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BETA- SKEWED LAPLACE DISTRIBUTION AND ITS APPLICATION IN MODELLING SOME CLIMATIC PARAMETERS

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Abstract

Climate parameters affect virtually every aspect of human endavours ranging from food availability, clothing, housing and general well being of humans. Despite this not much has been done in the area of developing a probabilistic distribution for these parameters. This study therefore proposed a new distribution "Beta Skewed Laplace Distribution (BSLD)" where additional 2 shape parameters are injected into the existing 3- parameters Skewed Laplace Distribution (SLD) to make it a 5 parameters distribution. Some of the statistical properties of the proposed distribution were derived and estimation of parameters for the proposed distribution was by the maximum likelihood method. The flexibility of the proposed distribution over the SLD and Skewed Generalized t- Distribution (SGTD) with the same parameters as the proposed distribution is illustrated by applying it to monthly temperature, rainfall, sunshine and wind speed in Port Harcourt, Rivers State, Nigeria between 2001-2017. The data was obtained from Nigerian Metrological Agency (NIMET). The log- likelihood function and Akaike Information Criterion (AIC) were used as model performance criteria and results were generated using BFGS optimization routine in R. The result shows that the BSLD gave better fit than SLD and SGT distributions. Hence, the use of this proposed distribution is recommended as it will help in better understanding of these parameters which would result in food sufficiency and better wellbeing and thus ameliorate the possible devastating effect of these climatic parameters.

Keywords: Climatic parameters, Beta Skewed Laplace distribution, maximum likelihood method.

I. INTRODUCTION

Climate is the statistics of weather over a long period of time. It can be measured by assessing the variation in temperature, rainfall, wind speed, sunshine, relative humidity and other metrological variables. Climate does not only affect socio-economic development but also affect the human existence [1] as it affect food and water resources that are critical for livelihood. The issue of climate is very critical in Africa where the majorities are poor and rely on the local supply systems that are sensitive to climate variation. Therefore, the role of climate on the survival of human, vegetations and other living things cannot be overemphasized.

The issue of climate change because of its far reaching on human existence has generated the attention of researchers in various fields of human endeavor. Although, climate often time is believed to be unpredictable but have a pattern. For instance, rainfall is usually high during wet season and low during dry season. This property makes climatic parameters to be studied using a probability distribution. Several probabilistic distributions have been used to study climatic parameters. This include: Generalized Extreme Value (GEV) distribution [2], distribution [3] among other distributions. This era we talk about climate change which means that there is a change in the pattern of these climatic variable which may mean that existing distributions may not able to capture the true pattern of these variables. This therefore emphasizes the need for more flexible distribution. Also, over the last decade several methods have been proposed for generating flexible distribution and one of which is the beta generated method proposed [4]. Hence, the paper proposes a new distribution called Beta Skewed Laplace (BSL) distribution and apply the proposed distribution to modelling some climatic variables (temperature, sunlight, rainfall and wind speed).

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This study would be of benefits in the sense that, It helps in the area of policy formulation most especially flood prevention. This study would be of significance to the public as it helps in better understanding of these climatic variables and with this information would contribute positively to food sufficiency and also ameliorate the harsh effect of climatic variables.

II. RESEARCH METHODOLOGY

Data for the study: The data used in the study is the monthly temperature, rainfall, sunshine and wind speed in Port Harcourt, Rivers State, Nigeria between 2001-2017. The data was obtained from Nigerian Metrological Agency (NIMET), Port Harcourt, Rivers State, Nigeria.

Skewed Laplace distribution

Let X be a random variable, the density function of a skewed Laplace distribution is given as:

$$f(X,\theta,\varepsilon,\delta)=\begin{bmatrix}\frac{1}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right), & x\le\mu\\ \left(\frac{1}{\theta+\varepsilon}\right)exp\left(-\frac{x-\mu}{\varepsilon}\right), & x>\mu\end{bmatrix} \quad (1)$$

While the cumulative density function is given as:

$$F(X,\theta,\varepsilon,\delta)=\begin{bmatrix}\frac{\theta}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right), & x\le\mu\\ 1-\left(\frac{\varepsilon}{\theta+\varepsilon}\right)exp\left(-\frac{x-\mu}{\varepsilon}\right), & x>\mu\end{bmatrix} \quad (2)$$

$-\infty<x<\infty,\theta>0,\varepsilon>0$

[5, 6]

The proposed distribution

According to [7] the probability density function of a beta generated distribution is:

$$g(x)=\frac{1}{B(a,b)}[F(x)]^{a-1}[1-F(x)]^{b-1}f(x), a<0, b>0 \quad (3)$$

In equation(3), $F(x)$ is the cumulative density function of the skewed Laplace distribution as defined in (2) and $f(x)$ is the probability density function of the distribution as defined in equation(1). Then, substituting (1) and (2) into (3), we have:

$$g(x,\theta,\varepsilon,\mu)=\begin{bmatrix}\frac{1}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\left[\frac{1}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\right]^{a-1}\left[1-\frac{\theta}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\right]^{b-1}, x\le\mu\\ \frac{1}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\theta}\right)\left[1-\frac{1}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right)\right]^{a-1}\left[\frac{\theta}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right)\right]^{b-1}, x\ge\mu\end{bmatrix} \quad (4)$$

$a<0, b>0, if\ a=1\ and\ b=1, we\ have\ the\ skew\ Laplace\ distribution\ but\ if\ a\neq 1\ and$ b $=1$, we have the Epxonentiated Skewed Laplace (ESL)distribution..

