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HEIGHT –DIAMETER MODELING OF *KHAYA SENEGALENSIS* (AFRICAN MAHOGANY)

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Abstract

The aim of this study is to examine the factors contributing to the height of Khaya Senegalensis tree using available tree characteristics which could be obtained by examining the adequacy of various H-D models in predicting height of Khaya Senegalensis. It also to investigate the adequacy of the addition of the breast height value of 1.3 meters in the tree models. The data used for this research work is a secondary data collected from the Forest Research Institute of Nigeria, Ibadan, Oyo state. The data contains; Height of the tree in meter (m), Debarked (l), Diameter at Breast Height (DBH), Diameter Above (m), Crown Height (m), Crown Length (m) and Crown Breadth (m). The methodology consists of both linear Regression model and non-linear regression models like Exponential model, Monomolecular model and Gompertz model. The results of the analysis shows that models with 1.3 constant value yielded the best fit having high R^2 and adjusted R^2 compared to those without 1.3. They also yielded low AIC, MSE, and BIC as compared to those without 1.3. A strong relationship between height and the corresponding independent variables was established.

***Keywords*: *Khaya Senegalengsis*- African Mahogany, Linear and non- Linear Regression Model, AIC- Akaike Information Criteria, MSE- Mean Square Error and BIC-Bayesian Information Criteria.**

1.0 Introduction

The relationship between Man, environment and Tree is very important for the survival of the Eco system. Presence of trees enhances the removal of Carbon dioxide from the atmosphere, which is called carbon sequestration, as Trees absorb the carbon dioxide being breath out by human being, and gives us Oxygen that we need. On the other hand, several disservices of urban trees exist, like litter fall, damage to foot paths, and fallen branches which cause additional costs and can endanger the public's safety. Due to these benefits and disadvantages, the growth of trees and space requirements are of high interest for planners in the urban areas.The Height of a tree is the most essential variable in forest inventories as they are frequently required for both routine forest economic management activities and for research purposes. As such, accurate data on this variable is highly important, thus the use of Height Diameter to predict tree height is more cost effective, easy and accurate than the traditional method. This study will also serve as an eye opener for the researchers and research institute in knowing the best model to be used in measuring height diameter of a tree.

This study examined sixteen tree height-Diameter (HD) models with eight of them being with or without the "1.3m" constants as noted from literature. Their ability to predict the height of trees using their corresponding independent variables was assessed with a view to identify the model with the highest potential. Trees show considerably variation and flexibility in their shape and size of crowns, height, and trunk diameters [1, 2]. These are governed by an inherited developmental tendency, which may in turn be modified by the environment where the tree grows. The size of a tree canopy and its height above the ground is significant to a tree because it determines the total amount of light that the tree intercepts for photosynthesis [3, 4]. Natural selection must generally be expected to favour trees that increase the amount of light that falls on the plant and since competition for light is often important in groups of trees. In the same respect, natural selection must tend to favour trees that grow high quickly [5].It has been shown, through a mathematical model that the higher a tree is, the

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more light it intercepts during the course of the day[6]. The tree trunk size also has its own adaptive significance to a tree. It must be strong enough to withstand the forces that act on it. These forces are the weight of the tree and the drag exerted on it by the wind, as demonstrated by [7]. Experimentally, wind has been found to be much more important than weight in determining what thickness of trunk is necessary for a tree. [8].

2.0 Methodology

Consider a general form of a non-linear regression model:

$Hi = f(Di, B) + e_i, i = 1, 2,,……,n$ (1)

Where H is the dependent or response variable, D is the independent or decision variable(s), B is a vector of the parameters βj to be estimated (β1, β2, …..βp) and e_i is the random error term. The estimator of are found by minimizing the sum of squares residual (SSR) function:

$\sum (Hi - f(Di, B))^2$ (2)

An iterative method will be employed to minimize the sum of square residual (SSR)

Since Hi and Di are fixed observations, the sum of squares residual is a function of B; these normal equations take the form of:

$\sum H_i - f(D_i, B)\left[\frac{df(Di,B)}{d3j}\right] = 0$ (3)

