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# HYBRIDIZED MULTI CRITERIA MODEL FOR BUSINESS DECISION MAKING

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***Abstract***

***Decision making is essential in managing business activities as without it, most managerial functions such as: strategizing, policy formation, forecasting and staffing cannot be properly carries out, especially when the data available for carrying out these functions are vague. In business, wrong decisions may not only lead to loss of capital and other resources, but also depicts the manager as incompetent. In this paper a fuzzy multi-objective approach is used to build a decision making model for organizing the business activities of a cocoa processing plant. The model was built, using a combination of fuzzy membership functions and linear programming techniques. At the testing stage, the comparison of the use of the fuzzy and the non-fuzzy approaching improving scientific objective functions reveal that the fuzzy approach is more efficient in building a model for decision making. It was also observed that the introduction of a technique for dealing with imprecise data into a linear programming model helps in producing a more accurate result when modeling the objective functions of the problems, involving imprecise data.***

***Keywords***: Business activities, Decision Making, Fuzzy Set, Linear Programming,

## 1. INTRODUCTION

Prior to the development of mathematically tailored systems of planning and conducting business related activities, most business decisions were made out of personal instinct and experience. Many of these decisions were often erroneous as they were usually based on vague information. In the modern day, some organizations still use intuitive methods in making business decisions, which often makes the decision inadequate and not well suited to solving serious problems.

A multiple objective decision process is crucial to planning a business activity. A decision can be represented by fuzzy numbers, since it is often imprecise. Models developed, using fuzzy programming methods should be regarded as new conventional decision making methods rather than as a new contribution to multiple objective decision making methods [1]. This research work aims at building multi objective decision methods for organizing business activities. To this end this paper examines which is better of applying the fuzzy and non-fuzzy multi objective decision model, (under fuzzy constraints), applying crisp and fuzzy objective functions respectively.

This paper is structured as follows: Section two provided a literature review of different types of multi objective decision models. Section three consists of relevant fuzzy set theory definition. Section four describes the non-fuzzy multiple objective decision model (MODM), while the fuzzy multiple objective decision model (FMODM) is elucidated in Section five. The outcomes of the two models are portrayed in the construction of a decision making model for a cocoa processing company, which produces, transports and delivers cocoa liquid at different cocoa product manufacturing sites.

## 2. LITERATURE REVIEW

One of the most important subjects in modern day decision making methods for businesses is the theory of decision making. This theory adopts the use of optimization methods linked with concepts of single and multiple criteria. Decision making models that deal with multiple criteria are more difficult to model. This is because; they have to do with human conviction or judgment. The points of preferences indicated by the human decision maker is what brings about or is referred to as human judgment [2].The idea of goal programming emanated from the first endeavors made in order to model decision processes for

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business, using the multiple criteria technique [3]. The method required the decision maker to pitch each of the objectives involved in the decision making, to a certain number of goals that need to be met [4]. Meeting these goals entails, providing a resolution to a multi criterion problem. In the end, the "ideal solution" confirms that the best solution to the problem has been established. This solution has to be that which optimizes all the criteria concurrently. This solution is however considered to be unattainable. As such, the decision maker deliberates on workable solutions which are very close to the ideal solution Generally, in goal programming, the preference of the decision maker are represented with objectives, weights, mutual benefit and stages of the goals, in order to originate a problem. Researchers in [5] proposed a fuzzy multicriteria model, which consists of linear mathematical programming and a comparison with stochastic programming. The advantages and disadvantages of the reviewed fuzzy mathematical programming techniques were illustrated using an optimal portfolio selection problem. In [6] a method for solving whole fuzzy linear programming problems was developed. They carried out numerical experimentations, showing the preference of the proposed method over the current ones. The transitional step towards fuzzy multi criteria models is models that consider some fuzzy values. Some of these models are linear mathematical formulation of multiple objective decision making presented by mainly crisp and some fuzzy values. Many authors studied such models [5, 7, 8, 12]. Interactive multiple objective system technique contributed to the improvement of flexibility and robustness of multiple objective decision making methodology. Lai considered several characteristic cases, which a business decision maker may encounter while its being ran. The cases could be defined as both non-fuzzy cases and fuzzy cases. These deal with notions relevant to fuzzy set theory. In [9], an approach to solve multi criteria problems with Pythagorean fuzzy information was developed. The researchers conducted simulation tests to analyze how the risk attitudes of the decision makers exert the influence on the results of multi criteria decision making, under uncertainty. Finally, a case study on selecting the governor of Asian Infrastructure Investment Bank is made to show the applicability of the proposed approach. The approach was found to scale well in solving multi criteria problems. Scientists in [10], analyzed research in international scientific journals and international conference proceedings that focus on green supplier selection. They proposed the following questions that will be answered: (i) which selection approaches are commonly applied? (ii) what environmental and other selection criteria for green supplier management are popular? (iii) and what limitations exist? Published research from 1997 to 2011 is structurally reviewed based on the first two questions. The researchers found that the applied techniques were "environmental management systems". A further critical analysis was completed and gaps in the current literature from 1997 to 2011 were identified. These gaps helped the researchers to identify improvements for green supplier selection process and possible future directions.

