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UNSTEADY HEAT FLOW OF A VISCOUS INCOMPRESSIBLE MHD FLUID OVER AN OSCILLATING PLATE PROVOKED BY RADIATION

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Abstract

An investigation of unsteady heat flow of a viscous incompressible magnetohydrodynamic fluid over an oscillating plate under the influence of radiation was carried out. The governing equations where solved by the method of undetermined coefficient and solutions were obtained. The features of the fluid flow were subjected to analysis using Wolfram 9 software and the results obtained showed clearly that velocity profile increases when Radiation, Reynolds, Prandtls, Grashofs number are increased but an increase in Hartmann number brought about a corresponding decrease in the velocity profile. When the radiation parameter is increased there is a rapid increase in the shear stress profile also when the Reynold's number is increased there is a significant decline in the shear stress profile but for the Prandtl's and the Grashof's number an increase in them brought about a minimal increase in the shear stress of the plate and an increase in the Hartmann's number led to a marginal decline in the shear stress of the plate.

***Keywords*:** Prandtl; Magnetohydrodynamic (MHD); Grashof; Reynolds; Hartmann; Nusselt.

1. Introduction

Magnetohydrodynamics (MHD) is the study of an electrically conducting fluid and its interaction with magnetic field. The basic concept of MHD is the dynamics of matter moving in an electromagnetic field where current is established in the matter by induction [1]. Examples of MHD fluid include mercury, liquid sodium, liquid metal and lithium e.t.c. MHD has applications in astrophysical and geophysical problems. Several scholarly work centered on MHD and its ancillary parameters, thus scholars have also thought it wise to pen down their findings.

Mass and heat transfer analysis of an unsteady MHD flow past an impulsively started vertical plate in the presence of thermal radiation was investigated by [2]. They obtained the exact solution to the governing equation in closed form by Laplace transform technique. From their result the mass diffusion tends to reduce the fluid velocity whereas a reverse effect was noticed for the radiation parameter on the velocity profile of the fluid.

Unsteady heat transfer of viscous incompressible boundary layer fluid flow through a porous plate with induced magnetic field was tackled by [3]. They noticed that where the Grashofs parameter is increased the velocity and temperature profile increases but the magnetic induction decreases with an increase in the Grashofs parameter. The effect of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid was worked on by [4]. From their findings they noticed that an increase in the variable viscosity parameter decreases the fluid and dust phase velocities whereas it causes an increase in both phases of the temperature profile. Also their result showed that increase in Prandtls number decreases the thermal boundary layer thickness.

Similarly, an investigation on the numerical study of heat and mass transfer MHD viscous flow over a moving wedge in the presence of viscous dissipation and heat function with convective boundary condition was carried out by [5]. They concluded from their findings that both mass injection and suction are important in separating the profiles of the hydrodynamics and thermal layers. Another interesting findings from their work is that the temperature profile and heat transfer coefficient depends upon the Prandtl number, viscous dissipation, heat sink and convective parameter. Also they observed that whenever the convection parameter is high the mass transfer rate are found to be high also.

In another development a research on radiation effects on MHD Couette flow between two plate with the transfer of heat was done by [6], and he observed that the velocity distribution is affected by the Hartmann number. Viscous flow of Incompressible MHD fluid flow over an accelerated plate with electroconductivity and chemical reaction in the presence of