Height- Diameter Tree Models

Table 1 Height Diameter Models

S/N	**Model**	**Models**	**Model Name**
1.	Mod 1	$H = 1.3 + \beta o + \beta_1$ DBH	Linear (simple)
2.	Mod 2	$H=1.3+\beta_0+\beta_1 Debark+\beta_2 Dbh+\beta_3 Dabove+\beta_4 Crh+\beta_5 CrL+\beta_6 CrB$	Linear (multiple)
3.	Mod 3	$H = 1.3 + \beta_0 e^{\beta 1 DBH}$	Exponential (simple)
4.	Mod 4	$H = 1.3 + \beta_0 e^{\beta 1 debarked + \beta 2 Dbh + \beta 3 Dabove + \beta 4 CrH + \beta 5 CrL + \beta 6 CrB}$	Exponential (multiple)
5.	Mod 5	$H = 1.3 + K - \beta 0 e^{-\beta 1 DBH}$	Monomolecular (simple)
6.	Mod 6	$H = 1.3 + K - \beta 0 e^{-\beta 1 debarked + \beta 2 Dbh + \beta 3 D-above + \beta 4 CrH + \beta 5 CrL + \beta 6 CrB}$	Monomolecular (multiple)
7.	Mod 7	$H = 1.3 + Ke^{-\beta 0 e^{-\beta 1 DBH}}$	Gompertz (simple)
8.	Mod 8	$H = 1.3 + Ke^{-\beta 0 e^{-\beta 1 Debarked + \beta 2 Dbh + \beta 3 D-above + \beta 4 CrH + \beta 5 CrL + \beta 6 CrB}}$	Gompertz (multiple)

H is the total tree height(m), K is maximum assymptom,β1 ,β2, β3, β4, β5 and β6 are parameter to be estimated, e is the base of natural logarithm (2.71828); 1.3 is a constant used to account that DBH is measured at 1.3 above the ground.

We considered the same height-Diameter functions listed above without the "1.3" part of the functions. The 1.3 is a constant used in indicating the fact that DBH was measured at 1.3m above the ground. This is done to investigate the adequacy of the addition of the '1.3' to the model. These functions still have two or more parameters.

Table 2 Adjusted Height-Diameter Models

S/N	**Model**	**Models**	**Reference**
1.	Mod 1	$H = \beta o + \beta_1 debarked$	Linear (simple)
2.	Mod 2	$H= \beta_0+\beta_1 Debark+\beta_2 Dbh+\beta_3 Dabove+\beta_4 Crh+\beta_5 CrL+\beta_6 CrB$	Linear (multiple)
3.	Mod 3	$H = \beta_0 e^{\beta 1 DBH}$	Exponential (simple)
4.	Mod 4	$H = \beta_0 e^{\beta 1 debarked + \beta 2 Dbh + \beta 3 Dabove + \beta 4 CrH + \beta 5 CrL + \beta 6 CrB}$	Exponential (multiple)
5.	Mod 5	$H = K - \beta 0 e^{-\beta 1 DBH}$	Monomolecular (simple)
6.	Mod 6	$H = K - \beta_0 e^{-\beta 1 debarked + \beta 2 Dbh + \beta 3 D-above + \beta 4 CrH + \beta 5 CrL + \beta 6 CrB}$	Monomolecular (multiple)
7.	Mod 7	$H = Ke^{-\beta 0 e^{-\beta 1 DBH}}$	Gompertz (simple)
8.	Mod 8	$H = Ke^{-\beta 0 e^{-\beta 1 Debarked + \beta 2 Dbh + \beta 3 D-above + \beta 4 CrH + \beta 5 CrL + \beta 6 CrB}}$	Gompertz (multiple)

H is the total tree height (m), K is maximum assymptom,β1 ,β2, β3, β4, β5 and β6 are parameter to be estimated, e is the base of natural logarithm (2.71828).

Models Selection Criterion

The following criteria will be considered in this study in determining the most appropriate model for fitting tree height and DBH

(i) Coefficient of determination R^2
(ii) Adjusted R^2
(iii) Mean Square Error(MSE)

(iv) Akaike Information Criterion (AIC)
(v) Bayesian Information Criterion (BIC) and
(vi) Residual Standard Error (RSE)

3.0 Result and Discussions

Scatter Plots Illustration of the variables considered.

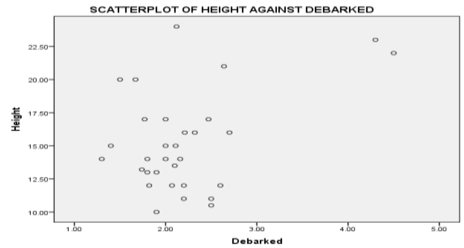


Figure 1: Scatter plot of the data set

Figure 1-6 are the scatter plot of the tree height against each of the independent variables. The scatter plot shows the relationship between the tree height and the factors. Thus some of the scatter plots show linear relationship while some does not.

