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ADMISSIBLE TOLERABLE NON-ORTHOGONAL CORRELATION POINT IN SEMIPARAMETRIC SEEMINGLY UNRELATED REGRESSION MODEL

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Abstract

Considerable attention has been given in the literature to the estimation of Semiparametric Seemingly Unrelated Regression (SSUR) estimator and the Ordinary Least Squares (OLS) estimator when the regressors are correlated. The former has been shown to be more efficient. This efficiency could be compromised depending on the degree of the correlation. However, in spite of the large number of studies on multicollinearity in SSUR model, there has been no attempt to investigate the tolerable level of correlation for which the SSUR estimator will still be more efficient when compared with the OLS estimator. This study was aimed at determining the Tolerable Non-orthogonal Correlation Point (TNCP) for the SSUR estimator. The TNCP was assessed using a modified Smith and Kohn four-equation SSUR model with a specified variance-covariance matrix (Σ). In a Monte Carlo experiment a pair of parametric and non-parametric regressors was simulated at varying degrees of positive and negative correlations with both defined on a uniform distribution. Data sample sizes of 25, 100 and 1000 were generated and replicated 5000 times in turn. The Cholesky decomposition was employed to partition Σ. Two triangular (upper and lower) matrices resulted from the Cholesky decomposition. Strong contemporaneous relationships among the SSUR equations were obtained by the product of the partitioned matrices and their error terms. The Average Mean Square Error (AMSE) and Bartlett test were applied to test the homogeneity of variances. The Barlett test showed that the efficiency of SSUR estimators was preserved at $\left|TNCP_{x_1,x_2}\right| \le 0.3$ for large sample sizes. The results indicate that the exclusion of variables when regressors are correlated can be circumvented when the tolerable correlation points lies between -0.3 and +0.3(or ± 0.3).

Keywords: Non-orthogonal correlation point, Cholesky decomposition, Multicollinearity, Semiparametric regression.

1. Introduction

Semiparametric model bridges the gap between a purely parametric structure and nonparametric structure; hence a semiparametric model is a flexible model. Researchers have used the Semiparametric Seemingly Unrelated Regression (SSUR) estimator to address both empirical and theoretical problems. For instance, [18] used the smoothing splines to simultaneously estimate a system of equations. In [15] a SSUR model was developed where data transformation may be required and/or outliers may exist in the data. Parametric SUR model was extended in [1, 9, 10] to SSUR and Geoadditive SUR models within a Bayesian context. Standard Linear Programming using the Newton Raphson steps was used by [8] to show that convergence is fast and there is efficiency improvement of SSUR over semiparametric OLS.

The SSUR have also been applied in exciting empirical studies which include [16] who used a kernel based approach to estimate functional relationship between a brand's unit sales and price discounts while modelling other predictors parametrically. The SSUR to sales of oranges using the cubic splines to show that SSUR is better than semiparametric OLS was applied by [10]. In [6] a semiparametric market share attraction model was used for the cubic smoothing splines to estimate price effects on a brand's market share. In these empirical applications, both semiparametric models performed better compared to rigid parametric models in terms of MSE and Bayesian Information Criterion (BIC). Several other estimation procedures proposed within the SSUR frame- work could be found in [6, 10, 13]. In all the SSUR estimation procedures reported above, Zellner's basic recommendation for contemporaneous correlation among the error vectors with uncorrelated explanatory variables within each response equation was maintained.

Multicollinearity is the result of strong correlations between the regressors which inflates the variances of the parameter

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estimates [4, 5]. However, in spite of the problems of multicollinearity, the selection method of covariates or remedy when this problem is encountered within the context of SSUR has not been adequately explored. Studies on multicollinearity in SUR model are still lacking or scarce in literature. However, a notable exception is [17] who investigated different correlation levels and determined the Tolerable Non-Orthogonal Correlation Points (TNCP). They found that at $|TNCP| \leq 0.2$, the efficiency of parametric SUR estimators would be preserved at all sample sizes. Although, the asymptotic property is achieved in both estimators, the parametric SUR superiority over the OLS is maintained whether or not some pairs of covariates in the system are correlated.

In the light of the above, it is worth investigating the TNCP in SSUR model. Although, this is relevant in practice, surprisingly to the best of our knowledge, such study is lacking in literature. Following the above background discussion, we raise this main research question: What is the tolerable admissible level of multicollinearity in SSUR model so that the efficiency of the estimator is not compromised compared to OLS?

2. The Model

The Semiparametric Seemingly Unrelated Regression Model considered in this study is specified as follows:

$$Y_{ij} = f_j(X_{ij}) + \beta_j(X_{ij}) + \varepsilon_{ij} \qquad i = 1,.......,N \quad j = 1,........,M \tag{1}$$

The model in (1) consists of M Seemingly Unrelated structural equations with N observations. Each structural equation is a component of both linear and unlinear functions.

where

y_{ij} is the $MT \times 1$ vector of response variables

x_{ij} is the $MT \times k$ explanatory variables

β_j is the $k \times 1$ unknown regression parameters for the linear terms

f_j are the unknown smooth functions for the nonlinear terms

ε_{ij} are the disturbance terms

The study by [15] specified and estimated a purely Nonparametric SUR model to the general case in (1) above. The model consists of four equations

$$\begin{aligned} y_{i1} &= \sin(8\pi x_{11}) + 0.0551618 + \varepsilon_{i1} \\ y_{i2} &= [\phi(x_{21}; 0.2, 0.05) + \phi(x_{21}; 0.6, 0.2)]/4 - 0.4530935 + \varepsilon_{i2} \\ y_{i3} &= 1.5x_{31} - 0.8380343 + \varepsilon_{i3} \\ y_{i4} &= \cos(2\pi x_{41}) + 0.1070571 + \varepsilon_{i4} \end{aligned} \tag{2}$$

The covariates values are i.i.d samples from

$$x_1 \sim U(0,1), \quad x_2 \sim U(0,1) \text{ and } \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \sim N\left(\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, 0.3 \begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix} \right)$$

The model in (2) is SSUR *iff* there is contemporaneous correlation among the error terms ε.

Therefore, these regressions are related through the correlated error term structure $\varepsilon \sim N(0, \Sigma \otimes I_n)$.

$\varepsilon \sim N(0, \Sigma)$ with

$$\Sigma = \begin{pmatrix} 1 & 0.96 & 0.64 & 0.93 \\ & 1 & 0.98 & 0.9 \\ & & 1 & 0.85 \\ & & & 1 \end{pmatrix} \tag{3}$$

In order to investigate the effect of multicollinearity on this model, it is modified as follows:

$$\begin{aligned} y_{i1} &= \sin(8\pi x_{11}) + 0.4x_{12} + 0.0551618 + \varepsilon_{i1} \\ y_{i2} &= [\phi(x_{21}; 0.2, 0.05) + \phi(x_{21}; 0.6, 0.2)]/4 - 0.4530935 + \varepsilon_{i2} \\ y_{i3} &= 1.5x_{31} - 0.8380343 + \varepsilon_{i3} \\ y_{i4} &= \cos(2\pi x_{41}) + 0.1070571 + \varepsilon_{i4} \end{aligned} \tag{4}$$

The two models differ in terms of y_1, a new parametric term $0.4x_{12}$ was added to the nonparametric term to make y_1 semiparametric.

