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CONCEPTUAL QUANTITATIVE MODELS ANALYSIS OF SOLAR DRYING EQUATION BASED ON THE PRINCIPLE OF HALF LIFE-1

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Abstract

In this study of half life model of solar drying equation, experiments were conducted to investigate drying rates of solar photovoltaic and thermal dryer using cassava chips. The experimental data obtained were used to model the drying rate quantitatively using the principle of half-life. The result enumerated the similarities and dereferences between radioactive half life, Solar drying half life, the associated theory. The Model can serve as veritable tool towards comparing different dryer devices hence their efficiencies. This model also allows us to directly calculate quantitatively the remaining wet or dry hence the efficiencies of devices at various life spans ($\frac{1}{4}, \frac{1}{2},, \frac{3}{4}$ life span and so on), when subjected to the same operating parameters thereby solving easily one of the deficiencies of some other drying equation and Models.

1.0 INTRODUCTION.

Food shortage in developing countries does not arise entirely from low agricultural production but partly due to inability to preserve available agricultural products. Farmers in developing economy usually dry agricultural products using solar energy, which can be harnessed in thermal or photovoltaic (PV) forms. Drying process is influenced by many factors including environmental parameters like temperature, rainfall, humidity, wind speed and pressure; equipment employed, type and properties of the materials being dried, such as moisture content and surface area among others. Many solar drying equations have been constructed on solar drying rates in the past but most of them were based on Univarate analysis which takes into consideration only Moisture Ratio (MR) and time (t).

Many experiments has been carried out over the years regarding Solar drying equations. It is on record that quitea numbers of these equations were based univarate analysis, taking into consideration only moisture ratio (MR) and time (T). In [1]experiment was conducted on mathematical modelling of solar drying curves by fitting nine drying curves equations into eight different models equations 1 to 8

Model name Model		References	
Newton $MR = exp^{-kt}$	(1)	Mujumdar, (1987)	[2]
Page $MR = exp^{-ktn}$	(2)	Diamante and Murro (1993)	[3]
Modified page $MR = exp^{(-kt)^n}$	(3)	White *et al* (1978.)	[4]
Modified page MR = exp[−(kt)^n]	(4)	Over hults*et al.*, (1972)	[5]
Henderson and Pabis $MR = aexp^{-kt}$	(5)	Zhang and Litchfield (1991)	[6]
Logarithmic $MR = exp^{(-ktA)+c}$	(6)	agcioglu *et al.*, (1990)	[7]
Two term $MR = aexp^{-k0t} +$ $MR = bexp^{-k1t}$	(7)	Henderson S.M (1974)	[8]
Wang and Singh $MR = 1 + at + bt^2$	(8)	Wang and Singh.,(1978)	[9]

In [10]experiment was carried out on Experimental Investigation of Drying Rates of Solar Photovoltaic and Thermal Dryers using Univariate analysis .In another experiment on Experimental Investigation of Drying Rates of Solar Photovoltaic and Thermal dryers using multivariate analysis, where various aspects of weather parameters like. Humidity, Temperature, Wind

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Speed and Moisture content that influence drying rates of agriculture commodities were directly taken into consideration in the derivation of the solar drying equation (Akinboro *et al.,* 2016)b[11]

In this studied attempt has also been made for the first time to analyse quantitatively Solar drying equation using the principle of Half Life .

,The moisture content was converted to moisture ratio (*MR*) using the following equation**:**

$MR = \frac{(M_t - M_0)}{(M_0 - M_e)}$(9)

Where M, M_0, M_e, M_t are the moisture content (kg water/kg dry matter) at a given time, beginning of the drying, when the equilibrium is reached and at time t (min), respectively.