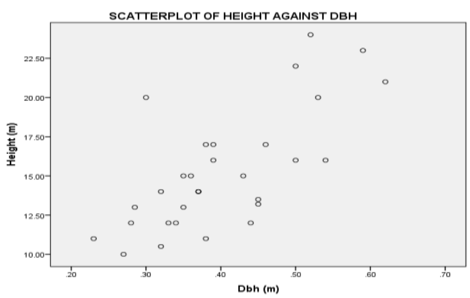


Fig 2 Scatter plots of the data set

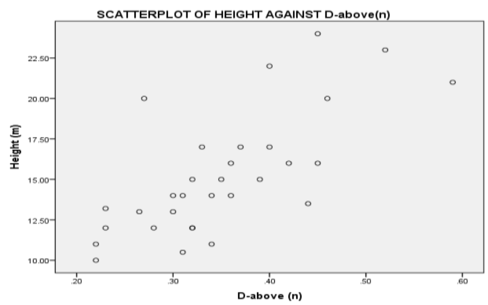


Fig 3: Scatter plots of the data set

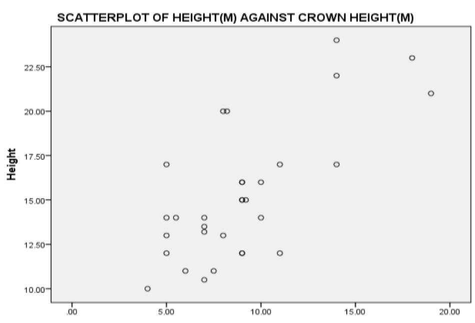


Fig 4. Scatter plots of the data set

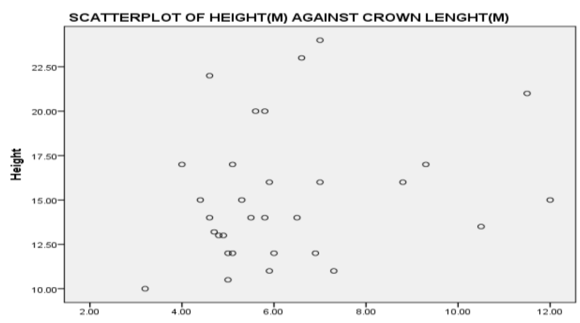


Fig 5. Scatter plots of the data set

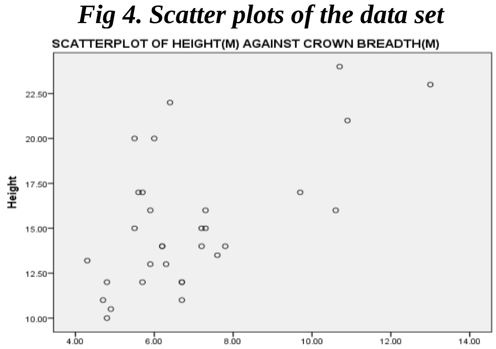


Fig 6. Scatter plots of the data set

Table 3: The Results from the Various Models considered with constant 1.3

Models	Parameters								ADJ						No of iterations
	A	B	C	D	E	F	G	Total	R^2	$\bar{R}^2$	RSS	MSE	AIC	BIC	
Model 1	2.881	27.675	-	-	-	-	-	2	0.509	0.4921	207.953	7.171	169.4567	188.5997	2
Model 2*	**5.175**	**-0.706**	**8.968**	**13.749**	**0.483**	**-0.565**	**0.163**	**7**	**0.658**	**0.5725**	**145.06**	**6.044**	**168.2916**	**178.395**	**2**
Model 3	6.323	1.934	-	-	-	-	-	2	0.523	0.5066	201.939	6.963	168.5469	187.8159	9
Model 4*	**7.901**	**-0.058**	**0.877**	**0.878**	**0.031**	**-0.049**	**0.007**	**7**	**0.665**	**0.5813**	**141.83**	**5.91**	**167.5935**	**177.6314**	**22**
Model 5	28.379	2.665	-	-	-	-	-	2	0.471	0.4528	223.939	7.722	171.7526	189.7719	8
Model 6	**24.389**	**-0.08**	**0.582**	**1.411**	**0.056**	**-0.048**	**0.025**	**7**	**0.621**	**0.5262**	**160.616**	**6.692**	**171.4495**	**181.4874**	**17**
Model 7	2.305	3.674	-	-	-	-	-	2	0.485	0.4672	217.958	7.516	170.9134	189.4379	7
Model 8*	**1.883**	**-0.101**	**0.894**	**2.109**	**0.073**	**-0.067**	**0.029**	**7**	**0.636**	**0.545**	**153.973**	**6.416**	**170.14**	**180.178**	**8**

