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OBSERVABILITY OF DYNAMICAL SYSTEMS WITH MULTIPLE CONSTANT DELAYS IN THE STATE

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Abstract

This paper deals with the observability of dynamical system with multiple delays in the state using the software packages for computer aided control system design MATLAB and SIMULINK. We discuss how to choose the observers initial conditions and how to setup the observers gain. Sufficient conditions are proposed which guarantees the existence of a Luenberger-like observer for the general system.

1. Introduction

Differential equations with delays (DDEs) are equations were dependent variable is found in the state and output. Differential equation with delays is seen in science in the areas of physiology, biology, mathematics, economics, finance and in engineering. Basically in sciences of life like nervous system, a time delay is noted because of axonal transfer velocities. The distances interval between the neurons is determinate in biology (cell biology) for the reason that of maturation cell times, also in biology of molecular the reason been that the time taking for translation and transcription. These delays contributed in the formation of time oscillations in physics, robust in control system and are usual affect physiological functions. The process of metallic cutting delays is accountable aimed at prattle uncertainties categorized in flamboyant noise, ferocious vibrations, and poor class of surface quality in manufacturing industry. For instant in laser, optical feedback defined as detrimental optical chain that has delayed feedback. There is a present feedback in laser when percentage of optical output is inserted in the device, as a result of optical elements in fiber-coupled modules like fiber combiners, fiber ends, micro-lenses, and particle emission from other bases and insignificant percentages of the reflected light that can knock off balance laser and yield not the same categories of irregular, regular energetic outputs in the system.
Concerning the concepts of observability and observer for dynamical systems with multiple delays, a direct adaptation of existing results observability and observers from regular systems to multiple systems is not immediate due to the fact that they involve both differential and algebraic equations. We clarify the observability using the software packages for computer aided control system design MATLAB and SIMULINK. We discuss also how to choose the observer initial conditions and how to set up the observer gains. Most of the existing works are based on the simple case with multiple delay in the state, i.e$\dot{x}(t) = \sum_{i=0}^{m} A_i\, x(t-h_i)$, where the input could be also involved. For this simple case, a general solution was derived in [1], based on which a sufficient condition for exact observability in finite time. For such a presentation, there exist lots of applications, such as LC electrical lines [2] and so on. More concrete applications can be found in [3]. In general, observability analysis and observer design become more difficult. Recently, inspired by the well-known Silverman and Molinari algorithm (see [4] and [5]) to analysis the observability for linear systems, a similar and sufficient condition was proposed in [6] to analysis the observability for this general multiple delay systems.
The contributions of this paper are as follows. Firstly, the class of the studied systems is quite general (multiple delays on the state). Actually, there exist some methods to eliminate the delay was employed in[7], [8] and [9] that the elimination of delay via a bicausal transformation with the same dimension. To take the advantage of good features of feedback[10], it is often assumed that all state variables are available for feedback, meaning that a feedback control input can be constructed as

$$u\big(x(t)\big) = -Fx(t) \qquad (1)$$

Where F is a constant feedback matrix of dimension $m \times n$. There is a difficulty of full-state feedback controllers. This difficulty is in the fact that all state space variables must be available for feedback, which in the case of higher-order dimensional systems; the variables are not available for feedback. Instead, an output signal that represents a linear combination of the state space variables is available only as

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$y(t) = Cx(t)$ (2)

The dimension of the output signal is much smaller than the dimension of the state space variable that is, $dim\{y(t)\} = l = C < n = dim\{x(t)\}, where\ C = rank\{c\}$.

2. Notations and Problem Statement

In this paper, we consider the following class of linear dynamical control systems with lumped, multiple, constant delays in the state described by the ordinary differential equation with a delay argument of the form:

$$\left.\begin{aligned} \dot{x}(t) &= \textstyle\sum_{i=0}^{m} A_i\, x(t-h_i) + Bu(t), \quad t \geq 0 \\ y(t) &= Cx(t) \end{aligned}\right\} \qquad (3)$$

where $x(t) \in \mathbb{R}^n$ is the instantaneous n-dimensional state vector, $u(t) \in \mathbb{R}^m$ is the admissible control vector for the dynamical system (3), $y(t) \in \mathbb{R}$ is the output, $A_i, i = 0,1, \dots \dots \dots m\ are\ (n \times n)$ −dimensional matrices with real elements, B, C are (n × n)-dimensional matrices with real elements, $h_i,\ i = 0,1, \dots \dots \dots m$ denote constant delays satisfying the following inequalities:

$$0 = h_0 < h_1 < \cdots < h_i < \cdots < h_{m-1} < h_m \qquad (4)$$

With the functional initial conditions $x(t) = \varphi(t), t \in [-h_m, 0]$, where $x(0) \in \mathbb{R}^n, u(0) \in \mathbb{R}$ and the Hilbert Space $\mathbb{R}^n \times L^2([-h_m, 0), \mathbb{R}^n)$ endowed with the scalar product defined by

$$< \{x(t), x_t\}, \{y(t), y_t\} > = \textstyle\sum_{i=1}^{n} x_t\,(t) y_i(t) + \int_{-h_m}^{0} < x_t(\tau), y_t(\tau) >_{\mathbb{R}^n} d\tau \qquad (5)$$

is denoted by $M_2([-h_m, 0], \mathbb{R}^n)$, where $x_t(\tau) = x(t+\tau)$ for $\tau \in [-h_m, 0]$ is the segment of the trajectory of length h_m, which is defined in the time interval $\tau \in [t - h_m, t]$. The symbol $\mathbb{Z}$ represents the set of integers, I is the field of real numbers and $\mathbb{R} \geq 0$ is associated with positive numbers $\{0 < a < +\infty\}$, for $a \in \mathbb{R}\ or\ \mathbb{C}$.

3. Definition

In this section we shall give the definitions of the various types of observability for dynamical systems with delays in the state. We also quote the definition and the necessary and sufficient condition for the asymptotic observability of the dynamical system.

Definition 1: The system (3) is said to be observable att_0 if $x(t_0)$ can be determined from the output function $y_{[t_0, t_1]}$ for $t_0 \in \tau$and $t_0 > t_1$, where t_1 is some finite time belonging to τ. If this is true for all $t_0$and $x(t_0)$, the system is said to be completely observable.

Definition 2: The system (3) is said to be backward observable on $[t_1, t_2]$ if and only if, for each $\in [t_1, t_2]$, there exist $\bar{t}_1$and $\bar{t}_2 < \tau$ such that $y(t) = 0$ for all $t \in [\bar{t}_1, \bar{t}_2]$ this implies that $x(\tau) = 0$. These backward observability is related to final observability see [11].

Definition 3: the system (3) is said to be relatively observable in the time interval $[0, t_1]$ from the complete state $y_0 = (x(0), x_0) \in M_2([-h_M, 0), \mathbb{R}^n)$ into set S if for every vector $\tilde{x} \in S$ there exists an admissible control $\tilde{u} \in L^2([0, t_1], U)$ such that the corresponding trajectory $x(t, y_0, \tilde{u})$ of the dynamical system (3) satisfies the condition

$x(t, y_0, \tilde{u}) = \tilde{x}$.

Definition 4: the system (3) is said to be relatively observable in the time interval $[0, t_1]$ into the set S if it is relatively observable in the interval $[0, t_1]$ into the set S for every initial complete state $y_0 = (x(0), x_0) \in M_2([-h_M, 0), \mathbb{R}^n)$.

4. Assumptions and Preliminary Result

According to Luenberger, any system driven by the output of the given system can serve as an observer for that system. The system output variables are available at all times and that information can be used to construct an artificial, dynamic system of the same order as the system under consideration order n, which will estimate the system state space variables at all times. Since the matrices A, B, C are known in (3).

$$\dot{y}(t) = C\dot{x}(t) \qquad (6)$$

$$= C \sum_{i=0}^{m} A_i\, x(t-h_i) + CBu(t)$$

One the other hand, for a given initial condition $(x(0), \Phi(t), t \in [-h_m, 0])$ and an admissible control $u \in \mathbb{R}^m$, for $t \geq 0$ for the system (3), there exists a unique, absolutely continuous solution x which is for the form

$$x(t) = e^{A_0 t}x(0) + \int_0^t e^{A_0(t-\tau)} \left[\textstyle\sum_{i=1}^{m} A_i x\,(\tau - h_i) + Bu(\tau)\right] d\tau \qquad (7)$$

Since the system matrices A, B and C in system (3) are known, it is rational to obtain the derivation of the output given by;

$$\left.\begin{aligned} \hat{x}(t) &= \textstyle\sum_{i=0}^{m} A_i\, \hat{x}(t-h_i) + Bu(t), \quad t \geq 0 \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned}\right\} \qquad (8)$$