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radiation was tackled by [7]. Also a work on MHD non-linear flow with heat flux passing through a porous sheet with heat generation and radiation was examined by [8], and they found out that the distribution of the temperature and wall temperature are enhanced for increasing numbers of magnetic field and heat generation parameters. A Studied on viscous dissipation influence and radiation on MHD Couette flow in a porous medium was carried out by [9]. They discovered that an increase in the temperature distribution is a fallout of an increase in the Reynolds number. Similarly, Time dependent MHD convection free past an infinite heat vertical plate in a medium that is porous with radiation influence, viscous dissipation and time dependent suction was studied by [10]. They observed that the magnetic number and radiation parameters gives a decrease in temperature.
Furthermore, a study was carried out by [11] where flow of viscous incompressible MHD fluid over a suddenly accelerated plate with variable electroconductivity and chemical reaction term induced radiation. He observed that an increase in the fluid parameters apart from the magnetic parameter brings about an increase in the shear stress at the plate. MHD radiation flow confined by a porous medium was examined by [12]. They observed that when Hartmann number is increased this decelerate the flow along the plate, also the velocity profile values are reduced strongly with increase in the Hartmann parameters. Our primary aim is to stretch the study of [13] by including Viscous and incompressible fluid flow in an oscillating plate.

NOMENCLATURE

T = Temperature
V = Fluid velocity
ρ = Fluid density
P = Fluid pressure
M = Absolute Viscosity
g = Acceleration due to gravity
V = Kinematic viscosity
$H_\circ$ = Magnetic field
a = Thermal diffusivity
q_y = Radiative term
c_v = Specific heat capacity at constant volume
Nu = Nusselt number
Λ = Planck's function
$\propto_k =$ Absorption coefficient
$R =$ Dimensionless radiation term
Nu = Nusselt number
θ = dimensionless temperature
M = Dimensionless magnetic parameter
Pr = Prandtl parameter
Gr = Grashof parameter
Re = Reynolds parameter

2. MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

Let a flat plate exist as shown in figure 1, extending to a large distance in the x and z directions. Let an incompressible viscous fluid exist over a half plate y = 0 (i.e xz - plane). Let the fluid extend to infinity and be at rest there. Further, let the plate be oscillating with constant amplitude and frequency with velocity ucoswt this generates a two dimensional parallel flow near the plate since the plate is situated in an infinite fluid, the pressure must be constant.

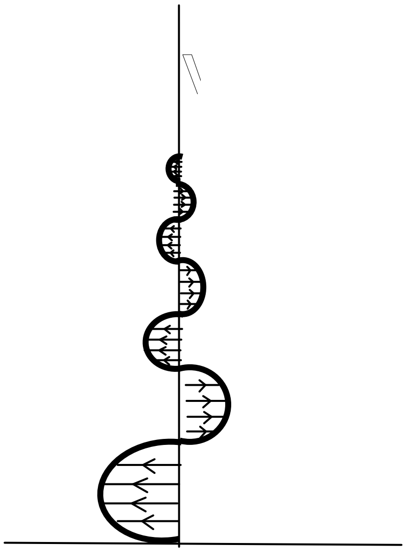


Figure1.The configuration of the problem and the coordinate system.

The hydrodynamics basic equations which the physics of the problem is governed by is drawn from the argument of [5] and [10]

$\nabla . V = 0$ (Continuity Equation) (1)

P $(\frac{\partial y}{\partial t} + (V.\nabla)V) = -\nabla P + \mu\nabla^2 V + \rho g - \frac{V\mu\delta_C H_0}{P_\propto}$ (Momentum Equation) (2)

$(\frac{\partial T}{\partial t} + (V.\nabla)T) = a^2\nabla^2 T - \frac{1}{\rho C_v}\nabla . q_y$ (Energy Equation) (3)

From equation (3) following [14] the radiation term reduces to

$\frac{\partial q_y}{\partial y} = 4\delta^2(T - T_\propto)$ (4)

Where $\delta^2 = \int_0^\alpha (\alpha_k \frac{\partial \lambda}{\partial T})dk'$ (5)

Substitute equations (4) into (3) having in mind that restricting the effect of variation of density under boussinesq approximation which is restricted with temperature exclusively to the body force term with these assumption, the physical description of the flow is given as

$\frac{\partial v}{\partial y} = 0$ (6)

$\rho\frac{\partial v}{\partial t} = -\frac{\partial P}{\partial y} + \mu\frac{\partial^2 v}{\partial y^2} + gP_0\boldsymbol{\xi}(T - T_0) - \frac{v\mu^2\delta_c H_0^2}{P_\alpha}$ (7)

$\frac{\partial T}{\partial t} = a^2\frac{\partial^2 T}{\partial z^2} - \frac{4\delta^2(T - T_O)}{PC_v}$ (8)

Where $\boldsymbol{\xi}$ is the coefficient of volume expansion perturbation.