The best 4 models are in bold while the asterisk model is the best model

In the above table 3 (models with 1.3. constant). The models that perform best are model 2 (Linear multiple regression), model 4 (multiple exponential), model 6 (multiple monomolecular) and model 8 (multiple Gompertz). The fitted models were found to give satisfactory results with high R^2 and Adjusted R^2 as well as low AIC, BIC and MSE.

Table 4: The results from the various Models considered without constant 1.3

Models	Parameters								ADJ						No of iterations
	A	B	C	D	E	F	G	Total	R^2	$\bar{R}^2$	RSS	MSE	AIC	BIC	
Model 1	10.266	2.269	-	-	-	-	-	2	0.17	0.1414	351.536	12.122	185.7317	188.5997	2
Model 2	**6.475**	**-0.706**	**8.968**	**13.749**	**0.483**	**-0.565**	**0.163**	**7**	**0.658**	**0.5725**	**145.06**	**6.044**	**168.2916**	**178.395**	2
Model 3	10.975	0.147	-	-	-	-	-	2	0.189	0.161	343.43	11.842	185.0085	187.8765	11
Model 4*	**9.022**	**-0.052**	**0.784**	**0.811**	**0.028**	**-0.044**	**0.007**	**7**	**0.665**	**0.5813**	**142.014**	**5.917**	**167.6337**	**177.6716**	**9**
Model 5	14.376	0.230	-	-	-	-	-	2	0.134	0.1041	366.959	12.654	187.0628	189.9307	15
Model 6	**23.813**	**-0.096**	**0.634**	**1.604**	**0.065**	**-0.055**	**0.030**	**7**	**0.612**	**0.515**	**164.330**	**6.847**	**172.1582**	**182.1916**	**18**
Model 7	0.873	0.301	-	-	-	-	-	2	0.142	0.1124	363.628	12.539	186.7801	189.648	13
Model 8	**1.735**	**-0.116**	**0.915**	**2.189**	**0.082**	**-0.072**	**0.033**	**7**	**0.627**	**0.5338**	**158.138**	**6.589**	**170.968**	**181.0054**	**11**

The best 4 models are in bold while the asterisk model is the best model

	Model 1		**Model 2**		**Model 3**		**Model 4**		**Model 5**		**Model 6**		**Model 7**		**Model 8**	
	A	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**		**A**	**B**
R^2	0.509	0.17	0.658	0.658	0.523	0.189	0.665	0.665	0.471	0.134	0.621	0.612	0.485	0.142	0.636	0.627
$\bar{R}^2$	0.9994	0.1414	0.5725	0.5725	0.99945	0.161	0.5813	0.5813	0.99939	0.1041	0.526	0.515	0.9994	0.1124	0.545	0.5338
MSE	7.171	12.122	6.044	6.044	6.963	11.842	5.910	5.917	7.722	12.654	6.692	6.847	7.516	12.539	6.416	6.589
AIC	169.4567	185.732	168.292	168.292	168.5469	185.009	167.594	167.634	171.67	187.063	171.45	172.16	170.91	186.780	170.14	170.968
BIC	188.599	188.599	178.395	178.395	187.816	187.877	177.631	177.672	189.77	189.930	181.49	182.19	189.437	189.648	180.178	181.005

Table 5: Table Showing the Comparison between Model with and without 1.3 Constant
AIC, BIC and MSE comparison

