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THE RADIUS OF GYRATION AND ACCELERATION DUE TO GRAVITY DETERMINED FROM A COMPOUND PENDULUM EXPERIMENT

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Abstract

Compound or physical pendulum is a set-up whose dimensions of its suspended body are significant. Its period and frequency depends on length of gyration, moment of inertia, and mass of pendulum as well as gravitational acceleration while the frequency and period of a simple pendulum depends on length of vibrating cord. The use of simple pendulum is flawed by its dependence on local gravity that do not support its use for defining seconds and its difficulty to measure accurately as a result of its significant variation with latitude and longitude. The compound pendulum is apt at addressing these shortcomings and present more accurate results. A solid body was mounted upon a horizontal axis so as to vibrate under the force of gravity in a compound pendulum. This experiment requires the acquisition of large amount of repetitive data and a complex analysis to determine both the position of centre of mass and moment of inertia. A bar pendulum made of 1m long metal of 5mm diameter perforation at every 5cm interval was suspended from a horizontal knife edge passing through each hole, the knife edge was fixed in a platform provided with screws for which the rear screws adjustment; the platform could be made horizontal. The bar pendulum with ends A and B was suspended with its knife edge on a rigid platform and set into 20 oscillations by displacing through a small angle to oscillate in a vertical plane with its period changing with respect to the position of the moveable axes. This was carried for the remaining holes. The process was repeated for opposite side of G. The values obtained varied at some point which could be attributed to the mass of the bar not been uniformly distributed. A graph of y^2 against yT^2 was plotted to obtain a slope of $10.6m/s^2$ and intercept of 0.28m for centre of gravity and radius of gyration respectively.

***Keywords*:** Acceleration due to gravity, Compound pendulum, Oscillations, Radius of gyration.

1.0 Introduction

The pendulum is a well-known object for which there exists a wide range of variation. The very first and the most common being the simple pendulum is characterized by its small amplitude. Neglecting the energy loss factors, there is no need for energizing this device through the forcing mechanisms [1]. To predict the behaviour of a pendulum is very limited in certain regimes, that is initial condition, because of the extreme sensitivity towards even small perturbations. In addition, [2] the pendulum is considered as a model system exhibiting deterministic chaotic behaviour and the motion is governed by equations. A simple pendulum consists of a small body called a "bob" (usually a sphere) attached to the end of a string, the length of which is great compared with the dimensions of the bob and the mass of which is negligible in comparison with that of the bob[3].Under these conditions, the mass of the bob may be regarded as concentrated at its centre of gravity, and the length of the pendulum is the distance of this point from the axis of suspension. When the dimensions of the suspended body are not negligible in comparison with the distance from the axis of suspension to the centre of gravity, the pendulum is called the compound, or physical pendulum. A rigid body mounted upon a horizontal axis so as to vibrate under the force of gravity is a compound pendulum [4].

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The compound pendulum is a standard topic in most intermediate physics courses, and is generally included in freshman laboratory experiment [5]. Traditionally, this experiment required the acquisition of a large amount of repetitive data, and a complex analysis to determine two parameters, the position of the centre of mass and the moment of inertia. Elementary physics courses tend to concentrate on static and particle dynamics and thus emphasize the importance of centre of mass, a concept which has many applications within the student's experience and is therefore readily accepted, however, the subtle effect of the mass distribution, which only manifest themselves in angular dynamics, are very much harder to comprehend. Furthermore, this research is usually introduced at a time when the student grasp of topic such as torque, moment of inertia, the simple harmonic motion, so that the experiment does not generally lead to an understanding of how the mass distribution affects the period of oscillation[6].
Consider an extended body of mass M with a hole drilled though it. Suppose that the body is suspended from a fixed peg, which passes through the hole, such that it is free to swing from side to side. This setup is known as a compound pendulum[7].

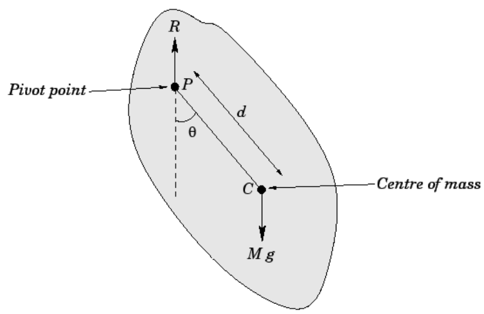

Fig.1: A compound pendulum
LetP be the pivot point, and let C be the body's centre of mass, which is located a distance d from the pivot. Let Θ be the angle subtended between the downward vertical (which passes through pointP) and the line PC. The equilibrium state of the compound pendulum corresponds to the case in which the centre of mass lies vertically below the pivot point: $i.e.\theta = 0$. The angular equation of motion of the pendulum is simply