## 3. FUZZY LOGIC APPROACH

Fuzzy set theory uses linguistic variables rather than quantitative variables to represent imprecise concepts. Linguistic variables analyze the vagueness of human language.

**Fuzzy set**: Let X be a universe of discourse. $\AA$ is a fuzzy subset of X if for all $x \epsilon X$, there is a number $\mu\AA(x) \;\epsilon\; /0, 1/$ assigned to represent the membership of x to $\AA$, and $\mu\AA(x)$ is called the membership function of Å [5].

***Fuzzy number:*** A fuzzy number Å is a normal and convex subset of X. normally implies

$\exists X \;\epsilon\; R \;v\; \mu\AA(x) = 1$

Convexity implies:

$\forall x_1 \;\epsilon\; X, \quad x_2 \;\epsilon\; X\,, \;\forall \alpha \epsilon\; /0.1/$

$\mu\AA(\alpha x_1 + (1-\alpha)\, x_2) \geq \min \mu\AA\,(x_1), \min \mu\AA\,(x_2)$

**Fuzzy decision**: The fuzzy set of alternatives resulting from the intersection of the fuzzy constraints and fuzzy objective functions [11]. A fuzzy decision defined in an analogy to non-fuzzy environments 'as the selection of activities which simultaneously satisfy objective functions and constraints". Fuzzy objective function is characterized by its membership functions. In fuzzy set theory the intersection of sets normally corresponds to the logical "and'. The 'decision' in a fuzzy environment can therefore be viewed as the intersection of fuzzy constraints and fuzzy objective functions. The relationship between constraints and objective functions in a fuzzy environment is fully symmetric [12].

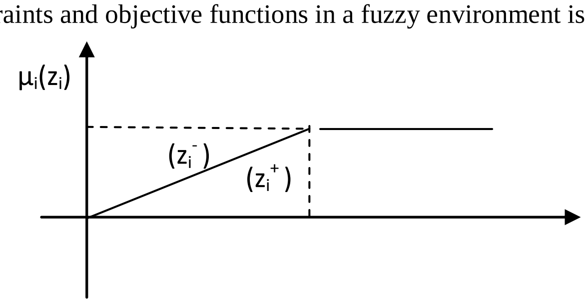



Figure 1: Objective function as a fuzzy number

## 4. NON-FUZZY MULTIOBJECTIVE PROBLEM

A general linear multiples criteria decision making model can be presents as:
Find a vector x written in the transformed form
$x^T = [\, x_1, x_2 \ldots\ldots x_n \,]$

This maximizes objective functions:

$$\max z_i = \sum_{j=1}^{n} c_{ij}.x_j \quad i = 1, 2, \ldots n \qquad (4.1)$$

With constraints,

$$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \qquad i = 1, 2 \ldots m,\ x \geq 0 \qquad (4.2)$$

where $c_{ij}$, $a_{ij}$ and $b_i$ are crisp (non) values. This problem has been studied and solved by many authors. Zimmermann has solved this problem by using the fuzzy linear programming [12]. He formulated the fuzzy linear program by separating every objective function $z_i$ its maximum $z_i^+$ value by solving.

