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ON SOME CLASSES OF MODIFIED DUAL TO RATIO ESTIMATORS OF FINITE POPULATION MEAN IN SAMPLE SURVEYS

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Abstract

In this article, some classes of modified ratio estimators are suggested using variable transformation (dual) for finite population mean of the study variable under simple random sampling scheme. The properties (Biases and Mean Square Errors $[MSEs]$) of the suggested classes of estimators t_{11}^* and t_{12}^* are derived up to first order approximation. The expression for the unknown weight α which optimized the efficiencies of t_{11}^* and t_{12}^* have been established. The optimum $MSEs$ of the suggested classes of estimators have also been obtained. The efficiencies of the suggested classes of estimators t_{11}^* and t_{12}^* has been compared theoretically and empirically with some existing related ratio estimators. The empirical comparison shows that the suggested classes of estimators have lower $MSEs$ and higher relative efficiency compared to the existing ones considered in the study under optimal conditions and thus most preferred over all the considered estimators for the use in practical application.

Keywords: **Dual-Ratio Estimator, Mean Squared Error , Efficiency, Auxiliary Variable.**

I.0 INTRODUCTION

In sampling theory, the role of auxiliary information is of prime importance. Auxiliary variables are used in survey sampling to obtain improved sampling designs and to achieve more precision in the estimates of the population mean of the variable under study. This information may be used either at the designing or at the estimation stages of survey sampling. The popular estimation methods that are widely discussed in sampling literature and used in practice comprise ratio, product and regression methods of estimation. When the correlation between study variable and auxiliary variable is (highly) positive, the ratio method of estimation proposed by [1] is preferred for estimating the population mean. On the other hand, if the correlation is negative, the product method of estimation introduced by [2] and revisited by [3] is used. Owing to this limitation of classical ratio and product estimators in recent past, statisticians concentrated their attention in developing modified ratio and product type estimators using some known population parameters of an auxiliary variable and this approach led to considerable reduction in the variance of the estimator. Moving along this direction, [4] suggested a modified ratio estimator using the coefficient of variation of auxiliary variable X for estimating the populations mean $\bar{Y}$ of the study variable *y*, [5] suggested another modified ratio estimator using the linear combination of coefficient of variation and coefficient of kurtosis and [6] used known correlation coefficient in ratio estimation of population $\bar{Y}$.The use of dual transformation for estimation of population parameters were first introduced by [7] and [8], and [9] considered a linear combination of ratio and dual to ratio estimator and suggested ratio-cum-dual to ratio estimator of finite population mean in simple random sampling. [10, 11] as well as [12] also utilized a dual technique to obtained different type of estimators for population mean of the study variable y in simple random sampling scheme. In this article, a dual to modified classes of ratio estimators by [13] have been suggested for finite population mean and the conditions for their efficiencies have also been established.

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II. NOTATIONS AND EXISTING ESTIMATORS

Let U denote a finite population consisting of N units $\{U_1, U_2, \dots, U_N\}$. Also let Y be study variable taking values $\{Y_1, Y_2, \dots, Y_N\}$ and X be auxiliary variable taking values $\{X_1, X_2, \dots, X_N\}$ on i^{th} unit U_i of the population U. Then

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ is the population mean of the study variable Y,

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ is the population mean of the auxiliary variable X,

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$ is the finite population variance of study variable Y,

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$ is the finite population variance of auxiliary variable X, $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)$ is the finite population covariance of X and Y,

$\rho_{yx} = \frac{S_{yx}}{S_x S_y}$ is the Pearson's moment correlation coefficient of X and Y,

$C_y = \frac{S_y}{\bar{Y}}$ is the coefficient of variation of Y, $C_x = \frac{S_x}{\bar{X}}$ is the coefficient of variation of X.

Let y_i and x_i be the values of population units of both the study and auxiliary variables included in the sample at i^{th} draw. The conventional unbiased estimator, which does not utilize auxiliary information is defined by

$$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

whose variance is given by

$$Var(t_0) = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2 \tag{2}$$

The variance given in (2) is sufficiently large, and the aim is to reduce this variance.