The moisture ratio$MR = \frac{M_t}{M_o}$may be taken instead of $MR = \frac{(M_t - M_0)}{(M_0 - M_e)}$for mathematical modelling of the solar drying curves because of the continuous fluctuation of the relative humidity of the drying air during both the forced and natural solar drying processes, Diamante and Munro, 1993; Yaldiz, [12]2001 and Sarsavadia (2007).[13]

MODEL 1

Table1. SIMILARITIES AND DIFFRENCES BETWEEN NUCLEAR HALF LIFE AND SOLAR DRYING HALF LIFE

NUCLEAR HALF LIFE	COMODITY SOLAR DRYING HALF LIFE
Graph of activities is Exponential	Graph display is Exponential
Emits α , β and γ particles	Emits water particle only
Radiation takes place from the atom	Diffusion takes place from the surface of commodity
Artificial and natural phenomenon independent of temperature and pressure	Phenomenon is forced operation which depend on temperature and pressure. Activities increases with temperature and pressure
Independent of Chemical composition	Dependent on Chemical composition
Ionising radiation	Activities are Non- Ionising
Activity dependent on quantum activity per second (Bq)	Activity dependent on amount of weight remaining per second (mg)
Disintegrate to smaller atoms without influence on the neighbouring atom, $\aleph = \frac{\Delta\aleph}{\Delta t} = \lambda\aleph$, (10)$Number\ of\ radioactive\ atons$ $\Delta\aleph$ =Rate of decay $\Delta t = Rate\ of Change\ in\ time\ in\ \Delta\aleph$	Does not disintegrate to smaller atoms. During Decay activity. The dimensionless drying rate (f) was also determined for each drying experiment as follows: $\mathrm{f}\left(\frac{\left(\frac{-\mathrm{dMR}}{\mathrm{dt}}\right)}{\left(\frac{-\mathrm{dMR}}{\mathrm{dt}}\right)_0}\right)$ (11) Where $\left(\frac{-\mathrm{dMR}}{\mathrm{dt}}\right)_0$is the initial drying rate. (Aghbashlo, et al., (2008)[14], thus Moisture Ratio was calculated as: $\mathrm{MR} = \frac{\mathrm{M_t}}{\mathrm{M_o}}$ $MR = \frac{M_t - M_e}{M_o - M_e} \approx \frac{M_t}{M_o}$ (12) $MR = Moisture$ ratio $M_t = Moisture\ content\ at\ the\ end\ of\ activity\ (Drying)$=Weight of commodity in time t M_e=Moisture content at equilibrium Monsture ratio $M_o = Moisture\ content\ at\ the\ begining\ of\ activity\ (Drying)$=Initial Moisture content

MODEL 2

Fick's diffusion equation for particles with slab geometry was used for calculation of effective moisture diffusivity by method of slopes. Since the betel leaves are having a flat surface geometry and. in this case, the average thickness of the leaves was 0.5 mm, the samples were considered of slab geometry (Rayaguru., et al.,2011).[15]The equation of Moisture Ratio (MR) relating to slab geometry was expressed according to (Lopez *et al*., 2000) [16]

$\mathrm{MR} = \frac{8}{\pi^2}\frac{e^{-\pi^2 D_{eff} t}}{4L^2}$ (13)

where*MR* is the dimensionless moisture ratio, D_{eff} is the effective moisture diffusivity in m^2/min,

where $k = \frac{\pi^2 D_{eff} t}{4L^2}$

is the time of drying in minutes and *L* is the slab thickness in meters,.

The expression obtained from equation 11 can simply be employed to have good understanding on how to calculate moisture diffusivity. Moisture diffusivity is an important determining factor in modelling equation of solar drying curves and it varies from one commodity to the other. From

Equation 18, taking the Log of both sides which gives us a straight line graph with an intercept, whereby k is the gradient. From equation 18, L is the thickness of the of the slab

Table 2 THEORY

$\aleph = N_o e^{-\lambda t}$(14) λ =Decay constant $\aleph = Number\ of\ radioactive\ atons.$ $N_o = Initial\ number\ of\ radioactive\ atom$ $t = Time\ taken\ for\ the\ radiactive\ activity$	$MR = \frac{M_t}{M_o} = e^{-kt}$(15) $\frac{M_t}{M_o} = e^{-kt}$ $M_t = M_o e^{-kt}$(16) $M_t = Moisture\ content\ at$ $the\ end\ of\ activity\ (Drying)$=Weight of commodity at that time $t = Time\ taken\ for\ the\ drying\ activity$ k=Decay constant M_e=Moisture content at equilibrium Monsture ratio $M_o = Moisture\ content\ at$ $the\ begining\ of\ activity\ (Drying)$=Initial Moisture content