The covariance matrix Σ reported in (3) turned out not to be positive definite, so we adopted Σ given by [1].

$$\Sigma = \begin{pmatrix} 1 & 0.7 & 0.6 & 0.9 \\ & 1 & 0.7 & 0.9 \\ & & 1 & 0.7 \\ & & & 1 \end{pmatrix} \quad (5)$$

3. The Design of the Monte Carlo Experiments

3.1 Derivation of the Error Structure

The inverse of $\sum$ was achieved through the use of Cholesky decomposition on the matrix. Since $\sum$ is a positive definite matrix, we decomposed it by a non-singular upper triangular matrix P, such that

$$\sum = PP' \quad (6)$$

If $$P = \begin{pmatrix} p_{11}^* & p_{12}^* & p_{13}^* & p_{14}^* \\ 0 & p_{22}^* & p_{23}^* & p_{24}^* \\ 0 & 0 & p_{33}^* & p_{34}^* \\ 0 & 0 & 0 & p_{44}^* \end{pmatrix} \quad (7)$$

Then

$$\sum = PP' = \begin{pmatrix} p_{11}^* & p_{12}^* & p_{13}^* & p_{14}^* \\ 0 & p_{22}^* & p_{23}^* & p_{24}^* \\ 0 & 0 & p_{33}^* & p_{34}^* \\ 0 & 0 & 0 & p_{44}^* \end{pmatrix} \begin{pmatrix} p_{11}^* & 0 & 0 & 0 \\ p_{21}^* & p_{22}^* & 0 & 0 \\ p_{31}^* & p_{32}^* & p_{33}^* & 0 \\ p_{41}^* & p_{42}^* & p_{43}^* & p_{44}^* \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix} \quad (8)$$

The following results were obtained for the lower and upper triangular matrices

$$\begin{pmatrix} 0.3551 & -0.2493 & -0.042 & 0.9 \\ 0 & 0.4247 & 0.098 & 0.9 \\ 0 & 0 & 0.7141 & 0.7 \\ 0 & 0 & 0 & 1 \end{pmatrix} \textit{and} \begin{pmatrix} 0.3551 & 0 & 0 & 0 \\ -0.2493 & 0.4247 & 0 & 0 \\ -0.042 & 0.098 & 0.7141 & 0 \\ 0.9 & 0.9 & 0.7 & 1 \end{pmatrix} \text{ respectively} \quad (9)$$

The four random disturbance series are established through the product of the partitioned matrix in (9) and their error terms, using

$$\varepsilon = P'e \quad (10)$$

So that using (10), we have

$$\begin{aligned} \varepsilon_{i1} &= 0.3551e_{i1} - 0.2493e_{i2} - 0.042e_{i3} + 0.9e_{i4} \\ \varepsilon_{i2} &= 0.4247e_{i2} + 0.098e_{i3} + 0.9e_{i4} \\ \varepsilon_{i3} &= 0.7141e_{i3} + 0.7e_{i4} \\ \varepsilon_{i4} &= e_{i4} \end{aligned} \quad (11)$$

3.2 The Simulation Procedure

In the standard computer algorithm written x_{11}, x_{12}, x_{21} were generated from a U [0, 1] distribution while x_{31}, x_{41} were generated from a normal distribution following [15].

$$\begin{pmatrix} x_{31} \\ x_{41} \end{pmatrix} : N\left(\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, 0.3\begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix}\right)$$

We then generated values for the four equations, $y_{i1}, y_{i2}, y_{i3}, y_{i4}$ as given in (2) (the dataset is available for interested readers). The generation of samples and random disturbances, imposition of correlation between the parametric and nonparametric functions in y_{i1}, computations and estimation were carried out by writing a program in STATA 9.0 Do-file editor (contact the authors for the program). The correlation between x_{11} *and* x_{12} in y_{i1} which is a semiparametric equation in the four- equation SSUR model was considered at various correlation levels $\pm0.9, \pm0.8, \pm0.7, \pm0.6, \pm0.5, \pm0.4 \pm 0.3 \pm 0.2, \pm0.1$ *and* 0.0 with each replicated 5000 times in turn for sample sizes 25, 50, 100 and 1000.

4.0 Analysis and Discussion of the Results

The performance of the estimators was evaluated by the Average Mean Square Error (AMSE). The plots of AMSE against various sample sizes in *Figs 1 and 2* show that despite the gain in efficiency by OLS estimators due to increased sample size, SSUR estimators were more efficient than OLS as shown by the AMSE plots for the positive and negative correlation.

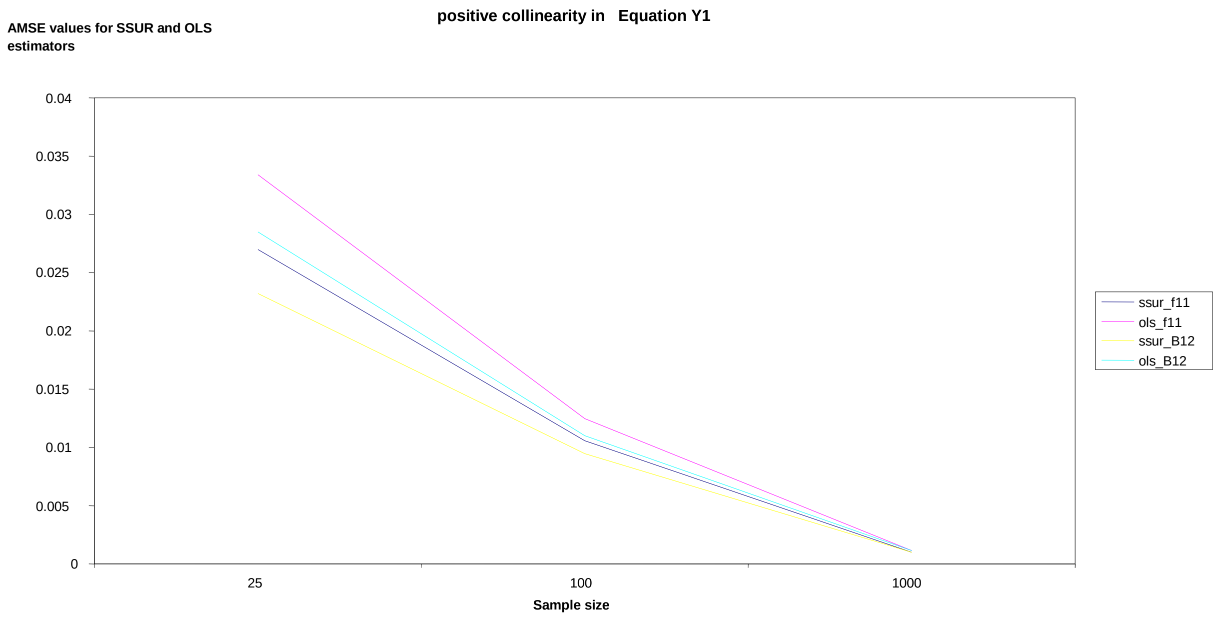

Fig 1: The plot of AMSE values against different sample sizes when there is positive correlation between X_{11} & X_{12} in Y_1