When the population mean $\bar{X}$ is known, [1] proposed conventional ratio estimator of finite population mean by incorporating both sample and population means of auxiliary variable into equation (1). The suggested estimator and its properties are given below;

$$t_1 = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{3}$$

$$bias(t_1) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_x^2 - \rho_{yx}C_yC_x\right) \tag{4}$$

$$MSE(t_1) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right) \tag{5}$$

In his work, it was established that t_1 is more efficient than t_0 if $\rho_{yx} > \frac{1}{2}\frac{C_y}{C_x}$.

Consider a transformation $x_i^* = \frac{N\bar{X} - nx_i}{N-n}$, where $i = 1, 2, 3, \dots, N$. Here, $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n}$ is an unbiased estimator of the population mean $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ of the auxiliary variable X and the correlation between $\bar{y}$ and $\bar{x}^*$ is negative.

Using the transformation x^* on the auxiliary variable X, [7] obtained dual to ratio estimator as

$$t_2 = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right) \tag{6}$$

To the first degree of approximation, the bias and the MSE of t_2 are respectively given by

$$bias(t_2) = -\bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)g\rho_{yx}C_yC_x \tag{7}$$

$$MSE(t_2) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_y^2 + g^2C_x^2 - 2g\rho_{yx}C_yC_x\right) \tag{8}$$

where $g=\dfrac{n}{(N-n)}$.

[13] utilized power transformation and linear combination to modify equation (3) which led to the establishment of some classes of modified ratio estimators for finite population mean of the study variable y .The suggested estimators are given below:

$$t_3=t_1\left[H(u)\right]^{\delta_1} \tag{9}$$

$$t_4=\frac{t_1}{\delta_2 H(u)+(1-\delta_2)} \tag{10}$$

where $u=\dfrac{\bar{x}}{\bar{X}}$, δ_i where $i=1,2$ are unknown functions to be estimated to minimize the $MSEs$ of t_3 and t_4 respectively and $H(.)$ is a parametric function satisfying certain regularity conditions as given in [14], such that

(i) $H(1)=1$

(ii) The first and second order derivatives of H with respect to u exist and are known constant at a given point $u=1$.

The biases and $MSEs$ of t_3 and t_4 to the first degree of approximation are respectively given as

$$bias(t_3)=\theta\bar{Y}\left[C_x^2-C_{yx}+\frac{\delta_1}{2}H_2C_x^2+\frac{\delta_1(\delta_1-1)}{2}H_1^2C_x^2-\delta_1H_1C_x^2+\delta_1H_1C_{yx}\right] \tag{11}$$

$$bias(t_4)=\theta\bar{Y}\left(C_x^2-C_{yx}+\delta_2^2H_1^2C_x^2-\frac{\delta_2}{2}H_2C_x^2+\delta_2H_1C_x^2-\delta_2H_1C_{yx}\right) \tag{12}$$

and

$$MSE(t_3)=\theta\bar{Y}^2\left[C_y^2+C_x^2-2C_{yx}-2\delta_1H_1C_x^2+2\delta_1H_1C_{yx}+\delta_1^2H_1^2C_x^2\right] \tag{13}$$

$$MSE(t_4)=\theta\bar{Y}^2\left[C_y^2-2C_{yx}-2\delta_2H_1C_{yx}+C_x^2+2\delta_2H_1C_x^2+\delta_2^2H_1^2C_x^2\right] \tag{14}$$

where $\theta=\left(\dfrac{1}{n}-\dfrac{1}{N}\right)$ and $C_{yx}=\rho_{yx}C_yC_x$.

Estimators t_3 and t_4 attained optimality at $\delta_1^{(opt)}=\dfrac{1-\rho_{yx}C_y}{C_xH_1}$ and $\delta_2^{(opt)}=\dfrac{\rho_{yx}C_y-1}{C_xH_1}$ respectively and the minimum $MSEs$ of t_3 and t_4 up to first order approximation, at optimum values of $\delta_1^{(opt)}$ and $\delta_2^{(opt)}$ are given respectively as:

$$MSE(t_3)=\theta\bar{Y}^2C_y^2\left(1-\rho_{yx}^2\right) \tag{15}$$

and

$$MSE(t_4)=\theta\bar{Y}^2C_y^2\left(1-\rho_{yx}^2\right) \tag{16}$$

which are equal to approximate MSE of the usual regression estimator given as

$$\bar{y}lr=\bar{y}+b\left(\bar{X}-\bar{x}\right) \tag{17}$$

where b is the estimate of coefficient of linear regression of y on x.