If we compare the outputs $y(t)$ and $\hat{y}(t)$, in general, they will be different since in the first case the initial condition of (3) is unknown, and the initial condition of the proposed observer (7) can be chosen arbitrarily by a control engineer. The difference between these two outputs defines an error signal

$$y(t) - \hat{y}(t) = Cx(t) - C\hat{x}(t) = C\varepsilon(t) \quad (9)$$

Which can be used as the feedback signal to the observer such that the estimation (observation) error $\varepsilon(t) = x(t) - \hat{x}(t)$ is reduced, theoretically goes to zero at steady state. The observer that takes into the account feedback information about the observation error given by [12].

$$\hat{x}(t) = \sum_{i=0}^{m} A_i\,\hat{x}(t-h_i) + Bu(t) + k(y(t) - \hat{y}(t))$$

$$\hat{x}(t) = \sum_{i=0}^{m} A_i\,\hat{x}(t-h_i) + Bu(t) + kC\varepsilon(t) \quad (10)$$

Where k is the observer gain that has to be chosen such that the observation error tends to zero as time increases form (3) and (8) is used to get an expression for dynamics of the observation error as

$$\dot{x}(t) - \hat{x}(t) = \sum_{i=0}^{m} A_i\,[x(t-h_i) - \hat{x}(t-h_i)] + Bu(t) + kC\varepsilon(t)$$

$$\dot{\varepsilon}(t) = \sum_{i=0}^{m} A_i\,[\varepsilon(t-h_i)] - kC\varepsilon(t)$$

$$\dot{\varepsilon}(t) = (A_i - KC)\varepsilon(t),\ \ \varepsilon(t_0) = unknown\ \ where\ h = 0 \quad (11)$$

If the observer gain k is chosen such that the feedback matrix $A - kC$ is asymptotically stable, then the estimation error $\varepsilon(t)$ will delay to zero for any initial condition $\varepsilon(t_0)$. This stabilization requirement can be achieved if the pair (A, C) is observable. By taking the transpose of the estimation error feedback matrix, that is $A^T - C^T k^T$, it can be seen that the pair (A^T, C^T) must be controllable (6). The controllability of the pair (A^T, C^T) is equal to observability of the pair (A, C), which is the condition needed for the observability(7).

Theorem 1: the system of (3) is point-wise observable if and only if

$$rank\left[\varphi(0,t_1) \equiv rank \int_0^{t_1} \{\textstyle\sum_{i=0}^{m} A_i e^{-sh_i}\}^T \times C^T C \sum_{i=0}^{m} A_i e^{-sh_i}\, d\tau\right] = n \quad (12)$$

Where $\varphi(0, t_1)$ is the observabilityGramian of the system.

With theorem 1 and (5), we can conclude the following.

Corollary 1: The system of (3) is point-wise observable if and only if all columns of the matrix

$$C \sum A_i\, e^{-sh_i} C_i^N \quad (13)$$

are linearly independent.

Since the Laplace transform is a one-to-one linear operator, we then obtain the following corollary.

Corollary 2: The system of (3) is point-wise observable if and only if all columns of the matrix

$$C(sI - A - A_d e^{-sh})^{-1} \quad (14)$$

are linearly independent except at the roots of the characteristic equation of (3).

Proof: In (5), in order to transfer $x(t)$to $0\ at\ t_1$, substitute an input obtained with the inverse of the observabilityGramian in (10).

$$u(t) = -C^T\{k(t_2, t_1)\}^T \varphi^{-1}(0, t_1) M(t_1, g, x_0) \quad (15)$$

where M is the free solution to (3) and comparing (5)and (15) yields

$M(t_1, g, x_0) \equiv \sum_{i=0}^{m} A_i\, e^{-sh_i} C_i^N$(16)

Then, $x(t_1) = 0$

Necessity. Given any $g$and x_0, suppose there exist $t_1 > 0$ and a control $u_{(0,t_1)}$ such that $x(t_1) = 0$, but (10) does not hold. The latter implies that there exists a nonzero vector $x_1 \in \mathbb{R}^n$ such that $x_1^T k(t_2, t_1)B = 0, 0 \le t \le t_1$ due to the following fact. Let F be an$n \times p$ matrix.