To show that the proposed Beta Skewed Laplace (BSL) distribution is a probability density function

Let

$$g_1(x)=\frac{1}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\left[\frac{1}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\right]^{a-1}\left[1-\frac{\theta}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\right]^{b-1}$$

and

$$g_2(x)=\frac{1}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\theta}\right)\left[1-\frac{1}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right)\right]^{a-1}\left[\frac{\theta}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right)\right]^{b-1}$$

$$\int_{-\infty}^{\mu}g_1(x)dx=\frac{1}{\theta+\varepsilon}\int_{-\infty}^{\mu}exp\left(\frac{x-\mu}{\theta}\right)\left[\frac{1}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\right]^{a-1}\left[1-\frac{\theta}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\right]^{b-1}dx \quad (5)$$

$$Let\ y=\frac{\theta}{\theta+\varepsilon}exp\left(\frac{x-\mu}{\theta}\right)\ and\ z=1-\frac{\epsilon}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right) \quad (6)$$

Following the rule of integration by substitution and simplifying the resulting term, we have $\int_{-\infty}^{\mu}g_1(x)dx=$

$$\frac{1}{\theta+\varepsilon}\int_0^{\frac{\theta}{\theta+\varepsilon}}exp\left(\frac{x-\mu}{\theta}\right)y^{a-1}(1-y)^{b-1}\frac{\theta+\varepsilon}{exp\left(\frac{x-\mu}{\theta}\right)}dy=\int_0^{\frac{\theta}{\theta+\varepsilon}}y^{a-1}(1-y)^{b-1}dy \quad (7)$$

Similarly,

$$\int_{-\infty}^{\mu}g_2(x)dx=\frac{1}{\theta+\varepsilon}\int_{\mu}^{\infty}exp\left(-\frac{x-\mu}{\varepsilon}\right)\left[1-\frac{\varepsilon}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right)\right]^{a-1}\left[\frac{\varepsilon}{\theta+\varepsilon}exp\left(-\frac{x-\mu}{\varepsilon}\right)\right]^{b-1}dx$$

$$\int_{-\infty}^{\mu}g_2(x)dx=\int_{\frac{\theta}{\theta+\varepsilon}}^{1}z^{a-1}(1-z)^{b-1}dz \quad (8)$$

Hence,

$$\int_{-\infty}^{\infty}g(x)dx=\frac{1}{B(a,b)}\left(\int_{-\infty}^{\mu}g_1dx+\int_{\mu}^{\infty}g_1dx\right)=\frac{1}{B(a,b)}\left[\int_0^{\frac{\theta}{\theta+\varepsilon}}y^{a-1}(1-y)^{b-1}dy+\int_{\frac{\theta}{\theta+\varepsilon}}^{1}z^{a-1}(1-z)^{b-1}dz\right] \quad (9)$$

$$\int_{-\infty}^{\infty} g(x)dx = \frac{1}{B(a,b)}\left[\int_0^1 y^{a-1}(1-y)^{b-1}dy\right] = \frac{B(a,b)}{B(a,b)} = 1 \qquad (10)$$

Hence, the Beta Skewed Laplace distribution defined in (4) is a probability density function

Estimation of Parameters

The parameters of the distributions given in (4) are estimated by the method of maximum likelihood. Therefore, the log-likelihood functions of Beta Skewed Laplace (BSL) ids given as:

$if\ x \le \mu$

$$l = -nlog(\theta+\varepsilon) - nlogB(a,b) + \sum\left(\frac{x-\mu}{\theta}\right) + (a-1)log\left[\left(\frac{\theta}{\theta+\varepsilon}\exp\left(\frac{x-\mu}{\theta}\right)\right)\right] + (b-1)log[\left(1-\frac{\theta}{\theta+\varepsilon}\right)exp\left(\frac{x-\mu}{\theta}\right)] \qquad (11)$$

$if\ \ x > \mu$

$$l = -nlog(\theta+\varepsilon) - nlogB(a,b) + \sum\left(-\frac{x-\mu}{\varepsilon}\right) + (a-1)log\left[\left(1-\frac{\varepsilon}{\theta+\varepsilon}\right)exp\left(-\frac{x-\mu}{\varepsilon}\right)\right] + (b-1)log[\left(\frac{\varepsilon}{\theta+\varepsilon}\right)exp\left(-\frac{x-\mu}{\varepsilon}\right)] \qquad (12)$$