III. SUGGESTED MODIFIED CLASSES OF ESTIMATORS

Having studied the work of [13] and motivated by [7], the following estimators of finite population mean are suggested;

$$t_{11}^*=t_2\left[H(u^*)\right]^{\alpha_1} \tag{18}$$

$$t_{12}^*=\frac{t_2}{\alpha_2H(u^*)+(1-\alpha_2)} \tag{19}$$

where $t_2=\bar{y}\dfrac{\bar{x}^*}{\bar{\bar{X}}}$, $u^*=\dfrac{\bar{x}^*}{\bar{X}}$, α_1 and α_2 are unknown functions to be estimated to minimize the $MSEs$ of t_{11}^* and t_{12}^* respectively.

IV. BIAS AND MEAN SQUARE ERROR OF THE MODIFIED CLASSES OF ESTIMATORS

In order to obtain these properties (bias and MSE), the following error terms e_0 and e_1 are defined as;

$e_0=\dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1=\dfrac{\bar{x}-\bar{X}}{\bar{X}}$ such that $|e_0|\cong 0$, $|e_1|\cong 0$ and $\bar{y}=\bar{Y}(1+e_0)$, $\bar{x}=\bar{X}(1+e_1)$

$$\left.\begin{array}{l} E(e_0)=E(e_1)=0,\ E(e_0^2)=\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2 \\ E(e_1^2)=\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2, E(e_0e_1)=\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yx}C_yC_x \end{array}\right\} \tag{20}$$

Employing Taylor's series expansion under usual assumption, $H(u^*)$ is expanded by the value 1 in the second order Taylor's series as

$$H(u^*)=H\left[1+(u^*-1)\right]=H(1)+(u^*-1)(\frac{\partial H}{\partial u^*})_{u^*=1}+\frac{1}{2}(u^*-1)^2(\frac{\partial^2 H}{\partial u^*})_{u^*=1}+... \tag{21}$$

Assuming $|(u^*-1)| < 1$, the higher order terms can be neglected and equation (18) and (19) can be re-written respectively as

$$t_{11}^*=t_2\left[1+(u^*-1)H_1^*+\frac{1}{2}(u^*-1)^2H_2^*\right]^{\alpha_1} \tag{22}$$

$$t_{12}^*=t_2\left[1+\alpha_2(u^*-1)H_1^*+\frac{\alpha_2}{2}(u^*-1)^2H_2^*\right]^{-1} \tag{23}$$

where $(\frac{\partial H}{\partial u^*})_{u^*=1}=H_1^*$ and $(\frac{\partial^2 H}{\partial u^*})_{u^*=1}=H_2^*$ denote the first and second order partial derivatives of H with respect to u^* and are known as constants.

In terms of error terms e_0 and e_1, equation (22) and (23) can be written respectively as

$$t_{11}^*=\bar{Y}(1-ge_1+e_0-ge_0e_1)\left[1-gH_1^*e_1+\frac{1}{2}g^2H_2^*e_1^2\right]^{\alpha_1} \tag{24}$$

$$t_{12}^*=\bar{Y}(1-ge_1+e_0-ge_0e_1)\left[1-\alpha_2gH_1^*e_1+\frac{\alpha_2}{2}g^2H_2^*e_1^2\right]^{-1} \tag{25}$$