$$I\ddot{\Theta} = T \quad (1)$$

Where I is the moment of inertia of the body about the pivot point, and T is the torque. Using similar arguments to those employed for the case of the simple pendulum (recalling that all the weight of the pendulum acts at its centre of mass), we can write

$$T = -M\,g\,d\sin\Theta \quad (2)$$

Note that the reaction, R, at the peg does not contribute to the torque, since its line of action passes through the pivot point. Combining the previous two equations, we obtain the following angular equation of motion of the pendulum:

$$I\ddot{\Theta} = -M\,g\,d\sin\Theta \quad (3)$$

Finally, adopting the small angle approximation $Sin\theta \cong \theta$, we arrive at the simple harmonic equation:

$$I\ddot{\Theta} = -M\,g\,d\,\Theta \quad (4)$$

It is clear, by analogy with our previous solutions of such equations, that the angular frequency of small amplitude oscillations of a compound pendulum is given by

$$\omega = \sqrt{\frac{Mgd}{I}} \quad (5)$$

It is helpful to define the length

$$L = \frac{I}{Md}. \quad (6)$$

Equation 5 reduces to

$$\omega = \sqrt{\frac{g}{L}}, \quad (7)$$

Equation (5) is identical in form to the corresponding expression for a simple pendulum. We conclude that a compound pendulum behaves like a simple pendulum with effective lengthL.

2.0 MATERIALS

A bar pendulum, knife–edge with a platform, precision stop watch, meter scale and telescope.
The bar pendulum is a metallic bar of about one meter length, with series holes each approximately 5mm in diameter, made along the length of the bar. The bar is suspended from a horizontal knife-edge passing through any of the holes. The knife-edge, in turn, is fixed in a platform provided with the screws. By adjusting the rear screw the platform can be made horizontal.

When the pendulum is suspended with its knife-edge on a rigid platform and set into oscillation, its period of oscillation changes depending on the position of the movable mass. The experiment is on studying the relationship between the distances of the movable mass from the axis of oscillation with the periodic time of the pendulum.

3.0 METHOD

The compound bar pendulum of ends A and B is suspended by passing a knife edge through the first hole at the end A. The pendulum is pulled aside through a small angle and released, whereupon it oscillates in a vertical plane with small amplitude. The time for 20 oscillations is taken. From this the period T of oscillation of the pendulum is determined.
In a similar manner, periods of oscillation are determined by suspending the pendulum through the remaining holes on the same side of the centre of mass G of the bar. The bar is then inverted and periods of oscillation are determined by suspending the pendulum through all the holes on the opposite side of G. The distances d of the top edges of different holes from the end A of the bar are measured for each hole. The position of the centre of mass of the bar is found by balancing the bar horizontally on a knife edge.
A graph is drawn with the distance d of the various holes from the end A along the X-axis and the period T of the pendulum at these holes along the Y-axis. The graph has two branches, which are symmetrical about G.

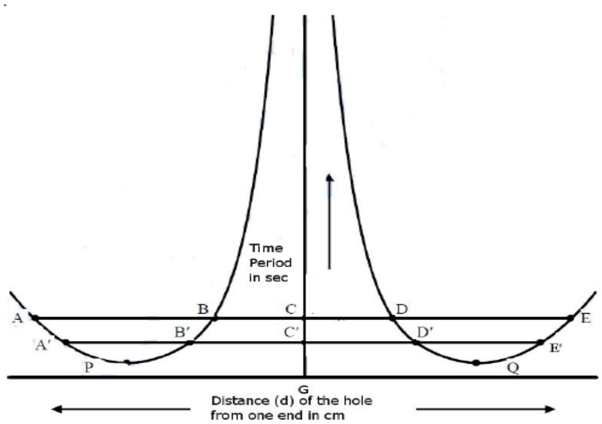

Fig. 2: Plot of Period T vs distance

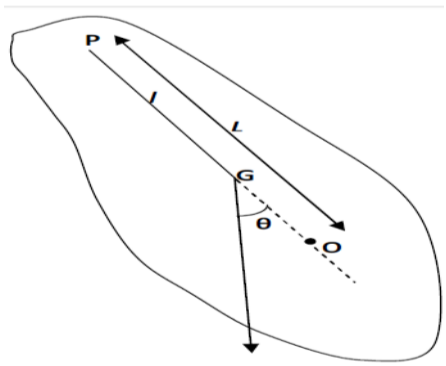

Fig. 3: Irregular shape body

To find the acceleration due to gravity and the radius of gyration of a compound pendulum, a rigid body, free to oscillate in a vertical plane about a horizontal axis through 0. I_G = Moment of Inertia of the body with respect to an axis through G, the centre of gravity. Then from the figure:

$$I_o = I_G + \frac{W}{g} y^{-2} \quad (8)$$

Equation of motion for body:

$$-W y \sin\theta = I_0 \frac{d^2\theta}{dt^2} \quad (9)$$

$$\frac{d^2\theta}{dt^2} + \frac{Wy}{I_o}\theta = 0 \quad (10)$$

Since sinθ = 0 for small angles. Hence motion is simple Harmonic, of period.

$$T = 2\pi\sqrt{\frac{K_G^2 + y^2}{gy}} \quad (11)$$

This may be written in the form:

$$y^2 = \frac{g}{4\pi^2}(yT^2) - K_G^2 \quad (12)$$

This is a straight-line graph of intercept K^2_G.