Define $$P_{(t_2,t_1)} \equiv \int_{t_1}^{t_2} F(t)\, F^T(t) dt \quad (17)$$

Then, the rows of F are linearly independent on (t_2, t_1) if and only if the $n \times n$ constant matrix $P_{(t_2,t_1)}$ is nonsingular (5). Then, from (5)

$$x_1^T x(t_1) = x_1^T M(t_1, g, x_0) + \int_0^{t_1} x_1^T\ \ k(t_2, t_1) Bu(\tau) d\tau \quad (18)$$

and $0 = x_1^T M(t_1, g, x_0)$. By hypothesis, however, g and x_0 can be chosen such that $M(t_1, g, x_0) = x_1$. Then, $x_1^T x_1 = 0$ which contradicts the assumption that $x_1 \neq 0$. Complete.

Having analyzed the existing system, it is now convenient for us to consider a particular problem and solve manually which will serve as a general case for the method selected. We shall observe that during the process of applying Fault Detection Filter Design on a Delay Linear System, we have to determine a function whose Laplace transform is already known. This is the reverse of determining the Laplace transform. The fault detection filter design process for delay linear system consists of three ways; firstly, the state estimation error connected to the fault in the subspace that is detection. The fault detection filters determine which filters will detect and identify fault. The ability to differentiate a fault from another and identify needs an observable system independent of detection space. The size of the state space limit the number of faults detected and identified by the fault detection filter. The health monitoring system considers nine system faults which comprise of two actuator faults and seven sensor faults. The design fault detection uses eigenstructure assignment and eigenvectors embedded in the filter dynamics. The contribution of this paper lies in three aspects. First, we address the delay-dependent filtering problem for switched linear neutral systems with time-varying delays, which appear not only in the state, but also in the state derivatives. The resulting filter is of the Luenberger-observer type. Second, by using average dwell time approach and the piecewise Lyapunov function technique, we derive a delay-dependent sufficient condition, which guarantees exponential stability of the filtering error system. Then, the corresponding solvability condition for a desired filter satisfying a weighted performance is established. Here, to reduce the conservatism of the delay-dependent condition, we introduce some slack error dynamic and a new integral inequality recently proposed, So far the observers employed in the context of fault detection are either full order or reduced order observers, however, as can be seen in chapter three, only an output observer - that will estimate only the output of the system - is sufficient since the estimation of the remaining state variables are not required as far as fault - detection is concerned. It is obvious that full discuss only the application of output observer in the context of fault detection. Order observers for fault detection purposes are computationally expensive especially for high order systems the following theorem was prove.

5. MODEL WITH DELAYS FOR OBSERVER SYSTEM

We study the cascade connection completely filled mixers following from the input concentration to the mixer and so on according to the scheme obtainable in Fig1. But in each reactor a delay is observed as a result of pressure or heat present, where $c_{in1}(t),\ c_{in2}(t)\ and\ c_{in3}(t)$are the input concentrations of the product $Q_1^*,\ Q_2^*\ and\ Q_3^*$ which represent the constant flow of concentrations. Then $V_1, V_2\ and\ V_3$ are the volumes of the mixers 1, 2 and 3. $c_1(t)$, $c_2(t)\ and\ c_3(t)$represent the strength of solutions in mixers 1, 2 and 3 respectively. The length of the reactor is L and h_i is the multiple delays in the reactors.

Assuming$V_1 = V_2 = V_3 = V$, the state equations of the above chemical system is in the form of:

$$V\frac{dc_1(t)}{dt} = Q_1^* c_{in1}(t) - Q_1^* c_1(t)$$

$$V\frac{dc_2(t)}{dt} = Q_1^* c_1(t-h_1) + Q_2^* c_{in2}(t) + c_2(t-h_2) - (Q_1^* + Q_2^*)c_2(t)$$

$$V\frac{dc_3(t)}{dt} = Q_2^* c_2(t-h_2) + Q_3^* c_{in3}(t) + c_3(t-h_3) - (Q_1^* + Q_2^* + Q_3^*)c_3(t)$$