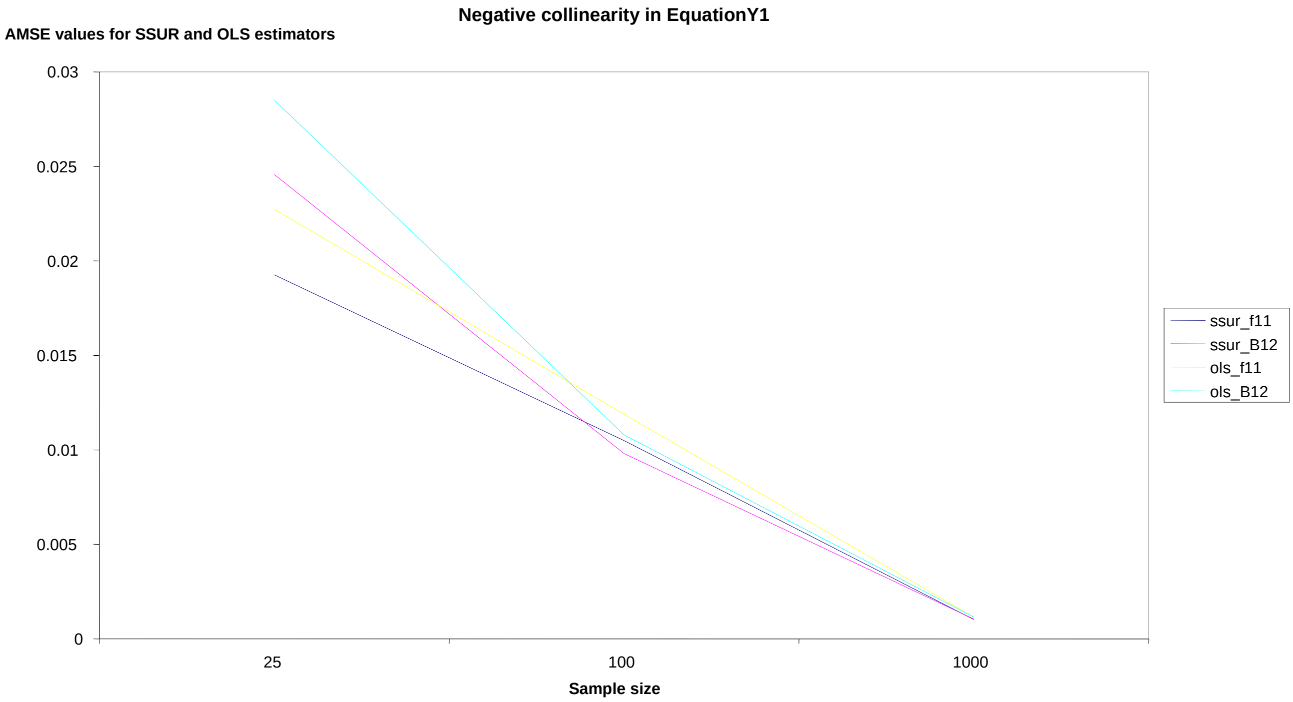

Fig 2: The plot of AMSE values against different sample sizes when there is negative correlation between X_{11} & X_{12} in Y_1

The Box-and-Whisker plots shown in *Figs 3 and 4* of the median AMSE values also show lower AMSE values for SSUR estimators than that of the OLS.

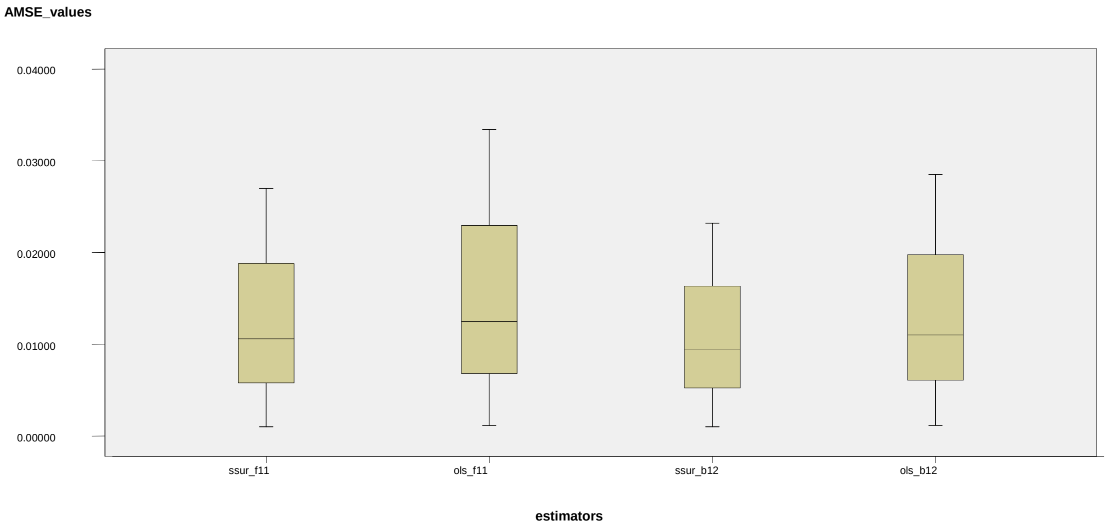

Fig 3: The Box- and Whisker plot for positive correlation.

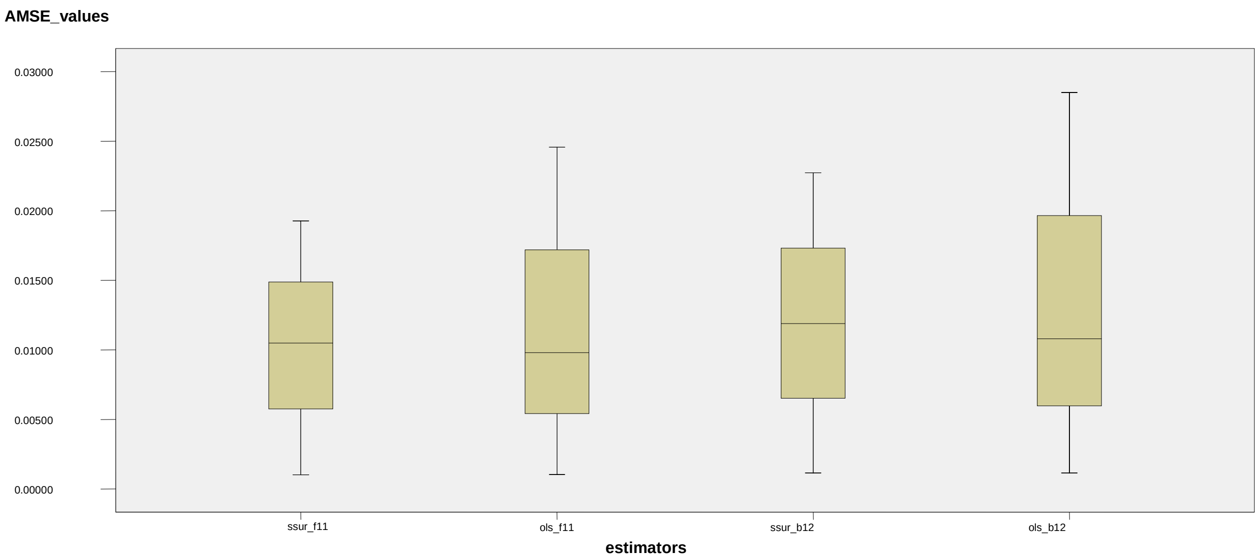

Fig 4: The Box- and Whisker plot for negative correlation.