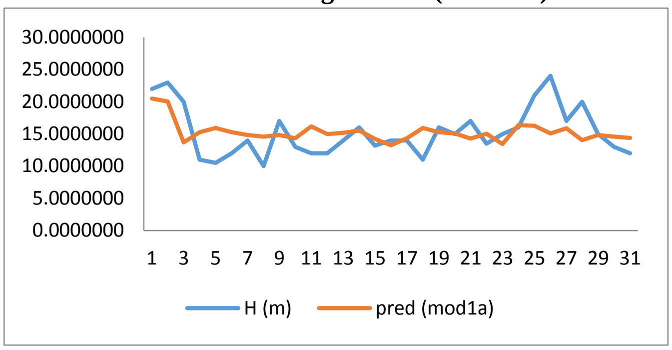
A is the model with '1.3' constant; while B is the model without '1.3' constant

In table 4 without 1.3 above, the models that perform best are model 2 (Linear multiple regression), model 4 (multiple exponential), model 6 (multiple monomolecular) and model 8 (multiple Gompertz). The fitted models were found to give satisfactory results with high R^2 and Adjusted R^2 as well as low AIC, BIC and MSE.

In table 5 above, it can be clearly seen that the models with 1.3 constant value yielded the best fit having high R^2 and adjusted R^2 compared to those without 1.3, and also low AIC, MSE, and BIC compared to those without 1.3, except in the models 1 and 2 having the same R^2, adjusted R^2, AIC, MSE and BIC. In general terms, half of the fitted models were found to give satisfactory results with high R^2 and Adjusted R^2 as well as low AIC, BIC and MSE.

Graphical Illustration of Observed Height and Predicted Height Relationship

Observed Vs Predicted Using Model 1(With 1.3)



Observed Vs Predicted Using Model 1(Without 1.3)

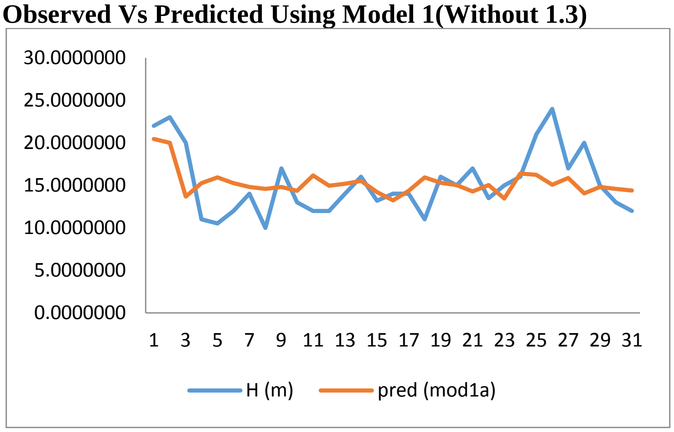
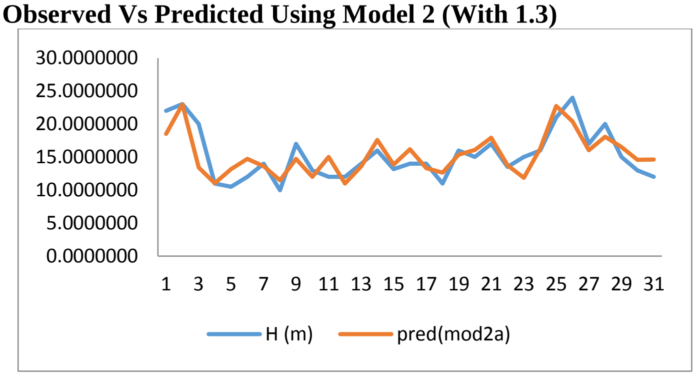


Fig 7 Observed vs Predicted using model 1with and without 1.3

Observed Vs Predicted Using Model 2 (With 1.3)



Observed Vs Predicted Using Model 2(Without 1.3)