Simplifying equation (24) and (25) to first order approximation, equations (26) and (27) are respectively obtained as

$$t_{11}^*=\bar{Y}\left[\begin{array}{l}1-\alpha_1gH_1^*e_1+\frac{\alpha_1}{2}g^2H_2^*e_1^2+\frac{\alpha_1(\alpha_1-1)}{2}g^2H_1^{*2}e_1^2- \\ ge_1+\alpha_1g^2H_1^*e_1^2+e_0-\alpha_1gH_1^*e_0e_1-ge_0e_1\end{array}\right] \tag{26}$$

$$t_{12}^*=\bar{Y}\left[\begin{array}{l}1+\alpha_2gH_1^*e_1-\frac{\alpha_2}{2}g^2H_2^*e_1^2+\alpha_2^2g^2H_1^{*2}e_1^2-ge_1- \\ \alpha_2g^2H_1^*e_1^2+e_0+\alpha_2gH_1^*e_0e_1-ge_0e_1\end{array}\right] \tag{27}$$

Subtract $\bar{Y}$ from both sides of equation (26) and (27) after ignoring the e's term with power two or higher, then taking expectations and apply the results in equation (20); the bias of t_{11}^* and t_{12}^* are obtained respectively as:

$$bias(t_{11}^*)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\begin{array}{l}\frac{\alpha_1}{2}g^2H_2^*C_x^2+\frac{\alpha_1(\alpha_1-1)}{2}g^2H_1^{*2}C_x^2+\alpha_1g^2H_1^*C_x^2- \\ \alpha_1gH_1^*\rho_{yx}C_yC_x-g\rho_{yx}C_yC_x\end{array}\right] \tag{28}$$

$$bias(t_{12}^*)=\bar{Y}\left(\frac{1}{n}-\frac{1}{N}\right)\left[\begin{array}{l}\alpha_2gH_1^*\rho_{yx}C_yC_x-g\rho_{yx}C_yC_x-\frac{\alpha_2}{2}g^2H_2^*C_x^2- \\ \alpha_2g^2H_1^*C_x^2+\alpha_2^2g^2H_1^{*2}C_x^2\end{array}\right] \tag{29}$$

Squaring both sides of equation (26) and (27) after ignoring the e's term with power two or higher, then taking expectation and apply the results in equation (20), the *MSE* of t_{11}^* and t_{12}^* to first order approximation are obtained respectively as,

$$MSE(t_{11}^*)=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[\begin{array}{l}C_y^2+g^2C_x^2+2\alpha_1g^2H_1^*C_x^2-2g\rho_{yx}C_yC_x- \\ 2\alpha_1gH_1^*\rho_{yx}C_yC_x+\alpha_1^2g^2H_1^{*2}C_x^2\end{array}\right] \tag{30}$$

$$MSE(t_{12}^*)=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[\begin{array}{l}C_y^2+g^2C_x^2+\alpha_2^2g^2H_1^{*2}C_x^2-2\alpha_2g^2H_1^*C_x^2- \\ 2g\rho_{yx}C_yC_x+2\alpha_2gH_1^*\rho_{yx}C_yC_x\end{array}\right] \tag{31}$$

Minimizing equation (30) and (31) with respect to α_1 and α_2 yield their optimum values as:

$$\alpha_1^{(opt)}=\frac{1}{H_1^*}\left[\frac{K}{g}-1\right], \qquad \alpha_2^{(opt)}=\frac{1}{H_1^*}\left[1-\frac{K}{g}\right]$$

where $K = \frac{\rho_{yx} C_y}{C_x}$

By substituting $\alpha_1^{(opt)}$ for α_1 in (30) and $\alpha_2^{(opt)}$ for α_2 in (31), the minimum MSE values of t_{11}^* and t_{12}^* denoted by $MSE\left(t_{11}^*\right)_{\min}$ and $MSE\left(t_{12}^*\right)_{\min}$ are given respectively as;

$$MSE\left(t_{11}^*\right)_{\min} = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2\left[1-\rho_{yx}^2\right] \quad (32)$$

$$MSE\left(t_{12}^*\right)_{\min} = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2\left[1-\rho_{yx}^2\right] \quad (33)$$

which are equal to the MSE of the linear regression estimator for population mean. Hence for optimum value of $\alpha_1^{(opt)}$ and $\alpha_2^{(opt)}$, the suggested modified classes of estimators are equally efficient as the linear regression estimator.

V. FUNCTIONS OF $H(u^*)$ TO BE USED

In this section, we defined the function $H(u^*)$ to be used in this study.