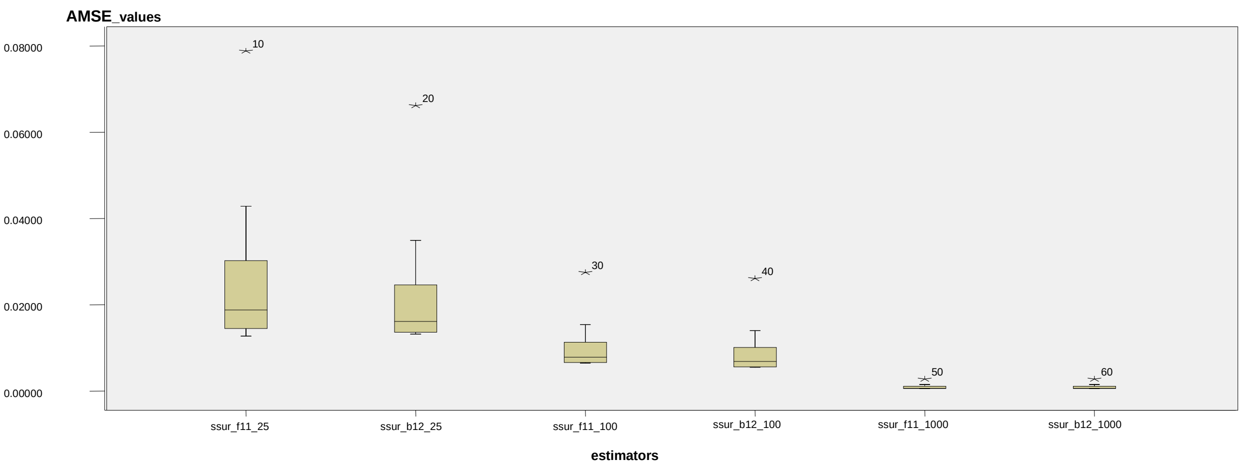

Fig 5: The Box- and –Whisker plot of AMSE values for SSUR estimator at different sample sizes for Y_1 depicting decreasing trend in the median AMSE as the sample size increases when there is positive correlation.

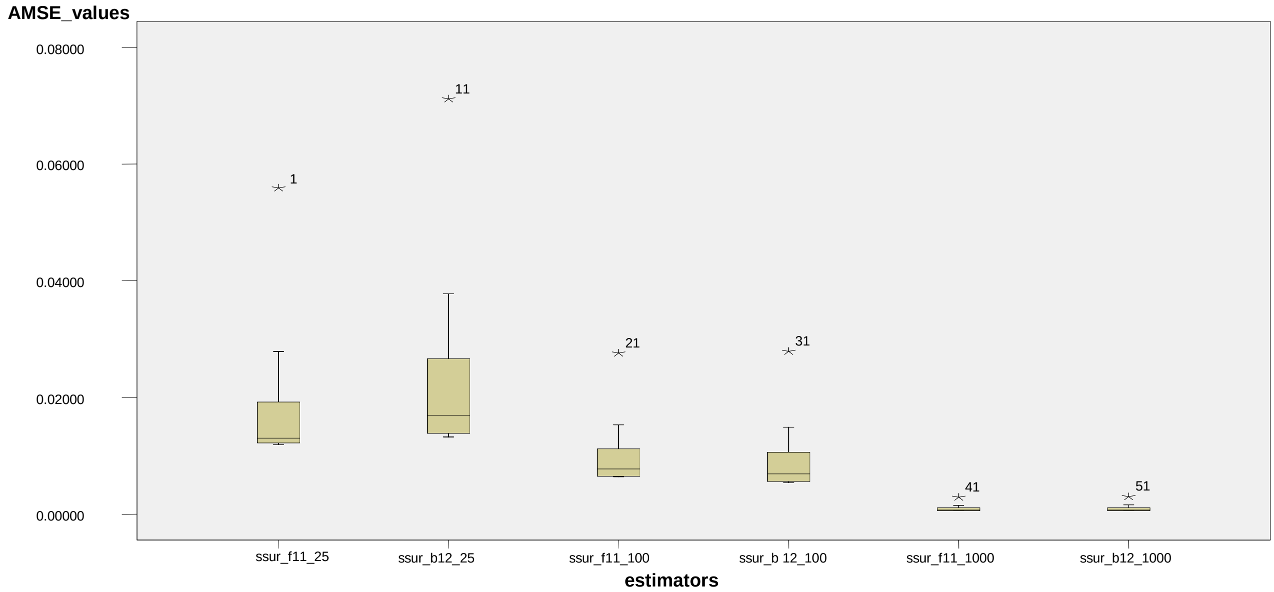

Fig 6: The Box- and –Whisker plot of AMSE values for SSUR estimator at different sample sizes for Y_1 depicting decreasing trend in the median AMSE as the sample size increases when there is negative correlation.

Figs 5- 8 also show that the AMSE values for the SSUR estimators decrease as sample size increases.