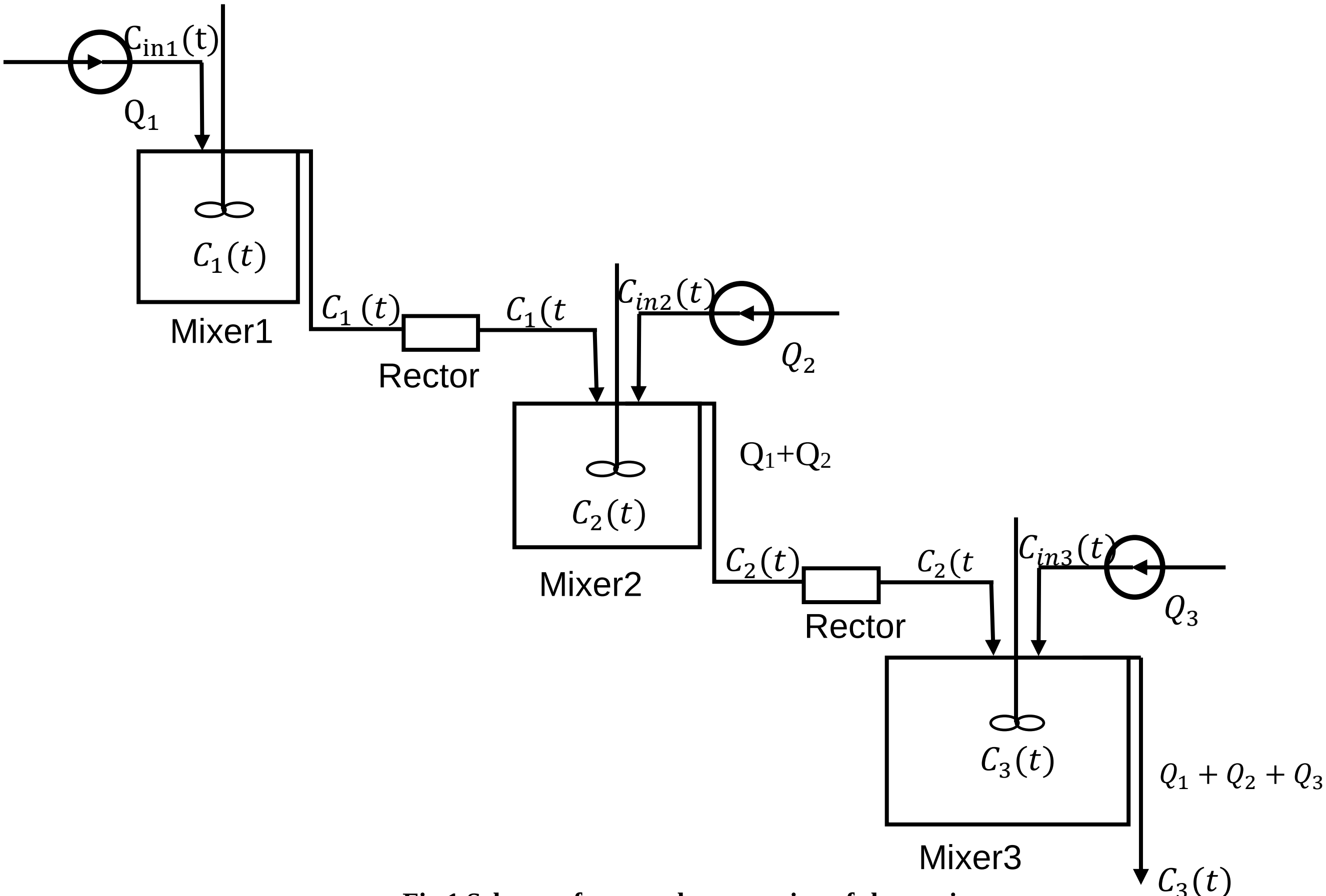

Fig.1 Scheme of a cascade connection of three mixers

After a transformation, we

$$\frac{dc_1(t)}{dt} = \frac{Q_1^*}{V}c_{in1}(t) - \frac{Q_1^*}{V}c_1(t),$$

$$\frac{dc_2(t)}{dt} = \frac{Q_2^*}{V}c_{in2}(t) + \frac{Q_1^*}{V}c_1(t-h_1) + \frac{Q_2^*}{V}c_2(t-h_2) - \frac{(Q_1^* + Q_2^*)}{V}c_2(t)$$

$$\frac{dc_3(t)}{dt} = \frac{Q_3^*}{V}c_{in3}(t) + \frac{Q_2^*}{V}c_2(t-h_2) + \frac{Q_3^*}{V}c_3(t-h_3) - \frac{(Q_1^* + Q_2^* + Q_3^*)}{V}c_3(t)$$