$$z_i^+ = \max z_i = \sum_j c_{ij}x_j, \text{ and } z_i = \min z_i = \sum_j c_{ij}x_j \qquad (4.3)$$

with constraints (4.2) solutions $z_i^+$ and $z_i$ are known as individual best and worst solutions respectively. Since for every objective function $z_i$ . Its value changes linearly from $z_i^-$ to $z_i^+$ it may be considered as a fuzzy number with the membership function $\mu_i(z_i)$ as shown in figure 1:

$$\mu_j(z_j) = \begin{cases} 0 & \text{for } z_i \leq z_i^- \\ \dfrac{(z_i - z_i^-)}{(z_i^+ - z_i^-)} & \text{for } z_i^- \leq z_i^+,\ 1, 2, \ldots n \\ 1 & \text{for } (z_i \geq z_i^+) \end{cases} \qquad (4.4)$$

According to Bellman-Zadeh's principle of decision making in the fuzzy environment the grade of membership of a decision j. specified by objective $z_i$ is obtained by:

$\alpha = \min \mu_i(z_i)$, j=1, 2,…k (4.5)

or

*Maxmin j*

Subject to

$\alpha \leq \mu_i(z_i)$ j=1,2,…k $0 \leq \alpha \leq 1$ (4.6)

According to this principle the optimal values of multicriteria optimization correspond to maximum of *j*. The auxiliary linear program is obtained by

$z_i^- = \max \alpha$ (4.7)

With constraints (4, 6) taking into account (4.1) and (4.4)

$$-\sum_{j=1}^{n} c_{ij} + (z_i^+ - z_i^-)\,\alpha \leq \frac{(z_i - z_i^-)}{(z_i^+ - z_i^-)} \qquad i = 1, 2, \ldots k \qquad (4.8)$$

$$0 \leq \alpha \leq 1,\ x_j \geq 0 \quad j = 1, 2 \ldots n$$

The original linear constraints (4.2) are added to these constraints. The problem can also be presented in a form [13]: Find a vector x subject to

$$z_i(x_i) \gtrsim z_i^0 \quad \forall_I,\ x \in X \qquad (4.9)$$

Where $z_i^0$, $\forall_I$ are corresponding goals, and $\gtrsim$ is a soft or quasi inequality. The objective functions are assured to be maximized

*max/min* $[z_i(x) \ldots. z_i(x)]$ (4.10)

$x \in X = \{x \mid g_s(x)\ \{\geq = \leq\}\ 0,\ s = 1 \ldots.. m\}$

*where* $z_i(x)$, $j \in J$ are maximization objectives, $z(x)$ $i \in I$ are the minimization objectives, $IUJ$ =

*{ 1,2,….n}* are considered as fuzzy constraints. All functions $z_j(x)$, $g_s(x)$, $(i = l,..n;\ s = l,…m)$ can be linear and nonlinear. With the tolerances of fuzzy constraints given, the membership functions $\mu_i\,(x)$, $\forall_i$ could be established. The feasible set solution obtained through min-operator is defined by interaction of the fuzzy objective set. The feasible set is presented by its membership

$\mu_i\,(x) = min(\mu_i\,(x)….\ \mu_i\,(x),)$

If a decision maker deals with a maximum $\mu_D\,(x)$ in the feasible set then the solution procedure is max $(\min_i \mu_i\,(x))$ $x\in X$, suppose the overall satisfactory level of compromise is $a = \min_i \mu_i\,(x)$ then the problem can be explained as:

Find max a subject to:

$$\alpha \leq \mu_i\,(x)\ \forall_I \qquad x\in X, \tag{4.11}$$

Assuming that membership functions, based on preference or satisfaction, are linear and non-decreasing between $z_i^+\,(x)$ and $z_i^-\,(x)$ for $\forall_i$

$$\mu_k\,(x) = \begin{cases} 1 & \text{if } z_i\,(x) > z_i^+ \\ \dfrac{(z_i(x) - z_i^-)}{(z_i^+ - z_i^-)} & \text{if } z_i^-\,(x) \leq z_i\,(x) \leq z^+ \\ 0 & \text{if } z_i\,(x) < z_i^- \end{cases} \tag{4.12}$$

The only feasible solution region is the area $\{x|z_i\,(x) \leq z_i\,(x) \leq z_i^+\}\ \forall_i$ and $x\in X$, hence we can write:

Find max a subject to

$$\mu_k(x) = [z_i\,(x) - z_i^-] / [z_i^+ - z_i^-] \geq \alpha \quad x\in X \tag{4.13}$$

This problem can be solved by using two-phase approach. The first phase relates to the search for the optimal value of $\alpha^0$ in order to find a possible solution $(x^0)$. If the possible solution is unique, $x^0$ is an optimal non-dominated solution. Otherwise, the second phase is introduced to search for the maximum arithmetic mean value of all membership restricted by original constraints and $\alpha \geq \alpha^0\ \forall$. That is,

$$\text{Max } (\textstyle\sum_i \alpha_i)/ I \tag{4.14}$$

$\alpha' \leq \alpha_i \leq \mu_i\,(x).\ \forall i,\ x\in X,$

for $i$ objective functions and $\alpha'$ solution (4.7). The objective functions (4.10) could be written

$\text{Max } [\sum_i \mu_i\,(x)] / I$

$$\alpha' \leq \mu_i\,(x),\ \forall I,\ x\in X, \tag{4.15}$$

By unifying both objective (4.7) and 4.11) the second step can be automatically solved after the first step following the solution procedure of the simplex method

$$\text{Max } \alpha + \delta\,[\textstyle\sum_i \mu_i\,(x)],\ / I \qquad \alpha \leq \mu_i\,(x),\ \forall i.\ x\in X, \tag{4.16}$$

*W*here $\delta$ is sufficiently small positive number. Since the weights between objectives are not equal we can write.

$$\max \alpha + \delta \textstyle\sum_i w_i \mu_i\,(x) \qquad \alpha \leq \mu_i\,(x),\ \forall i,\ x\in X, \tag{4.17}$$

For $w_i$ as the relative importance of the $i^{th}$ objective and $\sum_i \mu_i = 1$. The coefficient $\alpha$ represents the degree of acceptability or degree of possibility for the optimal solution. For construction industry activities the minimal value of the coefficient $\alpha_n$ can be prescribed. Hence two new constraints are added in this linear program:

$$\alpha \geq \alpha_1 \quad \alpha \leq \alpha_n \quad , \text{where } 0 \leq \alpha \leq I \quad 0 \leq \alpha, \tag{4.18}$$

Coefficient of satisfaction $(\varphi_I)$ in relation to the best individual solutions $z_i^+$ are:

$(\varphi_i) = \max z_i / z_i^+ \qquad i = 1, 2,…n$

From the aspect of fuzzy set theory the augmented max-min approach allows for compensation among objectives. Firstly one reaches the solutions at a large unit, and then by re-evaluating these solutions the compromise solutions at a smaller unit are obtained.

**5. FUZZY MULTIOBJECTIVE PROBLEM**

The Fuzzy Multiple objective Decision Model (FMODM) studied by Lai and Hwang [7, 8] states that the effectiveness of a decision makers' performance in a decision process can be improved as a result of the high quality of analytic information supplied by a computer. They propose an interactive Fuzzy Multiple objective Decision Model (IFMODM) to solve a specific domain of Multiple Objective Decision Model (MODM).

$Max\ (z_i(x) \ldots., z_n(x))$ (5.1)
Subject to
$g_j(x) \leq \sim b_j\ j=1,\ldots,m \quad x \geq 0$

where $b_j\ \forall_j$ are fuzzy resources available with corresponding maximal tolerances $t_i$. Their membership functions are assumed to be non-increasing linear functions between $b_j$ and $b_j + t_j$
The objective functions (5.1) are redefined into
$\text{Max } z_i(C_i, x) \quad i=1, 2\ldots.I$ (5.2)
Subject to:
$g_j(A, x)\ \{\leq\ =\ \geq\}\ b_i \quad j=1,2,\ldots..m,\ x \geq 0$
We present the model (5.2) limitations as fuzzy inequalities since the limitations prevent the objective functions from reaching their individual optimum.
Find x, subject to

$z_i(C_b\ x) \geq\sim z_i^0, \forall_i \quad g_j(A_j, x) \leq b_j, \forall_j\ x \geq 0$ (5.3)

where $z_i^0$, $\forall_i$ are the goals of the objectives and $\geq\sim$ is a soft or fuzzy inequality. With the known tolerances of fuzzy constraints the membership functions $\mu_i(z_i)$, $\forall_i$ to measure satisfaction levels of fuzzy objective constraint could be established. It is supposed that membership functions are based on a preference concept. The membership functions can be any non-decreasing functions for maximization objectives and non-increasing functions for maximization objectives such as linear, exponential, and hyperbolic. In [13], the researchers assume linear membership functions since the other types of membership functions can be transferred into equivalent linear forms.