MODEL 3

Table 3.RADIOACTIVE HALF LIFE AND SOLAR DRYING HALF LIFE

RADIOACTIVE HALF LIFE	COMODITY SOLARDRYING HALF LIFE
After sufficient time that radioactivity has elapse, and the number of remaining atoms is one half of initial atom $N_o$the time interval is known as $\frac{1}{2}$life. $\aleph = \frac{N_o}{2}$(17) $\aleph = Number\ of\ radioactive\ atons.$ $N_o = Initial\ number\ of\ radioactive\ atom$	After sufficient time that drying activity has elapse, and the weight of the remaining commodity $M_t$is one half of weight of initial commodity M_o the time interval is known as $\frac{1}{2}$life. $M_t = \frac{M_o}{2}$(18) $M_t = Moisture\ content\ at$ $the\ end\ of\ Drying\ activity\ (Dry/Wet)$=Weight of commodity in time t $M_o = Moisture\ content\ at$ $the\ begining\ of\ Drying\ activity\ (Dry/Wet)$Initial Moisture content

MODEL 4

Table 4.Half Life is closely related to Decay constant λand drying (Dry/Wet)constant k

$\aleph = N_o e^{-\lambda t}$ $\aleph = \frac{N_o}{2}$ $\frac{N_o}{2} = N_o e^{-\lambda \frac{t}{2}}$(19) $\frac{1}{2} = e^{-\lambda \frac{t}{2}}$ Log($\frac{1}{2} = e^{-\lambda \frac{t}{2}}$) (21) Log$\frac{1}{2}$=$-\lambda \frac{t}{2}$ $-0.693 = -\lambda \frac{t}{2}$ $\frac{t}{2} = \frac{0.693}{\lambda}$ $\lambda = \frac{0.693}{t}$ $\aleph = N_o e^{\frac{0.693}{t} / \frac{t}{2}}$ - (22)	$M_t = M_o e^{-Kt}$ $= M_t = \frac{M_O}{2}$ $\frac{M_O}{2} = M_O e^{-k\frac{t}{2}}$ (20) $\frac{1}{2} = e^{-k\frac{t}{2}}$ Log($\frac{1}{2} = e^{-k\frac{t}{2}}$) (22) Log$\frac{1}{2}$=$-k\frac{t}{2}$ $-0.693 = -k\frac{t}{2}$ $\frac{t}{2} = \frac{0.693}{\lambda}$ $k = \frac{0.693}{t}$ $\aleph = M_o e^{\frac{0.693}{t} / \frac{t}{2}}$ - (23)

Table 5: Experiment on Drying –Final Weight of Cassava (gms)Agaist Time,

S/N	WEIGHT/gms	TIME
	IBLK	Elapsed{SEC)
1	3344	0
2	2703	60
3	2466	195
4	2286	360
5	2123	619
6	2031	640
7	1940	790
8	1848	860
9	1690	980
10	1600	1100
11	1452	1220
12	1389	1370
13	1333	1490
14	1312	1610
15	1280	1880
16	1260	2000
17	1233	2060
18	1230	2190
19	1212	2330
20	1198	2570
21	1166	2630
22	1151	2980
23	1168	3220
24	1162	3620
25	1164	3940
26	1146	4120
27	1162	4300
28	1153	4930
29	1181	5320
30	1168	5980
31	1173	6550
32	1150	7300

Table 6: Experiment on Drying –Final Weight of Cassava (gms), Moisture Ratio and time

	IBLK Wt (gmS)	IBLK (MR)	Elapsed/Sec.
1	3344	1	O
2	2703	0.808313397	60
3	2466	0.737440191	195
4	2286	0.68361244	360
5	2123	0.634868421	619
6	2031	0.607356459	640
7	1940	0.580143541	790
8	1848	0.552631579	860
9	1690	0.505382775	980
10	1600	0.4784689	1100
11	1452	0.434210526	1220
12	1389	0.415370813	1370
13	1333	0.398624402	1490
14	1312	0.392344498	1610
15	1280	0.38277512	1880
16	1260	0.376794258	2000
17	1233	0.368720096	2060
18	1230	0.367822967	2190
19	1212	0.362440191	2330
20	1198	0.358253589	2570
21	1166	0.348684211	2630
22	1151	0.344198565	2980
23	1168	0.349282297	3220
24	1162	0.347488038	3620
25	1164	0.348086124	3940
26	1146	0.342703349	4120
27	1162	0.347488038	4300
28	1153	0.344796651	4930
29	1181	0.353169856	5320
30	1168	0.349282297	5980
31	1173	0.350777512	6550
32	1150	0.343899522	7300