4.0 RESULTS AND DISCUSSION

This section presents and discusses the results that were acquired after the data acquisition, processing and analysis that were carried out during this project in reflection to the objectives the following results were obtained:

TABLE 1: Readings of L(m) represent the length on the bar pendulum done serially until the length of 0.45m on one side of G was attained.

S/N	L(m)	OG = (y_m)	Number of oscillation	t_1 (sec)	T (Period)
1.	0.100	0.40	20	30	1.50
2.	0.150	0.35	20	28	1.40
3.	0.200	0.30	20	27	1.35
4.	0.250	0.25	20	28	1.40
5.	0.300	0.20	20	29	1.45
6.	0.350	0.15	20	31	1.55
7.	0.400	0.10	20	35	1.75
8.	0.450	0.05	20	47	2.35

TABLE 2: Readings of L(m) represent the length on the bar pendulum done serially until the length of 0.45m on other side of G was attained.

S/N	L(m)	OG = (y_m)	Number of oscillation	t_2 (sec)	T (Period)
1.	0.100	0.40	20	29	1.45
2.	0.150	0.35	20	28	1.40
3.	0.200	0.30	20	27	1.35
4.	0.250	0.25	20	28	1.40
5.	0.300	0.20	20	29	1.45
6.	0.350	0.15	20	31	1.55
7.	0.400	0.10	20	35	1.75
8.	0.450	0.05	20	48	2.40

TABLE 3: Combination of tables 1 and 2 showing two different time (sec)

S/N	L(m)	OG = (y_m)	Number of oscillation	t_1 (sec)	t_2(sec)
1.	0.10	0.40	20	30	29
2.	0.15	0.35	20	28	28
3.	0.20	0.30	20	27	27
4.	0.25	0.25	20	28	28
5.	0.30	0.20	20	29	29
6.	0.35	0.15	20	31	31
7.	0.40	0.10	20	35	35
8.	0.45	0.05	20	47	48

TABLE 4: Composite table built from table 3

S/N	L(m)	OG = (y_m)	Number of oscillation	t_1(sec)	t_2(sec)	t_{av} (sec)	T (Period)	y^2	T^2	yT^2
1.	0.10	0.40	20	30	29	29.50	1.475	0.16	2.176	0.870
2.	0.15	0.35	20	28	28	28.00	1.400	0.12	1.960	0.686
3.	0.20	0.30	20	27	27	27.00	1.350	0.09	1.823	0.547
4.	0.25	0.25	20	28	28	28.00	1.400	0.06	1.960	0.490
5.	0.30	0.20	20	29	29	29.00	1.450	0.04	2.103	0.421
6.	0.35	0.15	20	31	31	31.00	1.550	0.02	2.403	0.360
7.	0.40	0.10	20	35	35	35.00	1.750	0.01	3.063	0.306
8.	0.45	0.05	20	47	48	47.50	2.375	0.003	5.641	0.282

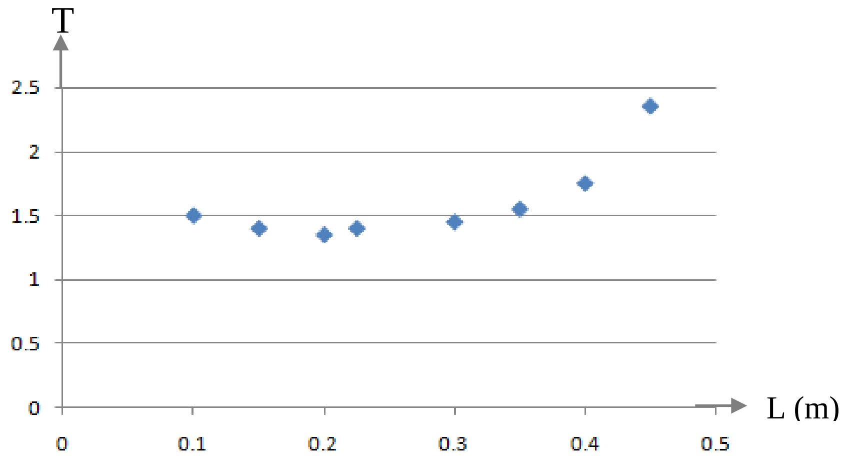

Fig 4:Plot of Period T vs L(m)