Velocity field, temperature field and pressure field are denoted by

$v' = v - v_e$

$P' = P - P_e$ (9)

$T' = T - T_e$

Equilibrium values is represented by the subscript e. Putting (9) in (6), (7) and (8) when unity terms are retained, the following linearized equations are obtained.

$\frac{\partial v'}{\partial y} = 0$ (10)

$\rho\frac{\partial v'}{\partial t} = -\frac{\partial P'}{\partial y} + \mu\frac{\partial^2 v}{\partial y^2} gP_0\boldsymbol{\xi}(\boldsymbol{T}' - \boldsymbol{T}_0') - \frac{\delta_c\mu^2 H_0 v'}{p_\alpha}$ (11)

$\frac{\partial T'}{\partial t} = a^2\frac{\partial^2 T'}{\partial y^2} - \frac{4\delta^2(T' - T_0')}{PC_v}$ (12)

Non-dimensional analysis

We substitute the following expressions below, for dimensionless homogeneity of the governing hydrodynamics equation.

$y = \frac{v' t}{d}$ $P = \frac{P'}{\rho V^2}\alpha^2 = \frac{4\delta^2 P_\alpha C_\alpha d^2}{\rho C_v V}$ $V = \frac{v'}{u}\beta^2 = \frac{\alpha^2 p'}{T_\alpha}$

g = $\frac{gd}{v^2}$ T = $\frac{T' - T_0}{T_1 - T_2}$ $\mathrm{R}e^{-1} = \frac{\mu}{v d_p} m^2 = \frac{\delta_c H_0' \mu v}{u^2 p_\alpha}$ Gr = g$\boldsymbol{\xi}\frac{(T - T_0)d^2}{v^2}$

Having employed the Rayliegh's technique into equations (10 - 12) it results into equations (13) – (15)

$$\frac{\partial v}{\partial y} = 0 \qquad (13)$$

$$\rho \frac{\partial v}{\partial t'} = -\frac{\partial p}{\partial y} + \mathrm{Re}^{-1}\frac{\partial^2 v}{\partial y^2} + \mathrm{Gr}\theta - m^2 v \qquad (14)$$

$$\frac{\partial \theta}{\partial t'} = \beta^2 \frac{\partial^2 \theta}{\partial y^2} - \alpha^2 \theta \qquad (15)$$

Boundary conditions given as,

U = ucosnt at y = 0

U = 0 at y = ∝ (16)

$\theta(0)$= 1 at $\theta(\alpha) = 0$

3. METHOD OF SOLUTION

We seek solution to equations (13 - 15) in the form

$$V(y,t) = h(y)\, e^{-nt} \qquad (17)$$

$$\theta(y,t) = \emptyset(y) e^{-nt} \qquad (18)$$

$$P(y,t) = h(y) e^{-nt} \qquad (19)$$

Where n is decay constant

The boundary conditions equation (16) transforms into

h = VCosnte^{nt} at y = 0 (20a)

h = 0 at y = ∞ (20b)

$\emptyset(0) = e^{nt}$ at $\emptyset(\infty) = 0$ (20c)