Each objective of equation (5.2) should have an individual best ($z_i^+$) and individual worst solution ($z_i^-$)
$Z_i^+ = \max z_i(C_b\ x), x \in X,$
$Z_i^- = \min z_i(C_b\ x), x \in X,$ (5.4)
The linear membership function can be defined as in (4.8). According to (4.18) and (4.19) the following augmented problem can be defined.
$max\ \alpha + \delta \sum_i w_i\ \mu i\ (x)$ (5.5)
$\alpha \leq$
$\alpha \leq \mu_i(x), \forall i, x\ \varepsilon\ X, \alpha \varepsilon\ [0,1]$
where $\delta$ is a sufficiently small positive number, and $w_i$ ($\sum_i w_i = 1$) is of relative importance or weight. If a decision maker wants to provide his/her goals $z_i^0$ and corresponding tolerances $t_i$ for objectives, than for $z_i^0 \leq z_i^+$ and $(z_i^0 - t_i)b \geq z_i$ the problem will become:

Find x, subject to
$z_i(C_i, x)\ ) \geq\sim z_i, \forall_i$ and $x \in X$, (5.6)
where $z_i^0$, $\forall_i$ as well as tolerances $t_i$ are given. Then
$\text{Max } \alpha + \delta \in_i w_i \mu_i(z_i)$
$\mu_i(z_i) = 1 - [\ z_i^0 - z_i(C_i, x)]/\ t_i \geq \alpha\ x \in X, \alpha \in [0,1]$ (5.7)
The problem can be further considered as:
$\text{Max } \alpha + \delta[\sum_i w_i \mu_i(z_i) + \delta \sum_i q_i \mu_i(g_i)]$ (5.8)
Subject to
$\mu_i(z_i) = [z_i(C_i, x) - z_i\ ]\ /\ [z_I^+ - z_I^-] \geq \alpha\ \forall_i$
$\mu_i(g_i) = 1 - [g_i(A_j, x) - b_j\ ]\ /\ t_I \geq \alpha\ \forall_j \quad x > \alpha \in [0,1]$
where $w_i$ and $g_i$, $\forall_i$ , j are of relative importance and $\sum_i w_i + \sum_j g_j = 1$

The computer program was written using MATLAB 2015. Input data are: number of objectives *k*, number of constraints *m*, number of unknowns *n*, goals $z_i$ (i=1,2,…k), elements $c_{ij}$ (i=1,2,…k; j =1,2,…n), $a_{ij}$ (i=1,2,…n), $b_i$ (i=1,2,…m), tolerances $t_i$ ( i= 1,2,…k) and $d_i$ (i=1,2,…m). The program determines the individual best $z_i^+$ solution and the individual worst solution $z_i^-$ for every objective $i_i$ (i=1, 2,…k). The objective functions are (4.3) and the constraints are (4.2). The obtained values $z_i^+$ and zj, based on the modified Zimmermann's procedure, are used to solve the linear program with the objective function (4.17) and constraints (4.2) (4.8) and 4.18). For the nonfuzzy problem, this program gives the values of unknown $x_j$ (j =1,2,…n) maximal values of objective function $z_i$ (i=1,2,…k),, coefficient of acceptability α and coefficients of satisfaction $\varphi_i$

(i=1,2,…k). For the fuzzy problem, the linear program with the objective function (5,3) and the constraints (5,6) gives: the optimal value of unknown $x_i$ (i=1,2,…n), objective function $z_i$ coefficients of satisfaction $\varphi_i$ (i=1,2,…k) and coefficient of acceptability α.