$H(u)$ is defined by [3] as

$$H(u) = \frac{\bar{x}}{\bar{X}} \quad (34)$$

To obtain the expression for H_1 and H_2, we determine the first and second order derivatives of equation (34) with respect to u provided that u =1 at a given point as;

$$H_1 = \frac{\partial H(u)}{\partial u}\Big|_{u=1} = 1 \quad (35)$$

and

$$H_2 = \frac{\partial^2 H(u)}{\partial u}\Big|_{u=1} = 0 \quad (36)$$

By transformation we have

$$H(u^*) = \frac{\bar{x}^*}{\bar{X}} = \frac{N\bar{X} - n\bar{x}}{\bar{X}(N-n)} \quad (37)$$

$$= \frac{\bar{X}(N - n\frac{\bar{x}}{\bar{X}})}{\bar{X}(N-n)}$$

$$= \frac{\left[N - nH(u)\right]}{N-n}$$

$$= \frac{(N-nu)}{N-n} \quad (38)$$

where $u = H(u) = \frac{\bar{x}}{\bar{X}}$

To obtain the expression for H_1^* and H_2^*, we find the first and second order derivatives of equation (38) with respect to u provided that u =1 at a given point as;

$$H_1^* = \frac{\partial H(u^*)}{\partial u}\Big|_{u=1} = \frac{-n}{N-n} \quad (39)$$

and

$$H_2^* = \frac{\partial^2 H(u^*)}{\partial u}\Big|_{u=1} = 0 \quad (40)$$

VI. THEORETICAL EFFICIENCY COMPARISONS

In this section, efficiency of the modified class of estimators are compared with those of other existing estimators and conditions are obtained under which modified classes of estimators are more efficient.

1. Modified Estimator t_{11}^* over some Existing Related Estimators

(i) t_{11}^* versus Sample mean t_0

$$MSE(t_{11}^*) - MSE(t_0) < 0 \quad (41)$$

$$\frac{1}{H_1^*}\left(\frac{2\rho_{yx}C_y}{gC_x} - 1\right) < \alpha_1 < -\frac{1}{H_1^*} \quad (42)$$

For the range of α_1 given in (42) the modified estimator t_{11}^* is better than t_0

(ii) t_{11}^* versus Ratio estimator t_1

$$MSE(t_{11}^*) - MSE(t_1) < 0 \tag{43}$$

$$\frac{1}{gH_1^*}(1-g) < \alpha_1 < -\frac{1}{gH_1^*}\left(1+g-\frac{2\rho_{yx}C_y}{C_x}\right) \tag{44}$$

For the range of α_1 given in (44) the modified estimator t_{11}^* is better than t_1

(iii) t_{11}^* versus Dual –Ratio estimator t_2 by [7]

$$MSE(t_{11}^*) - MSE(t_2) < 0 \tag{45}$$

$$\frac{2}{H_1^*}\left(\frac{\rho_{yx}C_y}{gC_x}-1\right) < \alpha_1 < 0 \tag{46}$$

For the range of α_1 given in (46) the modified estimator t_{11}^* is better than t_2

(iv) t_{11}^* versus estimator t_3 suggested by [13]

$$MSE(t_{11}^*) - MSE(t_3) < 0 \tag{47}$$

$$\frac{1}{gH_1^*}(1-\delta H_1 - g) < \alpha_1 < -\frac{1}{gH_1^*}\left(1+g-\delta H_1-\frac{2\rho_{yx}C_y}{C_x}\right) \tag{48}$$

For the range of α_1 given in (46) the modified estimator t_{11}^* is better than t_3

2. Modified Estimator t_{12}^* over some Existing Related Estimators

(i) t_{12}^* versus Sample mean t_0

$$MSE(t_{12}^*) - MSE(t_0) < 0 \tag{42}$$

$$\frac{1}{H_1^*} < \alpha_2 < -\frac{1}{H_1^*}\left(\frac{2\rho_{yx}C_y}{gC_x}-1\right) \tag{43}$$

For the interval of α_2 given in (43) the modified estimator t_{12}^* is better than t_0