We can rewrite equations (14) – (15) using equation (17 – 19) as

$$-h''(y) - \mathrm{Re}h'(y) + \mathrm{Re}(P_n - m^2)h(y) = -\mathrm{ReGr}\theta(y) \qquad (21)$$

$$\beta^2 \emptyset''(y) - (\alpha^2 - n)\emptyset(y) = 0 \qquad (22)$$

Solving equation (22), we impose the transformed boundary condition equation (20c) and substitute into equation (17), we have

$$\emptyset(y) = \mathrm{Coshfy} \qquad (23)$$

$$\mathrm{Nu} = \frac{\partial \theta}{\partial y} = fSinhfy \qquad (24)$$

were,

$$f = \left(\frac{\alpha^2 - n}{\beta}\right)^{1/2}$$

To solve equation (21), we put equation (23) into it and solve the resulting equation this is followed by imposing the boundary condition and substitute into equation (18) to get

$$V(y,t) = e^{-nt}\left[(\mathrm{UCosnt}e^{nt} - K)e^{T_1 y} + \mathrm{KCosh(fy)} + \mathrm{ISinh(fy)}\right] \qquad (25)$$

The shear stress at the plate gives

$$\sigma(y,x) = \mu\frac{\partial u}{\partial y} = \mu e^{-nt}\left[T_1(\mathrm{UCosnt}e^{nt} - K)e^{T_1 y} + Kf\mathrm{Sinh(fy)} + \mathrm{IfCosh(fy)}\right] \qquad (26)$$

were

$$I = \frac{Re^2 Gr^2 f - Re^2 f^2 Gr}{(f^2 + \mathrm{Re}(p_n + m^2))^3 - Re^2 f^2 (f^2 + \mathrm{Re}(p_n + m^2))}$$

$$K = \frac{-Re^2 f Gr}{(f^2 + \mathrm{Re}(p_n + m^2))^2 - Re^2 f^2}$$

$$T_1 = \frac{Re + \sqrt{(Re)^2 - 4Re(p_n + m^2)}}{2}$$

4. RESULTS AND DISCUSSION

For the purpose of getting a physical perspective and numerical validation of the problem we take the approximate value of velocity constant as (U = 5.36) and the decay constant (n = 0.0035) other parameter values used are