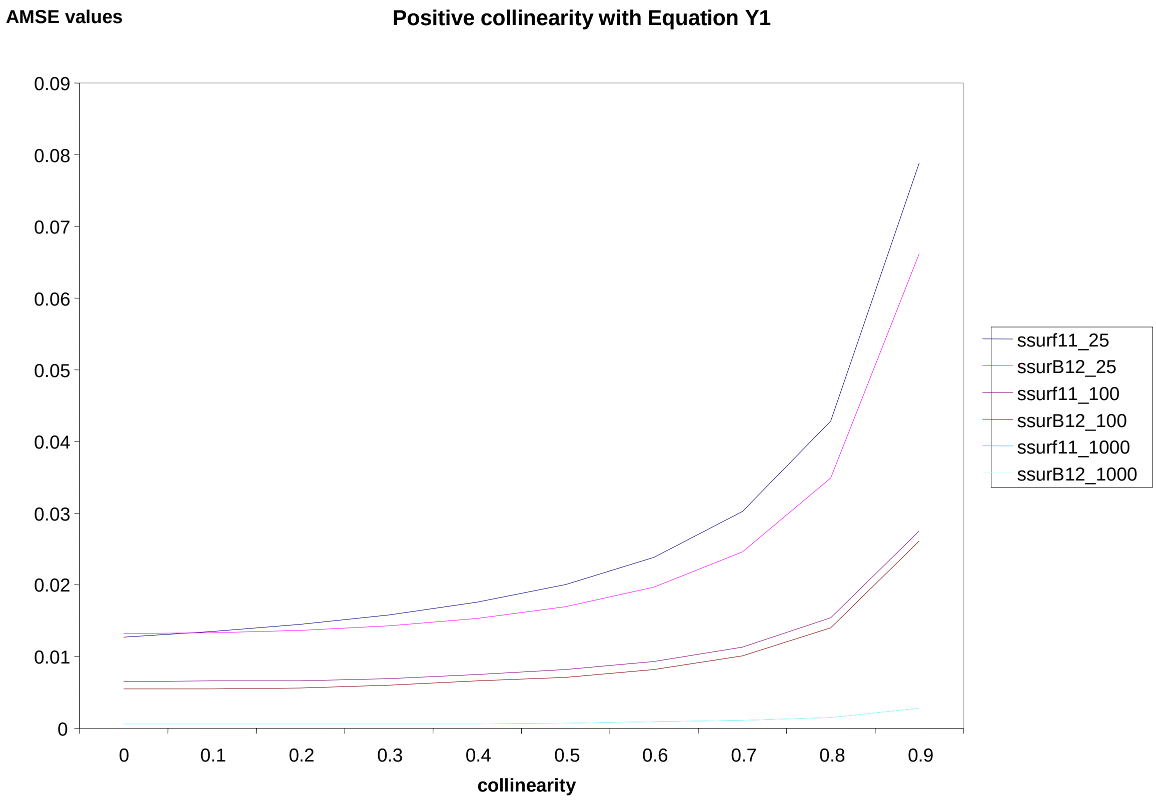

Fig 7: The SSUR estimators when there is positive correlation between X_{11} & X_{12} in equation Y_1

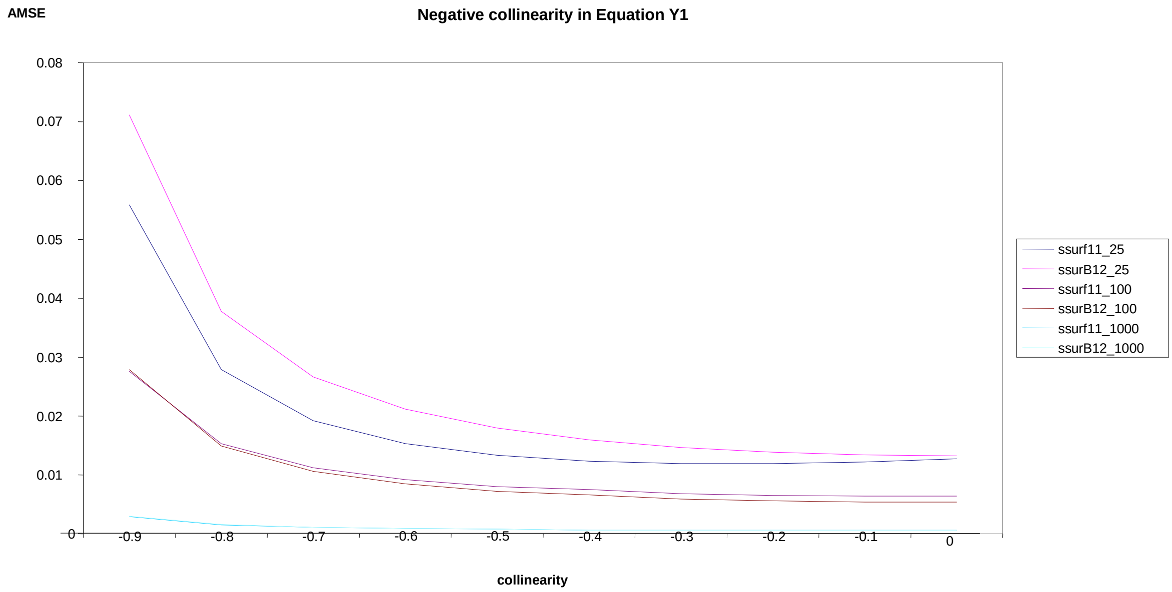

Fig 8: The SSUR estimators when there is negative correlation between X_{11} & X_{12} in equation Y_1

5 Discussion of Results

The different positive and negative correlation level values are given in Tables 1 and 2 respectively for sample size $n = 25$. In Table 1 below, the intercept AMSE values are fixed for both the SSUR and OLS estimators when replicated 5000 times in turn. That is, $\beta_{10}, \beta_{20}, \beta_{30}, \beta_{40}$ with AMSE values $0.1263, 0.0346, 0.0193, 0.1328$ for the SSUR estimator and $0.1661, 0.0451, 0.0210, 0.1467$ for the OLS estimators. Since we are interested in the nonparametric function f_{11} and parametric function β_{12} most of our discussions will centre on the two estimators. The $AMSE_{orthogonal}$ at which the predictors are purely uncorrelated for the SSUR estimators are $0.1328, 0.1115$ while for the OLS estimators $0.1430, 0.3886$ for 5000 replicates. The $AMSE_{orthogonal}$ at which the predictors are purely uncorrelated for the SSUR estimator is $0.1387, 0.1876$ while for the OLS estimators $0.1430, 0.3886$ for 5000. The AMSE values of the SSUR estimators were considerably lower than that of the OLS estimators. In Table 1, when $\rho_{x_{11},x_{12}} = 0.9$, AMSE values for SSUR and OLS estimators were 0.2163, 0.2867 and 0.2903,0.3886 which shows that SSUR is more efficient that OLS estimators. Similar results were also obtained for the AMSE values of the negative correlation levels. Another remarkable result is the loss in the efficiency of the SSUR estimators as the collinearity levels became severe. This is obvious from the following randomly selected values: $\rho_{x_{11},x_{12}} = 0.9, 0.6, 0.4, 0.0$ which are given as 0.2163, 0.2003, 0.1987, 0.1328 and 0.2867, 0.2712, 0.2119, 0.1115 for SSUR estimators while 0.2903, 0.2800, 0.2345, 0.1430 and 0.3886, 0.3886, 0.3886, 0.3886 for OLS estimators. For the negative correlation levels in Table 2, $\rho_{x_{11},x_{12}} = -0.9, -0.6, -0.4, -0.2$ the AMSE values are given as: 0.2081, 0.2021, 0.1975, 0.1889 and 0.2764, 0.2661, 0.2623, 0.2599 for SSUR estimators while 0.2903, 0.2800, 0.2200, 0.2345, 0.2800, 0.2903 and 0.3886, 0.3886, 0.3886, 0.3886. The different positive and negative correlation levels values are given in Tables 3 and 4 for sample size $n = 100$. The $AMSE_{orthogonal}$ at which the predictors are purely uncorrelated for the SSUR estimators are $0.0065, 0.0055$ while for the OLS estimators are $0.0074, 0.0060$ for 5000 replicates . In Table 3, when $\rho_{x_{11},x_{12}} = 0.9$, AMSE values for SSUR and OLS estimators were 0.0274, 0.0260 and 0.0336,0.0318 which shows that SSUR estimators are more efficient that OLS estimators. Similar results were also obtained for the AMSE values of the negative correlation levels. There was also loss in the efficiency of the SSUR estimators as the collinearity levels became severe. This is obvious from the following randomly selected values: $\rho_{x_{11},x_{12}} = 0.9, 0.6, 0.4, 0.0$ which are given as 0.0274, 0.0093, 0.0075, 0.0065 and 0.0260, 0.0082, 0.0066, 0.0055 for SSUR estimators while 0.0336, 0.0109, 0.0086, 0.0074 and 0.0318, 0.0094, 0.0072, 0.0060 for OLS estimators. For the negative correlation levels at $\rho_{x_{11},x_{12}} = -0.9, -0.6, -0.4, -0.2$ the AMSE values are given as 0.0276, 0.0092, 0.0075, 0.0064 and 0.0279, 0.0085, 0.0066, 0.0056 for SSUR estimators while 0.0319, 0.0103, 0.0082, 0.0074 and 0.0312, 0.0093, 0.0074, 0.0062 for OLS estimators.