## 6. CASE STUDY ANALYSIS AND MODELLING

Liquid cocoa is a raw material for producing cocoa products such as chocolates and cocoa butter, which are in high demand, both for local consumption and export to foreign countries. It is derived from cocoa beans after they have undergone the process of fermentation, drying, roasting and separation from their outer covering (skins).The beans are then grounded, to produce “cocoa mass”. This mass is melted to obtain liquid cocoa, which can be separated into cocoa solids (from which cocoa powder can be obtained) and cocoa butter. It can also be cooled and molded into blocks of raw chocolate. Data obtained from a cocoa processing plant has been analyzed for building our proposed decision making model. Cocoa liquids is shipped in barrels over a distance ranging1500m-3000m to two cocoa butter cream manufacturing sites and two cocoa beverage producing company sites (Sites A-D) . Three pumps and eleven interior vibrators are used for delivering the cocoa liquid at each manufacturing sites. Table 1, illustrates the manufacturing capacities of the plant, the operational capacity of the pumps and the labor requirements at the three sites. The analysis carried out, demonstrates the complexity of the variable and constraints of this liquid cocoa production plant and delivery system. The liquid cocoa producing company manager’s task shall be, to increase the profit, by using the maximum capacity of the cocoa processing plant while meeting the requirement of the three manufacturing sites for liquid cocoa, through a feasible schedule.

**Table 1:** Cocoa processing plant capacity and construction site’s resource demands

| | Cocoa Processing Plant | Site A | Site B | Site C | Site D | Remark |
|---|---|---|---|---|---|---|
| Plant Capacity | $60m^3/h$<br>$2520m^3$ Weekly | | | | | $200m^3$ (tolerance) |
| Transit mixers (total =3) | - | $8.45m^3/h$ | $9.26m^3/h$ | $7.26m^3/h$ | $10.57m^3/h$ | Operated by 7 Workers |
| Cocoa pumps (total =3) | | $16m^3/h$ | $22m^3/h$ | $26m^3/h$ | $28m^3/h$ | Operated by 6 Workers |
| Interior vibrators (total =11) | | $40m^3/h$ | | | | - |
| Worker requirement | 5 | 6 | 7 | 9 | 11 | |
| Minimal Cocoa requirement (tolerance) | | $14.0m^3/h$<br>588 $m^3$/ week<br>($47m^3$) | $18.0m^3/h$<br>756 $m^3$/ week<br>($60m^3$) | $21.5m^3/h$<br>903 $m^3$/ week<br>($72m^3$) | $24.8m^3/h$<br>1026 $m^3$/ week<br>($88m^3$) | |
| Weekly values are based on 42 working hours/week | | | | | | |

### 6.1 Objective Formulation

Success of any decision model will directly depends on the formulation of the objective function taking into account all the influential factors. We modeled the final objective function taking into the account independent factors (1) profit expressed as ₦/$m^3$ (2) index of work quality (performance) and (3) worker satisfaction

***Profit***: The expected profit as related to the volume of Cocoa to be manufactured is modeled as the first objective and is shown in Table 2. The minimal expected weekly profit as a fuzzy value is $Z^0$ = ₦982, 000 per week with tolerance $t_1$ = ₦ 760,000.

**Table 2:** Modeling profit as an objective

| Site Name | Site A | Site B | Site C | Site D |
|---|---|---|---|---|
| Expected profit (AUS/$m^3$) | 12 | 10 | 11 | 14 |

Index of quality: Equally or sometimes more important than the profit, quality plays an important role in every industry. We modeled the index of quality at construction sites as the second objective. The index is ranged from 5 points/$m^3$ (bad) quality to 10 points/$m^3$ (excellent) quality and the assigned values are shown in Table 3. The minimal expected total weekly number of points for quality, as fuzzy value, is $z^0_2$ = 21400 with tolerance, $f_2$ = 1700 points.

**Table 3:** Modeling index of quality as an objective

| Site Name | Site A | Site B | Site C | Site D |
|---|---|---|---|---|
| Index of Quality | 9 | 10 | 7.5 | 12 |

**Table 4:** Modeling worker satisfaction index as an objective

| Site Name | Site A | Site B | Site C | Site D |
|---|---|---|---|---|
| Worker satisfaction Index | 9 | 10 | 7.5 | 12 |

The fuzzy solution that gives higher profit with possibility of realization $\alpha = 0.852$

### 6.2 Variables that Optimize the Objective Function

After knowing the objective function our next task is to determine the variables that optimize the objective function. In our problem it is to find: the optimal value of unknowns *xi (i=1, 2, 3)* that represent quantities of Cocoa which have to be delivered to Site A, B and C respectively and corresponding optimal values of the objective functions *z1, z2, z3..* According to problem requirements and available data (Table 1, 2, 3 and 4) the objective functions can be modeled as follows:

- $\max z_1 = 12x_1 + 10x_2 + 11x_3 + 15x_4 (>,\sim)\ 27000$ with tolerance, $t1=2100$ ***(profit)***
- $\max z_2 = 9x_1 + 10_{x2} + 7.5x_3 + 13x_4 (>,\sim)\ 21400$ with tolerance, $t2=1700$ ***(index of quality)***
- $\max z_3 = 8x_1 + 7x_2 + 9x_3 + 12x_4 (>,\sim)\ 18000$ with tolerance, $t3=1400$ ***(worker satisfaction index)***
- $x1+x2+x3+x_4 (<,\sim)\ 2520$, tolerance $d_1=200$ ***(weekly capacity of the Cocoa plant)***
- $0.12\ x_1 + 0.11\ x_2 + 0.14\ x_3 + 0.17x_4 (\leq,\sim)\ 8x42 = 336\ h$, tolerance $d2=23\ h$ ***(weekly engagement of 7 transit mixers, taking into account of their working capacities)***
- $0.06\ x_1 + 0.05\ x2 + 0.04\ x3 + 0.02x_4\ (<,\sim)\ 4x42=168h$, tolerance $d3=10h$ ***(weekly engagement of 4 Cocoa pumps)***
- 6x1+7x2+9x3(<,~) 24x42=924, tolerance d4=84. ***(weekly engagement of 24 workers for interior delivering, placing and consolidating Cocoa at sites A,B,C and D).***
- Minimal weekly requests for Cocoa from the three construction sites:

Site A, $x1 \geq 588\ m3$, tolerance $d5=47m3$
Site B, $x2 \geq 756\ m3$, tolerance $d6=60\ m3$
Site C, $x3 \geq 756\ m3$, tolerance $d7=72\ m3$
Site D, $x4 \geq 756\ m3$, tolerance $d8=84\ m3$

- The minimal value of the degree of acceptability is $\alpha_1 \geq 0.80$. These constraints written in full are:
- $x1+x2+x3+x_4(<,\sim)2520$
- $0.118x1+0.108x2+0.139x3+ 0.126\ x_4(<,\sim)\ 294$
- $0.063x1+0.045x2+0.038x3+ 0.0267\ x_4(<,\sim)\ 126$
- $0.100x1+0.117x2+0.150x3 + 0.198\ x_4(<,\sim)\ 924$
- $0.033x1+0.033x2+0.055x3+ 0.077\ x_4(<,\sim)\ 294$
- $x_1(>,\sim)588\quad x_2(>,\sim)756\ x_3\ (>,\sim)903\ x_4\ (>,\sim)998$

By using linear programming technique we will be able to solve the above equations and the individual best and worst non-fuzzy solution for constraints (b) and individual objective functions (a) could be obtained. The obtained results are summarized in Table 5.

### 6.3. Solutions

Now we will try to solve the multiple objective functions using the results obtained for *zi+* and *zi -* as shown in Table 5 and using the modified Zimmermann's procedure as discussed in Section 4. We implemented the codes in MATLAB 2015 and were executed in a Windows 2010, Core i5 Computer System. The results obtained are summarized in Table 6. The simulations were repeated three times and found that the results are stable. Coefficient of acceptability of this solution was found to be $\alpha=0.941$. When the objective functions were modeled using the described fuzzy approach the obtained solutions are as summarized in Table 7.Coefficient of acceptability of this solution $\alpha = 0.852$.As depicted in Figures 2 and 3, the obtained results clearly shows the superiority of fuzzy approach. However it is also interesting to note that there is not much difference between fuzzy and non-fuzzy solutions for the three objective functions. The difference is being less that *2* percent. The coefficients of acceptability of the solutions $\alpha$, indicating the possibility of realizing these solutions, are very high. According to this, the decision maker could accept:

- the non-fuzzy solution that gives smaller profit with possibility of realization $\alpha=0.941$
- the fuzzy solution that gives higher profit with possibility of realization $\alpha=0.852$

