))}{(\bar{X}\omega_1+S_x^2)^3}Cov(\bar{x},\bar{y}) + \frac{1}{(\bar{X}\omega_1+S_x^2)^2}V(\bar{y}) \quad (65)$$

$$= \frac{1}{(\bar{X}\omega_1+S_x^2)^2}\left\{\frac{(\bar{Y}\omega_1+\beta(\bar{X}\omega_1+S_x^2))^2}{(\bar{X}\omega_1+S_x^2)^2}V(\bar{x}) - 2\frac{(\bar{Y}\omega_1+\beta(\bar{X}\omega_1+S_x^2))}{(\bar{X}\omega_1+S_x^2)}Cov(\bar{x},\bar{y}) + V(\bar{y})\right\} \quad (66)$$

Where $\beta = \frac{S_{xy}}{S_x^2} = \frac{\rho S_x S_y}{S_x^2} = \frac{\rho S_y}{S_x}$ note we omit the difference of $(E(b)-\beta)$ (67)

Because this assumes a line through the origin, as in the unbiased case for design-based ratio estimator. In [25] this condition for bias to 'vanish' for SRS makes sense because weighted least square (WLS) regression, and ordinary (homoscedastic) least squares (OLS) regression are both unbiased for b. a derivation of the design based ratio estimator which shows it is unbiased when we have a linear regression through the origin with the regression coefficient being homoscedastic is given in [25].

$$MSE(\bar{y}_{p1}) = \left(\bar{X}\omega_1+S_x^2\right)^2 E\left(\hat{R}_{p1}-R\right)^2 \quad (68)$$

$$\cong \frac{(\bar{Y}\omega_1+\beta(\bar{X}\omega_1+S_x^2))^2}{(\bar{X}\omega_1+S_x^2)^2}V(\bar{x}) - 2\frac{(\bar{Y}\omega_1+\beta(\bar{X}\omega_1+S_x^2))}{(\bar{X}\omega_1+S_x^2)}Cov(\bar{x},\bar{y}) + V(\bar{y}) \quad (69)$$

$$\cong \frac{(\bar{Y}\omega_1)^2+2\beta(\bar{X}\omega_1+S_x^2)\bar{Y}\omega_1+\beta^2(\bar{X}\omega_1+S_x^2)^2}{(\bar{X}\omega_1+S_x^2)^2}V(\bar{x}) - \frac{2\bar{Y}\omega_1+2\beta(\bar{X}\omega_1+S_x^2)}{(\bar{X}\omega_1+S_x^2)}Cov(\bar{x},\bar{y}) + V(\bar{y}) \quad (70)$$

$$\cong \frac{(1-f)}{n}\left\{\left[\frac{(\bar{Y}\omega_1)^2}{(\bar{X}\omega_1+S_x^2)^2} + \frac{2\beta\bar{Y}\omega_1}{(\bar{X}\omega_1+S_x^2)} + \beta^2\right]S_x^2 - \left[\frac{2\bar{Y}\omega_1}{(\bar{X}\omega_1+S_x^2)} + 2\beta\right]S_{xy} + S_y^2\right\} \quad (71)$$

$$\cong \frac{(1-f)}{n}(R_{p1}^2S_x^2 + 2\beta R_{p1}S_x^2 + \beta^2S_x^2 - 2R_{p1}S_{xy} - 2\beta S_{xy} + S_y^2) \quad (72)$$

$$MSE(\bar{y}_{p1}) \cong \frac{(1-f)}{n}(R_{p1}^2S_x^2 + 2R_{p1}\rho S_xS_y + \rho^2S_y^2 - 2R_{p1}\rho S_xS_y - 2\rho^2S_y^2 + S_y^2) \quad (73)$$

$$\cong \frac{(1-f)}{n}(R_{p1}^2S_x^2 - \rho^2S_y^2 + S_y^2) \cong \frac{(1-f)}{n}(R_{p1}^2S_x^2 + S_y(1-\rho^2)) \quad (74)$$

$$R_{p1} = \frac{\bar{Y}\omega_1}{(\bar{X}\omega_1+S_x^2)} \quad (75)$$

$$bias(\bar{y}_{p1}) \cong \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p1}^2 \quad (76)$$

$$MSE(\bar{y}_{p2}) \cong \frac{(1-f)}{n}(R_{p2}^2S_x^2 + 2R_{p2}\rho S_xS_y + \rho^2S_y^2 - 2R_{p2}\rho S_xS_y - 2\rho^2S_y^2 + S_y^2) \quad (77)$$

$$\cong \frac{(1-f)}{n}(R_{p2}^2S_x^2 - \rho^2S_y^2 + S_y^2) \cong \frac{(1-f)}{n}(R_{p2}^2S_x^2 + S_y(1-\rho^2)) \quad (78)$$

$$R_{p2}=\frac{\bar{Y}\omega_2}{(\bar{X}\omega_2+S_x^2)} \quad (79)$$

$$bias(\bar{y}_{p2})\cong\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2 \quad (80)$$