Taking $c_1(t) = x_1(t), c_2(t) = x_2(t),\ c_3(t) = x_3(t), c_{in1}(t) = u_1(t), c_{in2}(t) = u_2(t)\ and\ c_{in3}(t) = u_3(t)$ the mathematical model of the dynamical system with delay in the state is express in the following linear differential equation:

$$\dot{x}(t) = A_0x(t) + A_1x(t-h_1) + A_2x(t-h_2) + A_3x(t-h_3) + Bu(t),$$

Where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \qquad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

And $$A_0 = \begin{bmatrix} -\frac{Q_1^*}{V} & 0 & 0 \\ 0 & -\frac{(Q_1^*+Q_2^*)}{V} & 0 \\ 0 & 0 & -\frac{(Q_1^*+Q_2^*+Q_3^*)}{V} \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0 & 0 & 0 \\ \frac{Q_1^*}{V} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{Q_2^*}{V} & 0 \\ 0 & 0 & 0 \end{bmatrix} A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{Q_2^*}{V} \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ \frac{Q_3^*}{V} \end{bmatrix}$$

5.1 NUMERICAL EXAMPLE

$$A=\begin{bmatrix}2 & 1 & 3\\3 & 1 & 4\\1 & 0 & 1\end{bmatrix};\qquad B=\begin{bmatrix}1\\2\\0\end{bmatrix};\qquad C=\begin{bmatrix}1 & 0 & 1\end{bmatrix}$$

Frist eigenvalue/ vector decomposition

$$[\lambda I-A]=\begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix}-\begin{bmatrix}2 & 1 & 3\\3 & 1 & 4\\1 & 0 & 1\end{bmatrix}=0$$

$$=\begin{bmatrix}\lambda-2 & -1 & -3\\-3 & \lambda-1 & -4\\-1 & 0 & \lambda-1\end{bmatrix}$$

The eigenvalue $\lambda=4.24,-0.24,0$

$$[A-\lambda I]W\Rightarrow W=\begin{bmatrix}-0.6\\-0.78\\-0.18\end{bmatrix}$$

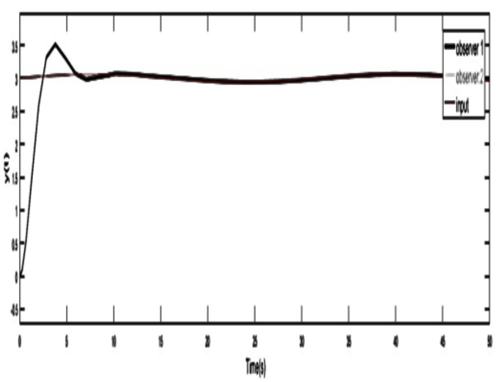

Fig2.Shows the observability of the system.

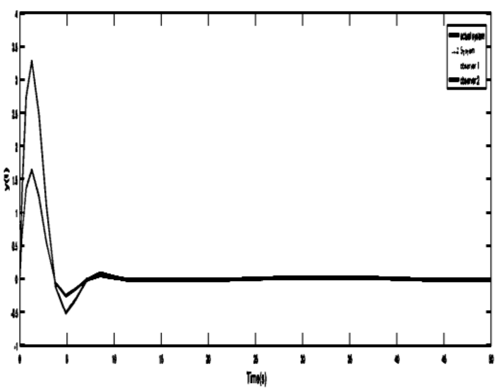

Fig3.Shown the observability of the State.

Used eigenvalue/vector decomposition to introduce concepts of observability which means the ability to determine $x(t)$ from the available measurements (3) with W ∧∨. The solve problem above shown matrix CW no zero columns which mean the system is observable.

5.3 Conclusion

Care should be taken on the types of inputs signal applied when considering a fault detection problem. Similarly, in the case of filter design, special care should be taken when choosing the filter gains depending on the stability of the signal generator. Again, one of the main reasons why output observers are almost inexistent in the literature is that the state space representation dose not easily lends itself to the design of such observers so therefore standard Luenberger or Kalman type of observer dose not seems suitable. By directing development of the project components in parallel and seeing significant progress in all areas, we are able to identify several important areas for future work: model refinement, robust fault detection filter design, health monitoring system evaluation, residual processing and delay network development, and platoon health monitoring.

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