R = 2.50, 3.50, 4.50, 5.50, 6.50

Re = 20, 30, 40, 50, 60

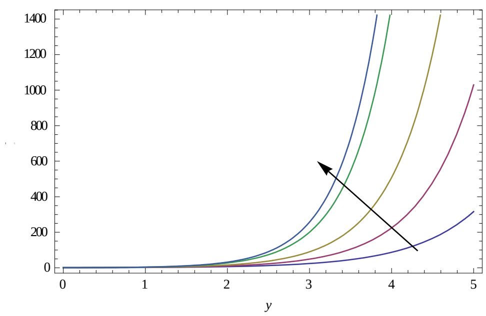
Pr = 0.51, 0.61, 0.71, 0.81, 0.91

Gr = 2, 4, 6, 8, 10

M = 20.0, 20.5, 30.0, 30.5, 40.0

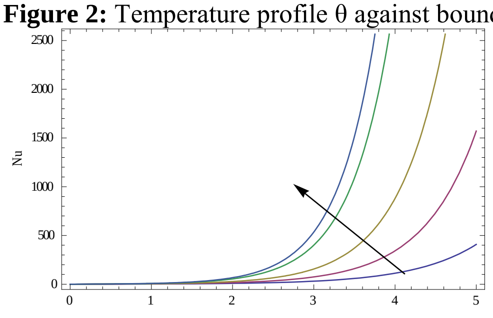
T = 1

For discussion of the results gotten we begin with the radiation parameter as shown in Figure 2, when the radiation number is increased it leads to a corresponding increase in temperature profile, this results is in tandem with the study of [5]. Figure 3 shows that an increase in the radiation number leads to a rise in the Nusselt profile. Also, figure 4 shows that when the radiation number is increased the velocity profile increases also. Figure 5 shows that an increase in the Reynolds number leads to an increase in the velocity profile, this results is consistent with the findings of [9]. From figure 6 we noticed that an increase in the Prandtl's number results in an increase in the velocity profile. This results corroborates the findings of [11]. Similarly, Figure 7 shows that when the thermal Grashofs number (Gr) is increased the velocity profile will also increase. The thermal Grashof number represents the relative effect of the thermal buoyancy force to the viscous MHD fluid. The flow is slightly accelerated due to the enhancement in buoyancy force corresponding to an increase in thermal Grashof number. The influence of the Hartmann number (M) on the velocity profile is shown in figure 8, we see that when the Hartmann parameter is increased the velocity profile drops. This study agrees with the study of [7] and [15]. Figure 9 represents typical shear stress in the boundary layers for various values of the radiation parameters. We observed clearly that the shear stress at the plate increases with an increase in the radiation parameters this is in line with [15]. Figure 10 shows that when the Reynolds parameter increases the shear stress at the plate will also decrease. Figure 11-12 shows that a marginal increase is experienced at the shear stress of the plate when the Prandtl and the Grashof parameters are increased respectively. Figure 13 shows clearly that increase in the Hartmann number will bring about a marginal decrease of the shear stress of the plate.



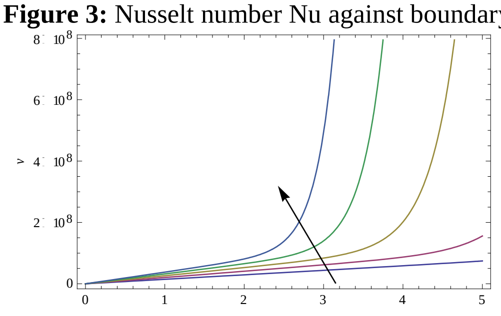
R = 2.5, 3.5, 4.5, 5.5, 6.5

Figure 2: Temperature profile θ against boundary layer y for varying Radiation R.



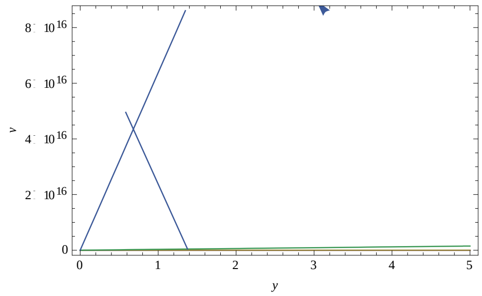
R = 2.5, 3.5, 4.5, 5.5, 6.5

Figure 3: Nusselt number Nu against boundary layer y for varying Radiation R.



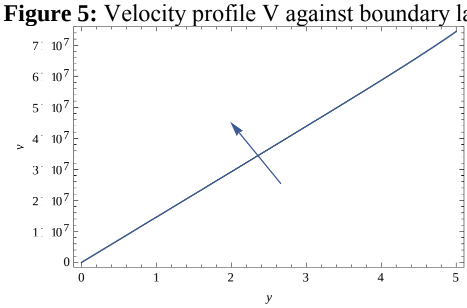
R = 2.5, 3.5, 4.5, 5.5, 6.5

Fig 4: Velocity profile V against boundary layer y for varying Radiation. R



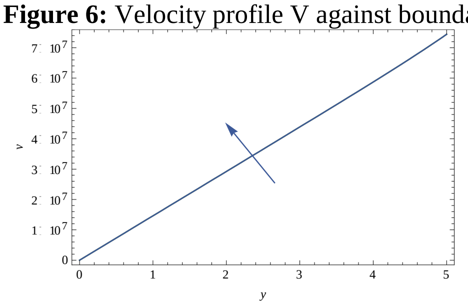
Re = 20, 30, 40, 50, 60, 80

Figure 5: Velocity profile V against boundary layer y for varying Reynold's number.