Table 1: AMSE of SSUR and OLS estimators for equation Y_1 with X_{11} and X_{12} positively correlated.(n= 25)

	AMSE (n=25, r =5000)																	
	SSUR									OLS								
$\rho_{x_{11},x_{12}}$	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}
0.9	0.1263	0.2163	0.2867	0.0346	0.9999	0.0193	0.1090	0.1328	1.1710	0.1661	0.2903	0.3886	0.0451	0.1377	0.0210	0.1361	0.1467	2.6761
0.8	0.1263	0.2157	0.2852	0.0346	0.9995	0.0193	0.1090	0.1328	1.1700	0.1661	0.2879	0.3327	0.0451	0.1377	0.0210	0.1361	0.1467	1.7230
0.7	0.1263	0.2010	0.2763	0.0346	0.9995	0.0193	0.1088	0.1328	0.9925	0.1661	0.2803	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	1.1533
0.6	0.1263	0.2003	0.2712	0.0346	0.9995	0.0193	0.1088	0.1328	0.8973	0.1661	0.2800	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	1.0999
0.5	0.1263	0.1999	0.2345	0.0346	0.9995	0.0193	0.1088	0.1328	0.8643	0.1661	0.2792	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	1.0567
0.4	0.1263	0.1987	0.2119	0.0346	0.9995	0.0193	0.1088	0.1328	0.6745	0.1661	0.2345	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	0.8998
0.3	0.1263	0.1967	0.1674	0.0346	0.9995	0.0193	0.1088	0.1328	0.5612	0.1661	0.2245	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	0.8003
0.2	0.1263	0.1943	0.1453	0.0346	0.9989	0.0193	0.1088	0.1328	0.4321	0.1661	0.2200	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	0.7556
0.1	0.1263	0.1674	0.1329	0.0345	0.9989	0.0193	0.1088	0.1328	0.3321	0.1661	0.1975	0.3886	0.0450	0.1377	0.0210	0.1360	0.1467	0.6785
0.0	0.1263	0.1328	0.1115	0.0345	0.9989	0.0193	0.1088	0.1328	0.1765	0.1661	0.1430	0.3886	0.0450	0.1377	0.0210	0.1360	0.1467	0.3452

Table 2: AMSE of SSUR and OLS estimators for equation Y_1 with X_{11} and X_{12} negatively correlated (n=25)

	AMSE (n=25, r =5000)																	
	SSUR									OLS								
$\rho_{x_{11},x_{12}}$	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}
-0.9	0.1195	0.2081	0.2764	0.0347	0.1004	0.0193	0.0936	0.1349	1.7752	0.1661	0.2903	0.3886	0.0451	0.1377	0.0210	0.1361	0.1467	2.6761
-0.8	0.1195	0.2049	0.2722	0.0347	0.1004	0.0193	0.0931	0.1349	1.1368	0.1661	0.2879	0.3886	0.0451	0.1377	0.0210	0.1361	0.1467	1.7230
-0.7	0.1195	0.2022	0.2677	0.0347	0.1004	0.0193	0.0931	0.1349	0.7382	0.1661	0.2803	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	1.1533
-0.6	0.1195	0.2021	0.2661	0.0347	0.1004	0.0193	0.0931	0.1349	0.5288	0.1661	0.2800	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	1.0999
-0.5	0.1195	0.1989	0.2638	0.0347	0.0995	0.0193	0.0927	0.1349	0.4149	0.1661	0.2792	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	1.0567
-0.4	0.1195	0.1975	0.2623	0.0347	0.0995	0.0193	0.0927	0.1349	0.3404	0.1661	0.2345	0.3886	0.0451	0.1377	0.0210	0.1360	0.1467	0.8998
-0.3	0.1195	0.1963	0.2606	0.0347	0.0987	0.0193	0.0925	0.1349	0.2925	0.1661	0.2243	0.3886	0.0451	0.1377	0.0209	0.1360	0.1467	0.8003
-0.2	0.1195	0.1889	0.2599	0.0347	0.0985	0.0193	0.0923	0.1349	0.2675	0.1661	0.2200	0.3886	0.0451	0.1377	0.0209	0.1360	0.1467	0.7554
-0.1	0.1195	0.1423	0.2595	0.0346	0.0964	0.0193	0.0923	0.1349	0.2550	0.1661	0.1975	0.3886	0.0450	0.1377	0.0209	0.1360	0.1467	0.6785
0.0	0.1195	0.1387	0.1876	0.0346	0.0962	0.0193	0.0923	0.1349	0.2427	0.1661	0.1430	0.3886	0.0450	0.1377	0.0209	0.1360	0.1467	0.3452

Table 3: AMSE of SSUR and OLS estimators for model Y_1 with X_{11} *and* X_{12} positively correlated.(n=100)