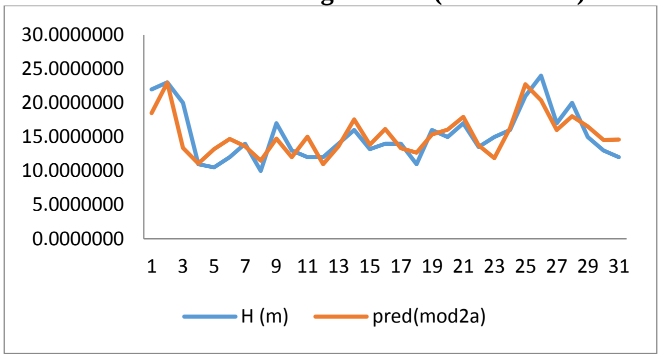
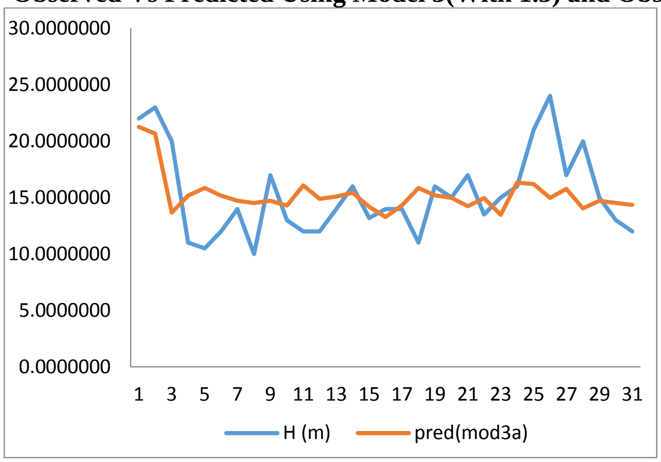


Fig 8: Observed vs Predicted using model 2 with and without 1.3

Observed Vs Predicted Using Model 3(With 1.3) and Observed Vs Predicted Using Model 3(Without 1.3)



Predicted Using Model 3(Without 1.3)

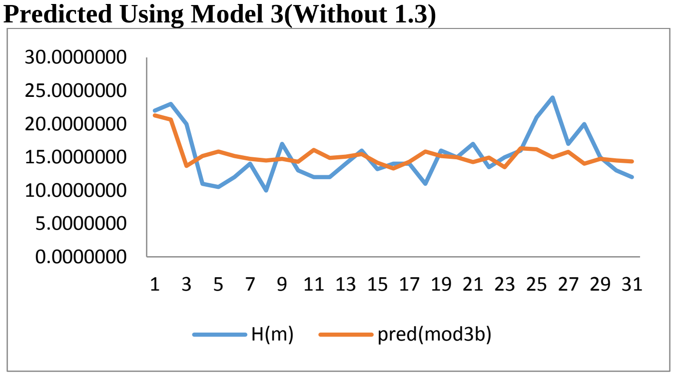
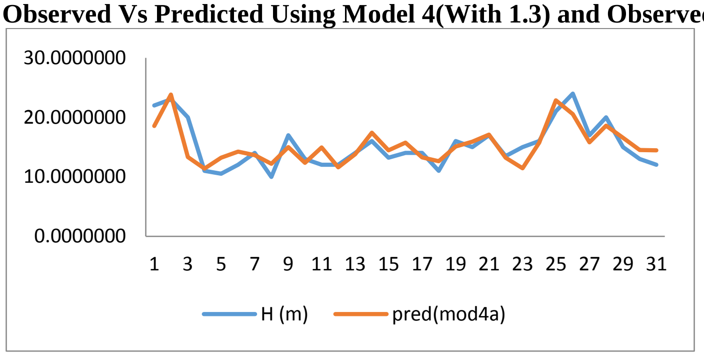


Fig 9. Observed vs Predicted using model 3 with and without 1.3

Observed Vs Predicted Using Model 4(With 1.3) and Observed Vs Predicted Using Model 4(Without 1.3)



Predicted Using Model 4(Without 1.3)

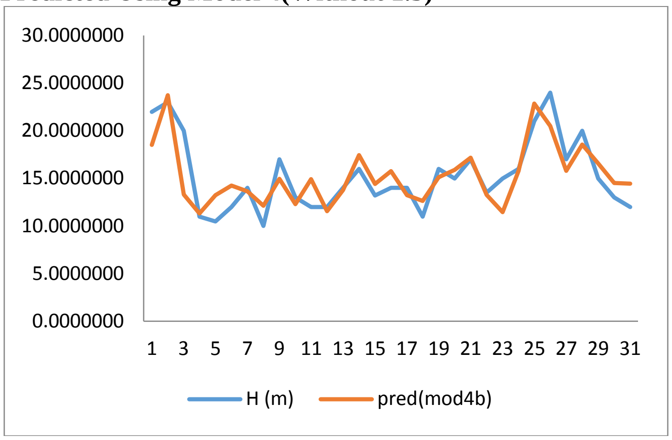
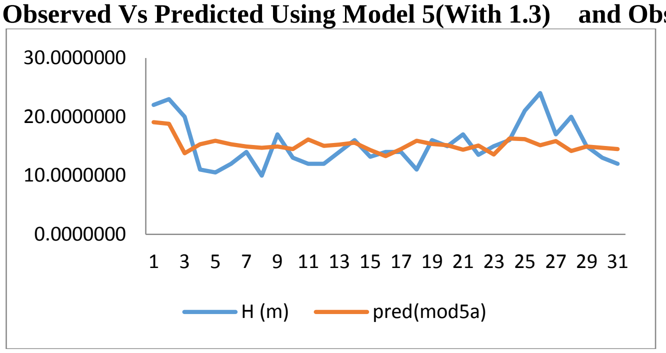