(ii) t_{12}^* versus Ratio estimator t_1

$$MSE(t_{12}^*) - MSE(t_1) < 0 \tag{44}$$

$$\frac{1}{gH_1^*}\left(1+g-\frac{2\rho_{yx}C_y}{C_x}\right) < \alpha_2 < -\frac{1}{gH_1^*}(1-g) \tag{45}$$

For the interval of α_2 given in (45) the modified estimator t_{12}^* is better than t_1

(iii) t_{12}^* versus Dual –Ratio estimator t_2 by [7]

$$MSE(t_{12}^*) - MSE(t_2) < 0 \tag{46}$$

$$0 < \alpha_2 < -\frac{2}{H_1^*}\left(\frac{\rho_{yx}C_y}{gC_x}-1\right) \tag{47}$$

For the interval of α_2 given in (47) the modified estimator t_{12}^* is better than t_2

(vi) t_{12}^* versus estimator t_4 suggested by [13]

$$MSE(t_{12}^*) - MSE(t_4) < 0 \tag{48}$$

$$\frac{1}{gH_1^*}\left(1+g+\delta H_1-\frac{2\rho_{yx}C_y}{C_x}\right) < \alpha_2 < -\frac{1}{gH_1^*}(1-g+\delta H_1) \tag{49}$$

For the interval of α_2 given in (49) the modified estimator t_{12}^* is better than t_4

VII. EMPIRICAL STUDY

To see the performance of the suggested modified classes of estimators t_{11}^* and t_{12}^* over the estimators t_0 , t_1, t_2 , t_3 and t_4 , we consider two (2) natural population datasets. The description of the populations are given in Table 1.

Table 1: Description of the population's parameters

Parameters	Population I	Population II
	Source: [15], Page 186 Y:Total no. of Member X:No. of Children	Source: [3], Page 228 Y:Output of 80 factories X:Number of workers
N	21	80
n	5	20
$\overline{Y}$	3.810	51.8264
$\overline{X}$	1.714	2.8513
C_y	0.3406	0.3542
C_x	0.7232	0.9484
ρ_{yx}	0.974	0.915
H_1^*	-0.3125	-0.3333
δ_1	0.5412912	0.6582739
δ_2	-0.5412912	-0.6582739

Table 2: Ranges of α_1 for which the modified class of estimator t_{11}^* is better than t_0 ,t_1, t_2 and t_3.

POPULATIONS	I	II
t_0	$(-6.194 < \alpha_1 < 3.2)$	$(-3.151 < \alpha_1 < 3.00)$
t_1	$(-7.037 < \alpha_1 < 4.046)$	$(-6.00 < \alpha_1 < 5.849)$
t_2	$(-2.994 < \alpha_1 < 0)$	$(-0.151 < \alpha_1 < 0)$
t_3	$(-5.417 < \alpha_1 < 2.422)$	$(-0.565 < \alpha_1 < 0.414)$
α_1^{opt}	**-1.497**	**-0.0756**

Table 3: Ranges of α_2 for which the modified class of estimator t_{12}^* is better than t_0 ,t_1, t_2 and t_4.

POPULATIONS	I	II
t_0	$(-3.2 < \alpha_2 < 6.194)$	$(-3.00 < \alpha_2 < 3.151)$
t_1	$(-4.046 < \alpha_2 < 7.037)$	$(-5.849 < \alpha_2 < 6.00)$
t_2	$(0 < \alpha_2 < 2.994)$	$(0 < \alpha_2 < 0.151)$
t_4	$(-2.422 < \alpha_2 < 5.417)$	$(-0.414 < \alpha_2 < 0.565)$
α_2^{opt}	**1.497**	**0.0756**

Table 2 and 3 provides the wide ranges of α_1 and α_2 along with its optimum values for which the suggested modified estimators t_{11}^* and t_{12}^* are more efficient than other estimators t_0 ,t_1, t_2 , t_3 and t_4 .The results of table 2 and 3 also indicates that there is enough scope for chosen the value of α_1 and α_2 to obtain better estimators than usual unbiased estimator , ratio estimator , dual to ratio estimator and estimators t_3 and t_4 suggested by [13].