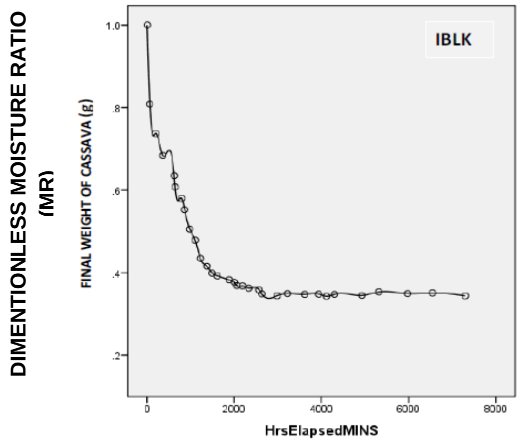

Figure1:Moisture Ratio against Time Integrated Black dryer. (IBLK)

Table 7. Measured Weight, Estimated Weight, Weight Error, and the Weight Error (%) for Integrated Black Dryer (Assisted IBLK)

Measured Weight	Estimated Weight	Weight Error	Weight Error(%)
3344	2604.861108	739.138892	-22.1
2703	2508.757372	194.242628	-7.19
2466	2208.56421	257.43579	-10.44
2286	2063.126684	222.873316	-9.75
2123	1674.467644	448.532356	-21.13
2031	1730.408647	300.591353	-14.8
1940	2314.805517	-374.805517	19.32
1848	1697.881392	150.118608	-8.12
1690	1604.326633	85.673367	-5.07
1600	1548.024577	51.975423	-3.25
1452	1696.096077	-244.096077	16.81
1389	1836.663995	-447.663995	32.23
1333	1756.193973	-423.193973	31.75
1312	1644.13288	-332.13288	25.32
1280	1523.667786	-243.667786	19.04
1260	1380.894676	-120.894676	9.59
1233	1396.224266	-163.224266	13.24
1230	1352.860021	-122.860021	9.99
1212	1545.771639	-333.771639	27.54
1198	1245.571343	-47.571343	3.97
1166	1150.837751	15.162249	-1.3
1151	1069.679006	81.320994	-7.07
1168	1109.394471	58.605529	-5.02
1162	917.143426	244.856574	-21.07
1164	1593.588612	-429.588612	36.91
1146	993.358825	152.641175	-13.32
1162	1517.44097	-355.44097	30.59
1153	1411.645479	-258.645479	22.43
1181	1218.205808	-37.205808	3.15
1168	1006.809965	161.190035	-13.8
1173	871.060212	301.939788	-25.74
1150	741.275269	408.724731	-35.54

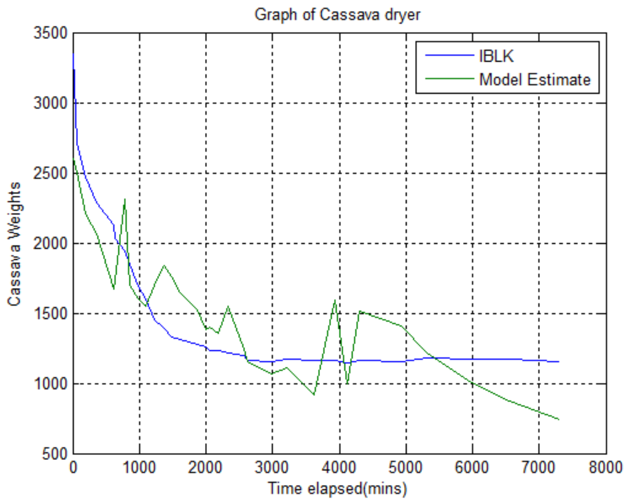

Figure2: Graph of Integrated Black Dryer (Assisted IBLK) Showing Fall In Weight against Time, for Experimental and Predicted values

DISCAUSION

QANTITATIVE ANALYSIS OF DRYING RATES OF SOLAR PHOTOVOLTAIC AND THERMAL DRYER BASED ON THE PRINCIPLE OF HALVE LIFE MODEL

MODEL 5

Table 8.