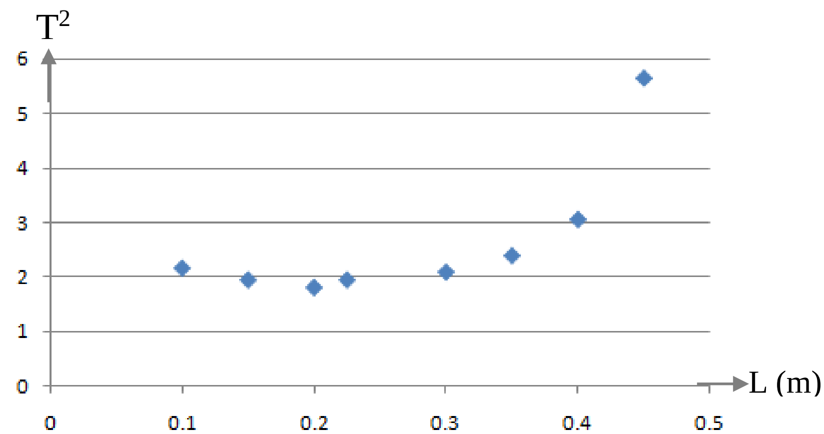

Fig 5: Plot of Period T^2vs L(m)

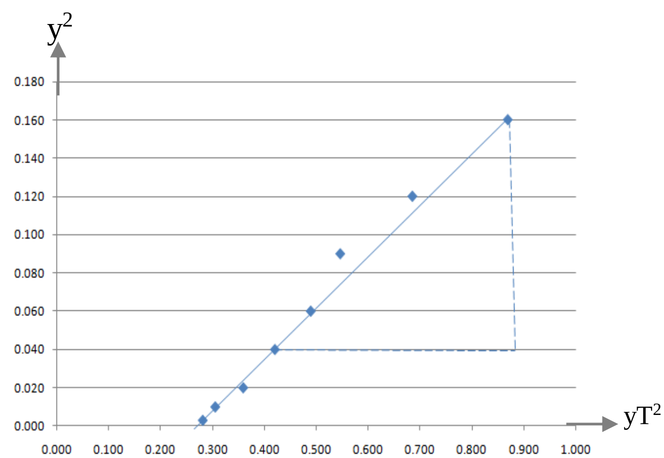

Fig 6: Plot of y^2 vs yT^2

To determine the acceleration due to gravity a plot of y^2 Vs yT^2(figure 6) was made where Simple harmonic, of the period is given as

$$T = 2\pi\sqrt{\frac{K_G^2+y^2}{gy}} \tag{13}$$

Therefore

$$T^2 = 4\pi^2\left(\frac{k_G^2+y^2}{gy}\right) \tag{14}$$

$gyT^2 = 4\pi^2 k_G^2 + 4\pi^2 y^2$ (15)

$4\pi^2 k_G^2 = gyT^2 - 4\pi^2 y^2$ (16)

$K_G^2 = \frac{gyT^2}{4\pi^2} - \frac{4\pi^2 K_G^2}{4\pi^2}$ (17)

Therefore;$y^2 = g\frac{yT^2}{4\pi^2} - K_G^2$ (18)

Comparing18 with Y=MX+C

$M = \frac{g}{4\pi^2}$ (19)

$C = K_G^2$ (20)

Where M = slope and C = Intercept

Slope $M = \frac{Y_2 - Y_1}{X_2 - X_1}$ (21)

$(yT^2)_1 = 0.42$

$(yT^2)_2 = 0.870$

y_1^2 = 0.04 y_2^2 = 0.16

Slope $M = \frac{0.16-0.04}{0.87-0.42} = \frac{0.12}{0.45}$

Therefore slope = 0.27

Since Slope $M = \frac{g}{4\pi^2}$

Therefore $g = M \times 4\pi^2$ (22)

$g = 0.27 \times 4 \times 3.142^2$

$g = 0.27 \times 4 \times 9.8596$

g = 10.6ms^{-2}

Therefore Acceleration due to gravity (g) = 10.6ms^{-2}

$C = K_G^2$ (23)

$K_G^2 = C$ (24)

K_G^2= 0.28

Therefore my Radius of Gyration =0.28m.

5.0 CONCLUSION

From the results of interpretation, acceleration due to gravity (g) is 10.6ms^{-2} and the radius of gyration is -0.28m.The period of the compound pendulum oscillation does not depend on the mass of the load, nor on the angle of revolution. Acceleration due to gravity can be found experimentally from the dependence y^2 Vs yT^2. The table and plot have been prepared to measure the length without the ruler, through the period of oscillation.

The time for 0.10m and 0.45m differs when taking the second reading for accurate result but the rest are approximately similar, this suggest in accuracy, air resistance or the mass of the compound pendulum not been uniform.

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