$$MSE(\bar{y}_{p3})\cong\frac{(1-f)}{n}(R_{p3}^2S_x^2+2R_{p3}\rho S_xS_y+\rho^2S_y^2-2R_{p3}\rho S_xS_y-2\rho^2S_y^2+S_y^2) \quad (81)$$

$$\cong\frac{(1-f)}{n}(R_{p3}^2S_x^2-\rho^2S_y^2+S_y^2)\cong\frac{(1-f)}{n}(R_{p3}^2S_x^2+S_y(1-\rho^2)) \quad (82)$$

$$R_{p3}=\frac{\bar{Y}\omega_3}{(\bar{X}\omega_3+S_x^2)} \quad (83)$$

$$bias(\bar{y}_{p3})\cong\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p3}^2 \quad (84)$$

$$PRE=\frac{MSE(existing-estimator)}{MSE(proposed-estimator)}\times 100 \quad (85)$$

3. DATA PRESENTATION AND RESULTS

To examine the efficiencies of the suggested estimators we are going to consider two natural population datasets. The descriptive statistics of the datasets are given below.

Population I:

The performances of the suggested ratio estimators are assessed and compared with the prevailing ratio estimators by using a population data given in [3]

Fixed capital which is denoted by X (auxiliary variable)

Output of 80 factories which are represented by Y (study variable)

N=80; n=20; M_d =757.5; S_y =1835.659; S_x =845.610; ρ =0.941; $\bar{Y}$ =5182.637; $\bar{X}$ =1126.463;

D_1 =360; β_2 =-0.063386; β_1 =1.050002; C_x =0.751; Q_d = 588.125; M_r =1795.5; D_{10} =3480; G=904.081

Population II:

The performances of the suggested ratio estimators are assessed and compared with the prevailing ratio estimators by using a population data given in [26],

Y : Output

x : fixed capital

N=34; n=20; M_d =150; S_y =733.1407; S_x =150.5059; ρ =0.4491; $\bar{Y}$ =856.4117; D_1 =70.3;

$\bar{X}$ =208.8823; β_2 =0.0978; β_1 =0.9782; C_x =0.7205; Q_d = 80.25; M_r =284.5; D_{10} =564.0;G=162.996.

The best performing modified estimators amongst the existing proposed modified estimators for each class is selected for comparison with the new class of proposed modified estimators

Population I

Table 1: The Statistical Analysis of the Estimators for the Population I

Estimators	Constant	Bias	MSE
$\hat{Y}_5=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}c_x+MR)}(\bar{X}c_x+MR)$	1.473	11.23	72582.52
$\hat{Y}_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+G)}(\bar{X}\rho+G)$	2.224	25.58	146971.80
$\hat{Y}_{*a}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D_{10})}(\bar{X}\rho+D_{10})$	1.074	5.970	45412.23
$\hat{Y}_{*b}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_{10})}(\bar{X}C_x+D_{10})$	0.899	4.185	36162.07
$\hat{Y}_{20}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+S_x)}(\bar{X}\beta_2+S_x)$	-0.424	0.931	19298.56

$\hat{Y}_{23} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}Q_d+D_{10})}(\bar{X}Q_d+D_{10})$	0.995	5.120	41005.92
$\hat{Y}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\omega_1+S_x^2)}(\bar{X}\omega_1+S_x^2)$	**-0.172**	**0.152**	**15260.00**
$\hat{Y}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\omega_2+S_x^2)}(\bar{X}\omega_2+S_x^2)$	**-0.287**	**0.426**	**16680.28**
$\hat{Y}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\omega_3+S_x^2)}(\bar{X}\omega_3+S_x^2)$	**-0.376**	**0.733**	**18271.50**

Table 1 shows results of the existing estimators and the proposed estimators when applied to the population data given in [3] the related constant, Bias and Mean Square Error (MSE) of the three proposed estimators are all smaller when compared to the existing ones.

Population II

Table 2: The Statistical Analysis of the Estimators for the Population II

Estimators	Constant	Bias	MSE
$\hat{Y}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}c_x+MR)}(\bar{X}c_x+MR)$	1.4185	1.096	9772.39
$\hat{Y}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+G)}(\bar{X}\rho+G)$	1.498	1.221	9880.25
$\hat{Y}_{*a} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D_{10})}(\bar{X}\rho+D_{10})$	0.585	0.186	8993.40
$\hat{Y}_{*b} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_{10})}(\bar{X}C_x+D_{10})$	0.864	0.406	9181.80
$\hat{Y}_{20} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+S_x)}(\bar{X}\beta_2+S_x)$	0.490	0.131	8946.13
$\hat{Y}_{23} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}Q_d+D_{10})}(\bar{X}Q_d+D_{10})$	4.081	9.070	16602.16
$\hat{Y}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\omega_1+S_x^2)}(\bar{X}\omega_1+S_x^2)$	**0.244**	**0.032**	**8862.02**
$\hat{Y}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\omega_2+S_x^2)}(\bar{X}\omega_2+S_x^2)$	**0.277**	**0.041**	**8869.86**
$\hat{Y}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\omega_3+S_x^2)}(\bar{X}\omega_3+S_x^2)$	**0.489**	**0.129**	**8945.46**

Table 2 shows results of the existing estimators and the proposed estimators when applied to the population data given in [26] the related constant, Bias and Mean Square Error (MSE) of the three proposed estimators are all smaller when compared to the existing ones.