VALUE OF THE MOISTURE REMAINING AFTER TIME (t), AND THE TIME TAKEN TO DRY UP 50% (HALVE LIFE)) OF MOISTURE FROM THE COMMODITY				
Type	Initial Weight (at T=0 Min.)(gms) $\{M_o\}$ (Table 5)	Value of Moisture Removed at Half Life 3344 $\{M_o\}$ $-1760(M_t)$ = (Table 5)	After sufficient time that drying activity has elapse, and the weight of the remaining commodity M_t is one half of weight of the initial commodity M_o the time interval is known as $\frac{1}{2}$life. $M_t = \frac{M_o}{2}$ (Table 5)	Time Taken to Remove 50% Moisture Content (Min) $t_{1/2}$) (Table 5)
IBLK	3344	1584	1760	1005Min.

Model 1 Table1explained the similarities and differences between radioactivehalf life and solar commodity drying half life. Model 2Table 2 explained the theory behind the modelling of radioactive half life along solar commodity drying half life. Model 3 Table 3. Indicates that after sufficient time that the radioactivity or solar commodity drying has elapse and the number of remaining atoms or commodity is one half of initial. ℵ orM_t the time interval is known as $\frac{1}{2}$life. Therefore ($\aleph = \frac{N_o}{2}$ or $M_t = \frac{M_o}{2}$).Model 4 Table 4, shows that half Life is closely related to decay constant λ *and* solar commodity drying constant k. Substituting ℵ and M_t in equations (13) and (14) $\frac{N_o}{2} = N_o e^{-\lambda\frac{t}{2}}$, radioactive half life or$\frac{M_O}{2} = M_O e^{-k\frac{t}{2}}$Solar commodity drying half life. For both radioactive half life and Solar commodity drying half life equations (20) and (21) gives $\frac{1}{2} = e^{-\lambda\frac{t}{2}}$ and $\frac{1}{2} = e^{-k\frac{t}{2}}$ *respectively* therefore Log$\frac{1}{2}$=$-\lambda\frac{t}{2}$ and Log($\frac{1}{2} = e^{-k\frac{t}{2}}$): $\lambda = \frac{0.693}{t}$ and $k = \frac{0.693}{t}$ respectively.

Table 5: is the experiment on solar commodity drying which recorded the final weight of Cassava (gms) against Time. Table 6: experiment on Drying on final weight of cassava (gms) against moisture ratio and time. Table 7**.is** the recordingof weight, estimated weight, and weight Error, and the Weight Error in (%) for Integrated Black Dryer (Assisted IBLK).Figure1: is the graph of Moisture Ratio (MR) against Time (t) for Integrated Black dryer. (IBLK).Figure 2: is the graph of Integrated Black Dryer (IBLK) showing fall In weight against **Time,** for experimental and predicted values. Table 8.Quantitative analysis of drying rates of solar photovoltaic and thermal dryer based on the principle of halve life model

CONCLUSION

The experimental data obtained were used to quantitatively model the solar dryingrate of cassavachips using the principle of half-life. Similarities and dereferences between radioactive half life, Solar commodity drying half life, associated theory and equation were enumerated I this study. This Models can serve as veritable tool towards comparing different dryer devices when we know the quantity $\aleph = Number\ of\ radioactive\ atons,\ M_t = Moisture\ content\ atthe\ end\ of\ activity\ (Drying)$ hence their efficiencies.We can use the result to determine directly the amount of commodity remaining at anytime or stage of activity. The graph of solar drying of cassava in fig. 1is exponential. The half life quantitative analysis of solar drying of cassava chip is recorded in table 8 where the initial amount or weight (M_0) of cassava chips at the beginning of the expedient was 3344gms and at the half life period (half time $M_{t/2}$) is 1670gms]and the half life ($t_{1/2}$) is 1005 minutes,

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