	AMSE (n=100, r = 5000)																	
	SSUR									OLS								
$\rho_{x_{11},x_{12}}$	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}
0.9	0.0059	0.0274	0.0260	0.0089	0.0251	0.0052	0.0308	0.0356	0.0753	0.0061	0.0336	0.0318	0.0099	0.0291	0.0053	0.0331	0.0364	0.0923
0.8	0.0059	0.0152	0.0140	0.0089	0.0251	0.0052	0.0304	0.0353	0.0741	0.0061	0.0184	0.0168	0.0099	0.0291	0.0053	0.0331	0.0360	0.0896
0.7	0.0059	0.0113	0.0101	0.0089	0.0251	0.0052	0.0299	0.0352	0.0689	0.0061	0.0134	0.0118	0.0099	0.0291	0.0053	0.0331	0.0359	0.0828
0.6	0.0059	0.0093	0.0082	0.0089	0.0251	0.0052	0.0295	0.0352	0.0651	0.0061	0.0109	0.0094	0.0099	0.0291	0.0053	0.0331	0.0359	0.0773
0.5	0.0059	0.0082	0.0071	0.0089	0.0251	0.0052	0.0291	0.0350	0.0622	0.0061	0.0095	0.0081	0.0099	0.0291	0.0053	0.0331	0.0359	0.0732
0.4	0.0059	0.0075	0.0066	0.0088	0.0251	0.0052	0.0287	0.0349	0.0599	0.0061	0.0086	0.0072	0.0099	0.0291	0.0053	0.0331	0.0358	0.0695
0.3	0.0059	0.0069	0.0059	0.0088	0.0250	0.0052	0.0284	0.0347	0.0582	0.0061	0.0080	0.0066	0.0099	0.0291	0.0053	0.0331	0.0354	0.0665
0.2	0.0059	0.0065	0.0056	0.0088	0.0249	0.0052	0.0280	0.0347	0.0564	0.0061	0.0076	0.0065	0.0099	0.0291	0.0053	0.0331	0.0354	0.0638
0.1	0.0059	0.0065	0.0055	0.0088	0.0249	0.0052	0.0278	0.0347	0.0551	0.0061	0.0075	0.0065	0.0099	0.0291	0.0053	0.0331	0.0354	0.0614
0.0	0.0059	0.0065	0.0055	0.0088	0.0249	0.0052	0.0275	0.0347	0.0543	0.0061	0.0074	0.0060	0.0099	0.0291	0.0053	0.0331	0.0354	0.0594

Table 4: AMSE of SSUR and OLS estimators for model Y_1 with X_{11} *and* X_{12} negatively correlated.(n=100)

	AMSE (n=100, r =5000)																	
	SSUR									OLS								
$\rho_{x_{11},x_{12}}$	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}
-0.9	0.0058	0.0276	0.0279	0.0089	0.0252	0.0052	0.0277	0.0345	0.3739	0.0060	0.0319	0.0312	0.0099	0.0289	0.0053	0.0329	0.0352	0.4492
-0.8	0.0058	0.0153	0.0149	0.0089	0.0252	0.0052	0.0270	0.0343	0.2463	0.0060	0.0174	0.0164	0.0099	0.0289	0.0053	0.0329	0.0352	0.3045
-0.7	0.0058	0.0112	0.0106	0.0089	0.0252	0.0052	0.0266	0.0342	0.1860	0.0060	0.0126	0.0116	0.0099	0.0289	0.0053	0.0329	0.0352	0.2322
-0.6	0.0058	0.0092	0.0085	0.0089	0.0251	0.0052	0.0266	0.0342	0.1518	0.0060	0.0103	0.0093	0.0099	0.0289	0.0053	0.0329	0.0352	0.1895
-0.5	0.0058	0.0080	0.0072	0.0089	0.0251	0.0052	0.0266	0.0340	0.1289	0.0060	0.0090	0.0079	0.0099	0.0289	0.0053	0.0329	0.0352	0.1602
-0.4	0.0058	0.0075	0.0066	0.0088	0.0251	0.0052	0.0265	0.0339	0.1116	0.0060	0.0082	0.0070	0.0099	0.0289	0.0053	0.0329	0.0351	0.1380
-0.3	0.0058	0.0068	0.0059	0.0088	0.0250	0.0052	0.0265	0.0338	0.0986	0.0060	0.0077	0.0065	0.0099	0.0289	0.0053	0.0329	0.0351	0.1212
-0.2	0.0058	0.0065	0.0056	0.0088	0.0249	0.0052	0.0265	0.0337	0.0887	0.0060	0.0074	0.0062	0.0099	0.0289	0.0053	0.0329	0.0351	0.1085
-0.1	0.0058	0.0064	0.0054	0.0088	0.0249	0.0052	0.0264	0.0337	0.0808	0.0060	0.0072	0.0060	0.0099	0.0289	0.0053	0.0329	0.0351	0.0982
0.0	0.0058	0.0064	0.0054	0.0088	0.0248	0.0052	0.0264	0.0337	0.0743	0.0060	0.0072	0.0059	0.0099	0.0289	0.0053	0.0329	0.0351	0.0898

Table 5: **AMSE of SSUR and OLS estimators for model Y_1 with X_{11} and X_{12} positively correlated. (n=1000)**

	AMSE (n=1000, r = 5000)																	
	SSUR									OLS								
$\rho_{x_{11},x_{12}}$	β_{10}	β_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	β_{41}	β_{10}	β_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	β_{41}
0.9	0.0006	0.0028	0.0028	0.0009	0.0026	0.0005	0.0026	0.0036	0.0079	0.0006	0.0032	0.0032	0.0010	0.0029	0.0005	0.0027	0.0036	0.0091
0.8	0.0006	0.0015	0.0015	0.0009	0.0026	0.0005	0.0026	0.0036	0.0076	0.0006	0.0017	0.0017	0.0010	0.0029	0.0005	0.0027	0.0036	0.0089
0.7	0.0006	0.0011	0.0011	0.0009	0.0026	0.0005	0.0025	0.0036	0.0071	0.0006	0.0012	0.0012	0.0010	0.0029	0.0005	0.0027	0.0036	0.0082
0.6	0.0006	0.0009	0.0009	0.0009	0.0026	0.0005	0.0025	0.0036	0.0066	0.0006	0.0010	0.0010	0.0010	0.0029	0.0005	0.0027	0.0036	0.0076
0.5	0.0006	0.0007	0.0007	0.0009	0.0026	0.0005	0.0025	0.0036	0.0062	0.0006	0.0008	0.0008	0.0010	0.0029	0.0005	0.0027	0.0036	0.0071
0.4	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0025	0.0036	0.0059	0.0006	0.0008	0.0008	0.0010	0.0029	0.0005	0.0027	0.0036	0.0065
0.3	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0024	0.0036	0.0056	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0061
0.2	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0024	0.0036	0.0053	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0058
0.1	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0051	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0054
0.0	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0048	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0051

Table 6: **AMSE of SSUR and OLS estimators for model Y_1 with X_{11} and X_{12} negatively correlated.(n=1000)**

	AMSE (n=1000, r =5000)																	
	SSUR									OLS								
$\rho_{x_{11},x_{12}}$	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}	β_{10}	f_{11}	β_{12}	β_{20}	β_{21}	β_{30}	β_{31}	β_{40}	f_{41}
-0.9	0. 0006	0.0029	0.0030	0.0009	0.0026	0.0005	0.0024	0.0036	0.0376	0.0006	0.0032	0.0032	0.0010	0.0029	0.0005	0.0027	0.0036	0.0435
-0.8	0.0006	0.0015	0.0016	0.0009	0.0026	0.0005	0.0023	0.0036	0.0255	0.0006	0.0017	0.0017	0.0010	0.0029	0.0005	0.0027	0.0036	0.0305
-0.7	0.0006	0.0011	0.0011	0.0009	0.0026	0.0005	0.0023	0.0036	0.0194	0.0006	0.0012	0.0012	0.0010	0.0029	0.0005	0.0027	0.0036	0.0235
-0.6	0.0006	0.0009	0.0009	0.0009	0.0026	0.0005	0.0023	0.0036	0.0157	0.0006	0.0010	0.0010	0.0010	0.0029	0.0005	0.0027	0.0036	0.0191
-0.5	0.0006	0.0008	0.0008	0.0009	0.0026	0.0005	0.0023	0.0036	0.0133	0.0006	0.0009	0.0009	0.0010	0.0029	0.0005	0.0027	0.0036	0.0161
-0.4	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0115	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0138
-0.3	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0101	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0121
-0.2	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0091	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0108
-0.1	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0083	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0098
0.0	0.0006	0.0006	0.0006	0.0009	0.0026	0.0005	0.0023	0.0036	0.0076	0.0006	0.0007	0.0007	0.0010	0.0029	0.0005	0.0027	0.0036	0.0089