Fig 10 Observed vs Predicted using model 4 with and without 1.3

Observed Vs Predicted Using Model 5(With 1.3) and Observed Vs Predicted Using Model 5(Without 1.3)



Predicted Using Model 5(Without 1.3)

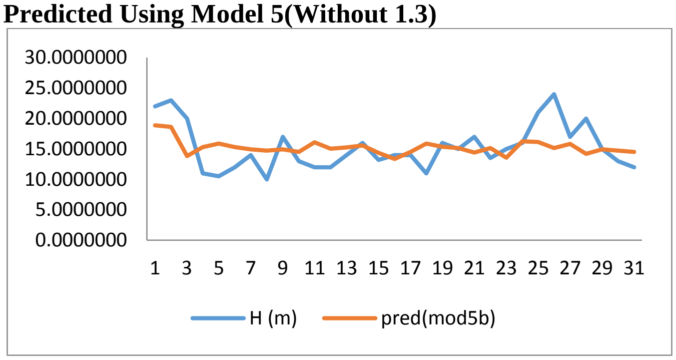
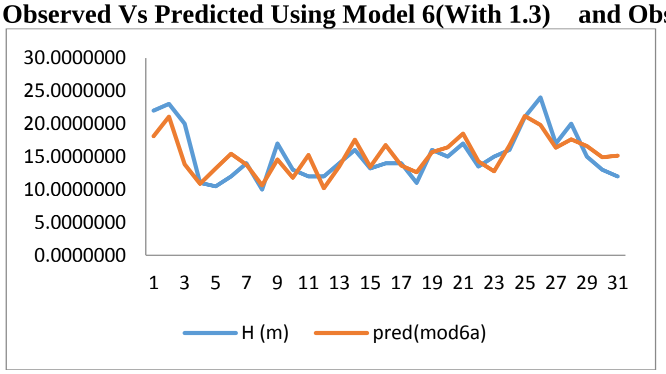


Fig 11 Observed vs Predicted using model 5 with and without 1.3

Observed Vs Predicted Using Model 6(With 1.3) and Observed Vs Predicted Using Model 6(Without 1.3)



Predicted Using Model 6(Without 1.3)

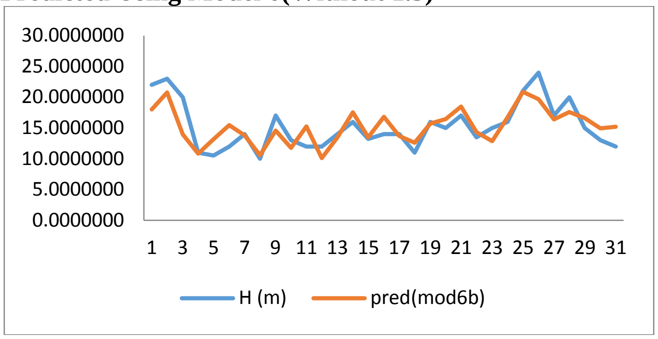
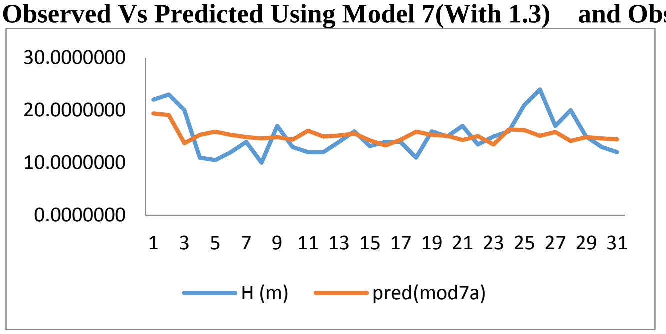