The parameters of the proposed distribution were estimated using the maxLik optimization function in R with the "BFGS" algorithm proposed [8] after supplying necessary information. The fitness of these distributions was compared using Log likelihood and AIC defined by:

$$\mathrm{AIC} = -\frac{2\log(\mathrm{L})+2\beta}{\mathrm{n}} \qquad (13)$$

III. Results And Discussion

Table 1 shows the Maximum Likelihood Estimates (MLEs) 0f the model parameters for the Skewed Laplace (SLD) distribution, Beta Skewed Laplace (BSL) distribution and Skewed Generalized-t (SGT) distribution. For temperature, sunshine and rainfall, the Beta Skewed Laplace (BSL) distribution is the best with the largest value of LL and smallest value of AIC. This is closely followed by Skewed Generalized-t (SGT) distribution with the exception of wind speed where the Skewed Generalized-t (SGT) distribution gave the highest LL value(782.33) and least value of AIC (-7.7189). This findings agrees with that of other studies on beta generated distribution where the proposed distribution was also found to be more superior than that respective parent distribution ([9, 10, 11])

Table 1: MLES of the Model parameters, the log-likelihood values and AIC for temperature, sunshine, wind speed and rainfall.

Climatic variables	Distributions	θ	ε	a	b	μ	LL	AIC
Temperature	SL	0.1684	0.165	-	-	27.11	329.032	-3.2650
	BSL	0.3882	1.010	2.128	3.118	27.22	**336.307**	**-3.3461**
	SGT	8.8719	0.4667	0.4056	9.7271	28.8167	334.328	-3.3267
Sunshine	SL	0.2198	1.9922	-	-	2.310	307.88	-3.0576
	BSL	0.3945	1.036	2.565	3.100	2.385	**467.275**	**-4.6301**
	SGT	1.4950	0.0128	2.1027	2.885	4.5382	371.45	-3.6907
Wind speed	SL	0.1691	4.090	-	-	87.90	678.09	-6.6872
	BSL	0.2662	2.021	1.755	2.161	87.22	776.05	-7.6574
	SGT	11.2101	0.0116	1.8414	3.2003	79.3549	**782.33**	**-7.7189**
Rainfall	SL	0.062	0.1540	-	-	87.51	623.15	-6.1485
	BSL	0.8948	0.260	2.892	2.892	87.12	**982.52**	**-9.6816**
	SGT	26.4168	0.5173	0.4773	9.90	87.56	630.90	-6.2343

LL- Log likelihood, AIC- Akaike Information Criteria. Bolded values are the highest loglikelihood and least value of AIC, SL- Skewed Laplace, BSL-Beta Skewed Laplace, SGT- Skewed Generalized t.

IV. Conclusion

The focus of this paper is to improve on the Skewed Laplace (SL) distribution using Beta generator to form a 5 parameter distribution. The proposed distribution was applied to four climatic variables (temperature, sunshine, wind speed and rainfall) and the result were compared with that of Skewed Laplace and Skewed Generalized -t(SGT) distribution. Result revealed that the proposed Beta Skewed Laplace (BSL) distribution outperformed other competing distributions with the exception of wind speed where the Skewed Generalized -t(SGT) distribution gave better result than Skewed and Beta Skewed Laplace(BSL) distribution.

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References

[1] Adejuwon SA 2004. Impact of climate variability and climate change on crop yield in Nigeria. *Contributed Paper to Stakeholders Workshop on Assessment of Impact and Adaptation to Climate Change (AIACC)*: 2-8.

[2] Kysley. J. (2002). Probability estimates of extreme value temperature events: Stochastic modelling approach versus extreme value distribution. Stud. Geophys. Geod., 46, 93-112.

[3] Lawab, S.M., Abidin, W.A. Z, Chai, W.Y, Baharun, A. and Masri, T. (2015). Statistical modelling of long- term wind speed data. American Journal of Computer Science and Information Technology, 3(1), 1-6.

[4] Eugene N., Lee, C., & Famoye F. (2002). The beta-normal distribution and its applications. *Communication Statistical Theory Methods*, *31*, 497–512.

[5] Akinsete, A. , Famoye, F., and Lee, C. (2008). The beta- Pareto distribution. *Statistics*, 42(6), 547-653.

[6] Theodossiou, P. (1998). Financial data and the skewed generalized t distribution. *Management Science*, 44, 1650-1661.

[7] Adepoju, K.A., & Chukwu, A.U. (2014). The beta power exponential distribution. *Journal of Statistical Science and Application*, *2*, 37-46.

[8] Henningsen, Arne and Toomer, Ott (2011). maxLik: A package for maximum likelihood estimation in R. Computational Statistics 26(2), 443-458

[9] Shittu, O. I., & Adepoju, K. A., & Adeniji, O. E. (2014). On beta skew t-distribution in modelling stock return in Nigeria. *International Journal of Modern Mathematical Sciences*, *11*(2), 94-102.

[10] Kysley. J. (2002). Probability estimates of extreme value temperature events: Stochastic modelling approach versus extreme value distribution. Stud. Geophys. Geod., 46, 93-112.

[11] Adepoju, K.A., & Chukwu, A.U. (2014). The beta power exponential distribution. *Journal of Statistical Science and Application*, *2*, 37-46.