Table 3: PREs of the New Class of Modified Ratio Estimators for Population I

Estimators	$\hat{Y}_{p1}$	$\hat{Y}_{p2}$	$\hat{Y}_{p3}$
$\hat{Y}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}c_x+MR)}(\bar{X}c_x+MR)$	475.64	435.14	397.24
$\hat{Y}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+G)}(\bar{X}\rho+G)$	963.12	881.11	804.38
$\hat{Y}_{*a} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D_{10})}(\bar{X}\rho+D_{10})$	297.59	272.25	248.54
$\hat{Y}_{*b} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_{10})}(\bar{X}C_x+D_{10})$	236.97	216.80	197.92
$\hat{Y}_{20} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+S_x)}(\bar{X}\beta_2+S_x)$	126.47	115.70	105.62
$\hat{Y}_{23} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}Q_d+D_{10})}(\bar{X}Q_d+D_{10})$	268.72	245.83	224.53

Table 3 shows the Percentage Relative Efficiency values of the proposed modified estimators to the existing ones when applied to the population data given in [3].The results shows that the new modified estimators are more efficient than the existing ones.

Table 4:PREs of the New Class of Modified Ratio Estimators for Population II

Estimators	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$
$\hat{\bar{Y}}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}c_x + MR)}(\bar{X}c_x + MR)$	110.27	110.18	109.24
$\hat{\bar{Y}}_{11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + G)}(\bar{X}\rho + G)$	111.49	111.39	110.45
$\hat{\bar{Y}}_{*a} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + D_{10})}(\bar{X}\rho + D_{10})$	101.48	101.39	100.54
$\hat{\bar{Y}}_{*b} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_{10})}(\bar{X}C_x + D_{10})$	103.61	103.52	102.64
$\hat{\bar{Y}}_{20} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + S_x)}(\bar{X}\beta_2 + S_x)$	100.95	100.86	100.01
$\hat{\bar{Y}}_{23} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}Q_d + D_{10})}(\bar{X}Q_d + D_{10})$	187.34	187.17	185.59

Table 4 shows the Percentage Relative Efficiency values of the proposed modified estimators to the existing ones when applied to the population data given in [6].The results shows that the new modified estimators are more efficient than the existing ones.

4. DISCUSSION

The accessibility of auxiliary data improves the productivity of the estimators. The mean ratio estimator has been suggested using well-known values of population variance, coefficient of kurtosis, first decile, quartile deviation and median of the auxiliary variable. The performance of the proposed modified mean ratio estimators and the existing estimators has been evaluated using the data given in [3] and [26]. The population statistics are shown in population I and II. The properties of the proposed and existing estimators have been studied through their related constants, bias and mean squared errors. The bias of the suggested estimators are approximately approaching zero which gives the suggested estimators an edge over the prevailing ones. The mean squared errors of the proposed estimators performed better than all the existing estimators proposed in [17-19] and [21-22] compared with in this study. The (PREs) of the suggested ratio estimators with respect to the existing estimators were calculated in (Table 3 and 4). It is clearly seen that the suggested estimators outperformed all prevailing estimators compared with, which further shows the efficiency of the proposed estimator. Also, the percentage relative efficiency of the proposed estimator revealed some amount of percentage efficiency over the existing ratio estimators. Practical studies revealed that the bias and the mean squared error of the suggested estimators are less than that of the existing estimators under the two different naturally occurring populations. Therefore, the new class of modified ratio estimators suggested in this study under simple random sampling should be used for improved and steadier results, and should be considered over the existing modified ratio estimators for empirical applications.

5. CONCLUSION

In this study, we proposed three new modified ratio estimators for estimation the population under simple random sampling when information on the auxiliary variable is known. The MSE of the proposed estimators were derived using the partial derivative method as given in [24]. Also the conditions for which the proposed estimators outperforms the existing ones have been provided theoretically and empirically. An empirical study was also carried out to evaluate the performance of the proposed estimators over some related existing estimators under simple random sampling using two different natural populations which are positively strongly related and positively moderately related respectively. Tables (1 to 4) shows the constants, bias, MSE and PRE of the proposed and existing estimators of which the proposed estimators outperforms the existing ones by emerging with the least mean squared errors and a higher percentage relative efficiency. Amongst the three proposed estimators, the one with the linear combination of the first decile, coefficient of Kurtosis and variance of the auxiliary variable performed best.

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