Table 4: Bias, MSE and PRE of t_{11}^*, t_{12}^* and some Related Existing Ratio Estimators using Data I and II

Estimators	**Data 1**			**Data II**		
	Bias	**MSE**	**PRE**	**Bias**	**MSE**	**PRE**
t_0	0	0.2566	100	0	12.6366	100
t_1	0.1644	0.3521	72.87	1.1507	41.3151	30.59
t_2	-0.0435	0.0379	677.03	-0.1991	2.0633	612.44
t_3	0.0377	0.0132	1948.41	0.1966	2.0569	614.35
t_4	0.1644	0.0132	1948.41	1.1507	2.0569	614.35
Modified t_{11}^*	0.0150	0.0132	1948.41	0.0041	2.0569	614.35
Modified t_{12}^*	-0.0435	0.0132	1948.41	-0.1991	2.0569	614.35

Table 4.4 shows the Biases, MSEs and PREs of the modified estimators with some existing related ratio estimators considered in the study using Data I, and II. It is observed that all the estimators with exception of sample mean t_0 are not unbiased. The results also revealed that estimators t_{11}^*, t_{12}^* and estimators t_3 and t_4 by [13] have the least MSEs and highest PREs among other estimators considered in the study.

Efficiency comparison of Modified Estimators t_{11}^*, t_{12}^* with [13] Ratio Estimators (t_3 and t_4)

Table 5: MSEs of t_{11}^* and t_{12}^* with [13] Estimators t_3 and t_4 using Data I

Estimators	Ranges of Weight(α)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
t_3	0.2385	0.1479	0.0805	0.0363	0.0151	0.0172	0.0423	0.0906	0.1620
t_4	0.4889	0.6489	0.8320	1.0382	1.2675	1.5200	1.7957	2.0944	2.4163
t_{11}^*	0.0413	0.0449	0.0488	0.2169	0.0572	0.0617	0.0664	0.0714	0.0766
t_{12}^*	0.0347	0.0317	0.0290	0.0529	0.0241	0.0220	0.0202	0.0185	0.0171

Table 6: MSEs of t_{11}^* and t_{12}^* with [13] Estimators t_3 and t_4 using Data II

Estimators	Ranges of Weight(α)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
t_3	30.2934	21.0838	13.6860	8.1003	4.32645	2.36458	2.21466	3.8767	7.3507
t_4	54.1486	68.7942	85.2517	103.5211	123.6025	145.4958	169.2011	194.7183	222.0475
t_{11}^*	2.0914	2.1418	2.21466	2.3099	2.42741	2.5673	2.7296	2.9143	3.1214
t_{12}^*	2.0576	2.0743	2.1133	2.1747	2.2584	2.3646	2.4931	2.6440	2.8172

Tables 5 and 6 exhibits that that the modified estimator t_{12}^* has minimum MSE compared to t_4 for all the values of weight used in all the two datasets. The results also revealed that there is considerable reduction in the variance of the suggested modified estimator t_{11}^* from that of the estimator t_3, if the values of α; (0.1, 0.2, 0.3, 0.8 and 0.9) and (0.1, 0.2, 0.3, 0.4, 0.5, 0.8, 0.9) are used for all the two datasets respectively.

Table 7: PRE of t_{11}^* , t_{12}^* and [13] Estimators t_3 and t_4 using Data I

Estimators	Ranges of Weight(α)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
t_3	107.61	173.46	318.65	707.56	1694.60	1495.57	606.47	283.22	158.36
t_4	52.48	39.54	30.84	24.72	20.24	16.88	14.29	12.25	10.62
t_{11}^*	621.08	570.86	525.76	485.24	448.77	415.91	386.25	359.42	335.11
t_{12}^*	739.32	808.57	885.35	970.08	1062.93	1163.68	1271.48	1384.60	1500.20

Table 8: PRE of t_{11}^* and t_{12}^* with [13] Estimators t_3 and t_4 using Data II

Estimators	Ranges of Weight (α)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
t_3	41.714	59.94	92.33	156.00	292.08	534.41	570.59	325.96	171.91
t_4	23.34	18.37	14.82	12.21	10.22	8.69	7.47	6.49	5.69
t_{11}^*	604.22	589.99	570.59	547.07	520.58	492.21	462.94	433.61	404.84
t_{12}^*	614.15	609.21	597.96	581.08	559.53	534.41	506.87	477.94	448.55