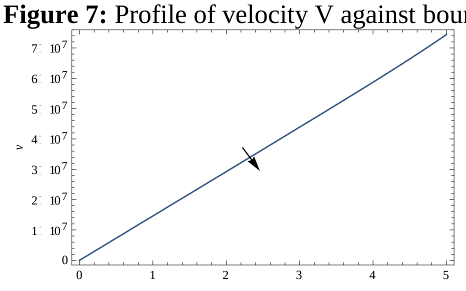
Pr = 0.51, 0.61, 0.71, 0.81, 0.91

Figure 6: Velocity profile V against boundary layer y for varying Prandtl number Pr



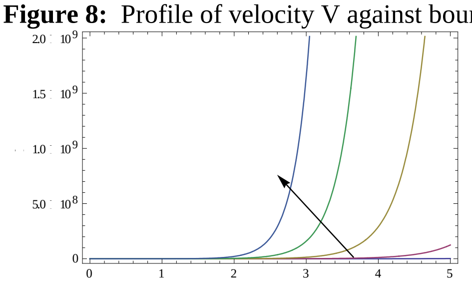
Gr = 2, 4, 6, 8, 10

Figure 7: Profile of velocity V against boundary layer y for varying Grashof number Gr



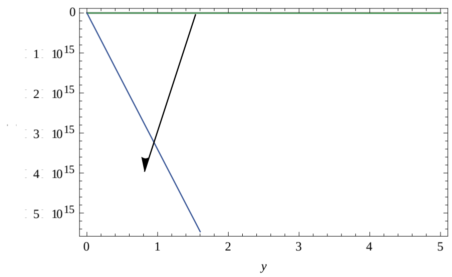
M = 20.0, 20.5, 30.0, 30.5, 40.0

Figure 8: Profile of velocity V against boundary layer y for different Hartmann number Mg



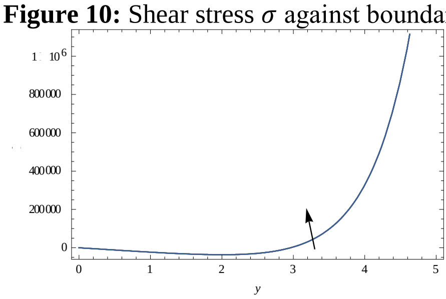
R = 2.5, 3.5, 4.5, 5.5, 6.5

Figure 9: Shear stress σ against boundary layer y for varying Radiation R.



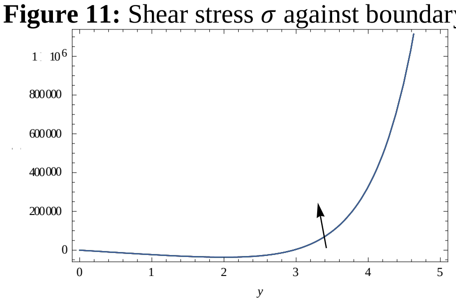
Re = 20, 30, 40, 50, 60

Figure 10: Shear stress σ against boundary layer y for varying Reynold's number R



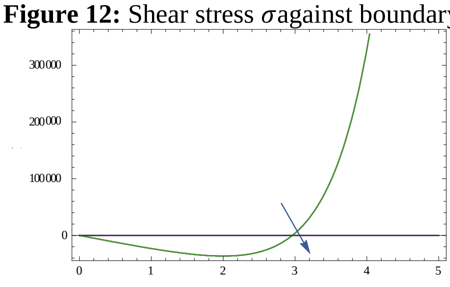
Pr = 0.51, 0.61, 0.71, 0.81, 0.91

Figure 11: Shear stress σ against boundary layer y for varying Prandtl number Pr



Gr = 2, 4, 6, 8, 10

Figure 12: Shear stress σagainst boundary layer y for varying Grashof number Gr



M = 20, 20.5, 30, 30.5, 40

Figure 13: Shear stress σ against boundary layer y for varying Hartmann number M

5. CONCLUSION

The study of MHD together with other parameters is very important as it helps to explain the behavior of the MHD fluid in various conditions which can lead to proper application. Conclusion drawn from this study are that;

1. Radiation parameter when increased led to an increase in the velocity and Nusselt profile of the fluid.
2. Parameter such as Prandtl, Raynolds and Grashofs when increases will bring about an increase in the velocity profile of the fluid
3. Hartmann parameter impedes on the flow of the MHD fluid as when the parameter is increase there is a corresponding decrease velocity profile of the fluid.

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