Fig 12 Observed vs Predicted using model 6 with and without 1.3

Observed Vs Predicted Using Model 7(With 1.3) and Observed Vs Predicted Using Model 7(Without 1.3)



Predicted Using Model 7(Without 1.3)

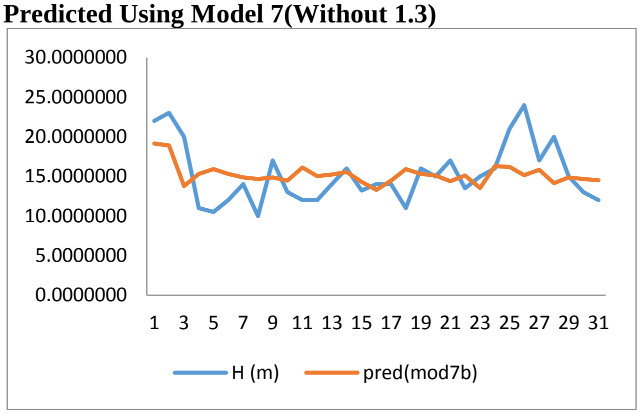
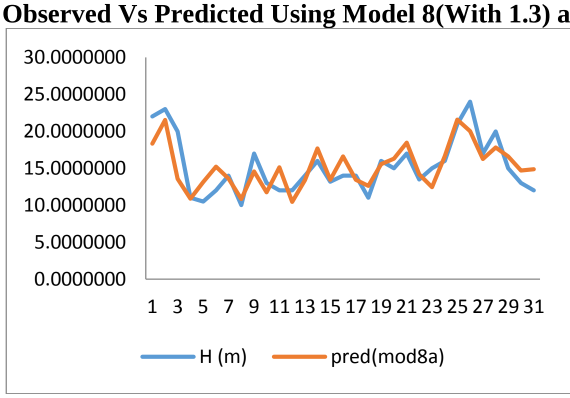


Fig 13 Observed vs Predicted using model 7 with and without 1.3

Observed Vs Predicted Using Model 8(With 1.3) and Observed Vs Predicted Using Model 8(Without 1.3)



Predicted Using Model 8(Without 1.3)

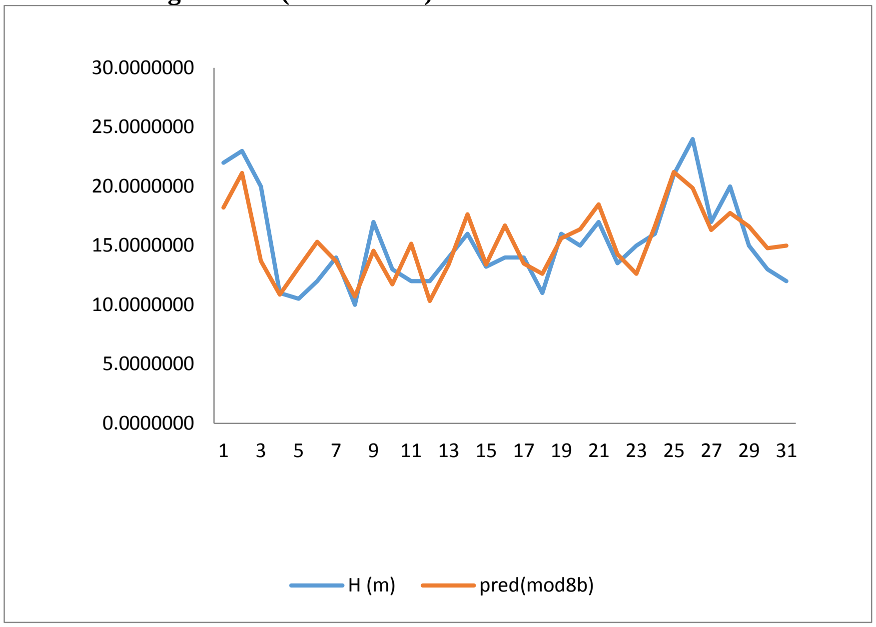


Fig 14 Observed vs Predicted using model 8 with and without 1.3

Conclusion

The study revealed clearly that models with 1.3 constant value yielded the best fit having high R^2 and adjusted R^2 compared to those without 1.3. This model with 1.3 constant also yielded low AIC, MSE, and BIC compared to those without 1.3. There is also a strong relationship between height of the tree and the corresponding independent variables.

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