Table 7 and 8 provides that larger gain in efficiency can be observed by using the modified class of estimator t_{12}^* over t_4 for all the values of α **.** Furthermore, the results also revealed that there is considerable gain in the efficiency of the suggested modified estimator t_{11}^* from that of the estimator t_3, for the ranges of α; $\alpha < 0.4$ and $\alpha > 0.7$ for Data I and $\alpha < 0.6$ and $\alpha > 0.7$ for Data II. This implies that even if the scalar α deviates from its exact optimum value (α_{opt}), the modified estimators t_{11}^* and t_{12}^* will still yield better estimators than estimators t_3 and t_4 suggested by [13].

VIII. CONCLUSION

From the efficiency comparison in section 6 and the empirical results of percent relative efficiency of the suggested classes of estimators in section 7, the suggested modified estimators t_{11}^* and t_{12}^* are more efficient than usual unbiased estimator t_0, ratio estimator t_1, dual to ratio estimator t_2 and estimators t_3 and t_4 suggested by [13] under the stipulated conditions and can produce better estimate if their respective optimum values of unknown weight are utilized.

REFERENCES

[1] W. G. Cochran, "The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce," *The Journal of Agricultural Science*, vol. 30, no. 2, pp. 262–275, (1940).

[2] D. S. Robson, "Applications of Multivariate polykays to the theory of unbiased Ratio-Type Estimation", *Journal of the American Statistical Association,* 52(280): 511-522, (1957).

[3] M. N. Murthy, "Product method of estimation", *The Indian Journal of Statistics,* Series A, 26, 69-74, (1964).

[4] B.V.S. Sisodia and V.K. Dwivedi, "A modified ratio estimator using coefficient of variation of auxiliary variable", *Journal of Indian Society of Agricultural Statistics,* 33(1): 13-18, (1981).

[5] L. Upadhyaya and H.P. Singh, "Use of transformed auxiliary variable in estimating the finite Population Mean", *Biometrical Journal*, 5: 627-636, (1999).

[6] H.P. Singh and R. Tailor, "Use of known Correlation Coefficient in Estimating the Finite Population Mean", *Statistics in Transition,* 6, 555-560, (2003).

[7] T. Srivenkataramana, "A dual to ratio estimator in sample surveys", *Biometrika,* 67(1):199-204, (1980).

[8] S. Bandyopadhyaya, "Improved ratio and product estimators", *Sankhya,* Series C, 42(2), 45-49, (1980).

[9] B. Sharma and R. Tailor, "A New Ratio-Cum-Dual to Ratio Estimator of Finite Population Mean in Simple Random Sampling", *Global Journal of Science Frontier Research,* Vol. 10, Issue1 (Ver 1.0), 27-31, (2010).

[10] H. P. Singh, R. Singh, M. R. Espejo, M. D. Pineda and S. Nadarajan, "On the efficiency of the dual to ratio-cum product estimator, Mathematical Proceedings of the Royal Irish Academy **105 A** (2) 51-56, (2005).

[11] S. Chaudhary and B. K. Singh, "An Efficient Class of Dual to Product-Cum-Dual to Ratio Estimator of Finite Population Mean in Sample Surveys", *Global Journal of Science Frontier Research*, 12(3), 25-33, (2012).

[12] R. Rashid, M. Noor ul Amin and M. Hanif, Exponential Estimators for Population Mean Using the Transformed Auxiliary Variables. *Applied Mathematics and Information Science,* Vol. 9, No. 4, 2107-2112, (2015).

[13] A.K.P. Swain and M. Das, "Some classes of modified ratio type estimators in sample surveys", *Statistics in Transition new series*, Vol. 16, No. 1, pp. 37–52, (2015).

[14] S.K. Srivastava, Generalized Estimator for Mean of a Finite Population using Multi-Auxiliary Information, *Journal of the American Statistical Association,* 66: 404−407, (1971).

[15] W.G. Cochran, "Sampling Techniques", Third Edition. John Wiley and Sons, New York, (1977).