**Table 5: Individual best and worst non-fuzzy solution**

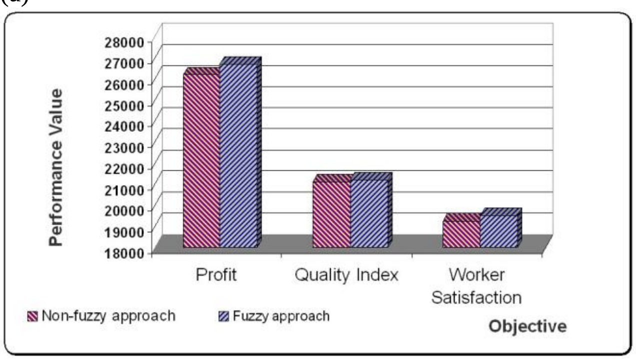
| Objective | $X_1$ ($m^3$/week) | $X_2$ ($m^3$/week) | $X_3$ ($m^3$/week) | $X_4$ ($m^3$/week) | $Z_1^+$($) | $Z_1^-$($) |
|---|---|---|---|---|---|---|
| 1 | 734.02 | 756.00 | 903.00 | 976.00 | 26301.29 | 0 |
| 2 | 588.00 | 915.95 | 903.00 | 950.00 | 21224.00 | 0 |
| 3 | 734.02 | 756.00 | 903.00 | 950.00 | 19291.00 | 0 |

**Table 6: Optimal results using non-fuzzy procedure**

| $X_1$ ($m^3$/week) | $X_2$ ($m^3$/week) | $X_3$ ($m^3$/week) | $X_4$ ($m^3$/week) | Max ($Z_1$) | Max ($Z_2$) | Max ($Z_3$) | Max ($Z_4$) |
|---|---|---|---|---|---|---|---|
| | | | | $\varphi$ | $\varphi$ | $\varphi$ | $\varphi$ |
| 635.94 | 863.43 | 903.0 | 1054.0 | 26,199 | 21,130 | 19,259 | 20,762 |
| | | | | 0.996 | 0.996 | 0.998 | 0.998 |

• φ is the coefficient of satisfaction

(a)




(b)

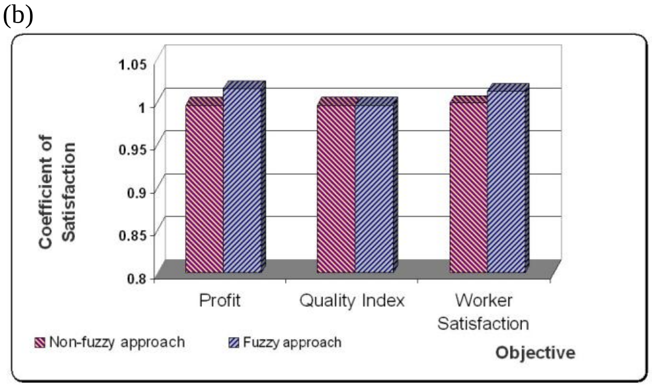



**Figure 2.**Comparison of fuzzy and non-fuzzy approach (a) showing the performance value of objective functions (b) showing the coefficient of satisfaction for the different objective functions.

The developed software will also help the decision maker to vary the values of the coefficient α in the interval [*0, 1*] and to receive the corresponding optimal values of production and profit with corresponding values of possibility. After careful study of the optimal values of the objective functions and the various constraints, very often expert problem domain knowledge will be required to understand the possibility of realization of the achieved results.

**7.0 Conclusion**

Organizing business activities involve making cogent decisions that can either bring profit or loss to the business. Decision making procedures that are capable of bolstering up the business activities that involve imprecise data can be analyzed using the multi-objective criteria fuzzy models. The modeling of the cocoa processing plant problem presented in this paper involves the combination of fuzzy linear objective functions and constraints. The results show how that the fuzzy method used, in terms of individual solution for the four objective functions and coefficients of satisfaction more efficient. It also shows that the difference between fuzzy and non-fuzzy objective functions for the individual best solutions. There is however less than 20% possibility of realizing optimal profit. The software developed is capable of calculating the optimal profit for a given possibility of realization coefficient. This research work has shown that a similar or better level of satisfaction for the obtained results can be achieved when membership functions are introduced into a linear programming model, either in constraints, or both as objective functions and constraints.

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