The different positive and negative correlation level values are given in Tables 5 and 6 for sample size $n = 1000$ respectively. In Table 5, the intercept AMSE values are also fixed for both the SSUR and OLS estimators when replicated 5000 times in turn. That is, $\beta_{10}, \beta_{20}, \beta_{30}, \beta_{40}$ with AMSE values $0.0006, 0.0009, 0.0005, 0.0036$ for the SSUR estimator and $0.0006, 0.0010, 0.0005, 0.0036$ for the OLS estimators. The $AMSE_{orthogonal}$ at which the predictors are purely uncorrelated for the SSUR estimators are $0.0006, 0.0006$ while for the OLS estimators $0.0007, 0.0007$.

Although, the difference in the AMSE values was generally low compared to other sample sizes, however the efficiency of the SSUR estimators reduced as the collinearity levels became severe. This is obvious from the following randomly selected values: $\rho_{x_{11},x_{12}} = 0.9, 0.7, 0.5, 0.2$ which are given as 0.0028, 0.0011, 0.0007, 0.0006 and 0.0028, 0.0011, 0.0007, 0.0006 for SSUR estimators while 0.0032, 0.0012, 0.0008, 0.0007 and 0.0032, 0.0012, 0.0008, 0.0007 for OLS estimators. Similar results were also obtained for the negative correlation values.

Table 7: AMSE for SSUR Estimators f_{11} *and* β_{12} **of model** Y_1 at TNCP = +0.3

	f_{11}			Barlett test		β_{12}			Barlett test	
Sample Size	SUR $AMSE_{orthogonal}$	SUR $AMSE_{Nonorthogonal}$	**difference**	χ^2	**p-value**	SUR $AMSE_{orthogonal}$	SUR $AMSE_{Nonorthogonal}$	**difference**	χ^2	**p-value**
25	0.1328	0.1967	0.0639	0.0923	0.041	0.1115	0.1674	0.0559	0.1056	0.027
100	0.0065	0.0069	0.0004	0.8341	0.657	0.0055	0.0059	0.0004	0.1231	0.985
1000	0.0006	0.0006	0.0000	0.9925	0.834	0.0006	0.0006	0.0000	0.1345	0.917

Table 8: AMSE for SSUR Estimators f_{11} *and* β_{12} **of model** Y_1 at TNCP = -0.3

	f_{11}			Barlett test		β_{12}			Barlett test	
Sample Size	SUR $AMSE_{orthogonal}$	SUR $AMSE_{Nonorthogonal}$	**difference**	χ^2	**p-value**	SUR $AMSE_{orthogonal}$	SUR $AMSE_{Nonorthogonal}$	**difference**	χ^2	**p-value**
25	0.1387	0.1963	0.0576	0.0811	0.032	0.1876	0.2606	0.0730	0.989	0.031
100	0.0064	0.0068	0.0004	0.5630	0.736	0.0054	0.0059	0.0004	0.1432	0.976
1000	0.0006	0.0006	0.0000	0.7914	0.814	0.0006	0.0006	0.0000	0.1568	0.889

6.0 Summary

As earlier stated, the objective of this study is to determine the admissible correlation level (TNCP) among the predictors in separate equation of SSUR model. We considered the TNCP point as the correlation point or a range of correlation values at which AMSE of SSUR estimator with non-orthogonal (correlated) covariates ($AMSE_{NonOrthogonal}$) do not differ from its AMSE values ($AMSE_{Orthogonal}$) at which the predictors are purely orthogonal (uncorrelated). The study revealed fairly stable results with large sample sizes (n= 100, 1000). Therefore, our judgments are based on large sample sizes. From Table 3, the $AMSE_{Orthogonal}$ values for f_{11}, β_{12} (n =100) SSUR estimators are 0.0065 and 0.0055 respectively. These are their AMSE values at $\rho_{x_{11},x_{12}} = 0.0$. Also, the $AMSE_{NonOrthogonal}$ values for f_{11}, β_{12} for the SSUR estimators at $\rho_{x_{11},x_{12}} = 0.3$ are 0.0069 and 0.0059 respectively. Therefore, the simple difference ($AMSE_{NonOrthogonal}$ - $AMSE_{Orthogonal}$) between these two sets of AMSE for each of the f_{11}, β_{12} gave 0.0004, 0.0004 $\approx$ 0.000 respectively. The $AMSE_{NonOrthogonal}$ at $\rho_{x_{11},x_{12}} = 0.3$ agrees with the $AMSE_{Orthogonal}$ up to three decimal places for SSUR estimators with the large sample sizes. However, there was obvious loss in the efficiency of the SSUR estimators when we take the difference between the $AMSE_{NonOrthogonal}$ and $AMSE_{Orthogonal}$ values at other higher correlation levels beyond the range $TNCP \geq 0.3$. We also examined the negative correlation values and found almost similar results that we got for the positive values. The results obtained supported the TNCP threshold levels of ±0.3. However, it was noted that at higher sample size n= 1000 that there was perfect agreement between the $AMSE_{NonOrthogonal}$ and $AMSE_{Orthogonal}$ even beyond the TNCP threshold levels of ±0.3. These results pointed out one of the remedies of multicollinearity, which is working with larger sample size(s). However, the higher correlation levels beyond ±0.3 at which such perfect agreement was achieved are ignored. For instance, the SSUR estimators remain efficient (both $AMSE_{NonOrthogonal}$ and $AMSE_{Orthogonal}$ remain the same) up to the collinearity level of ±0.6 at n =1000. This apparent efficiency is not, however, sustained at sample size n = 100 after the collinearity level of ±0.3 at each of these sample sizes. Since the SSUR estimator is found to be efficient in the presence of collinearity levels up to ±0.3 and at a sample size n = 100, shows that its efficiency will obviously improve at higher sample sizes even at collinearity levels beyond the TNCP threshold values. The equality or otherwise of the AMSE is further established at TNCP = ± 0.3 using the Bartlett's test for homogeneity of variances (see Table 7).

This study also reveals the efficiency of SSUR estimator over OLS method at different sample sizes. Therefore, taking cognizance of the contemporaneous correlation among the response variables improves the efficiency of estimators from SSUR models over OLS. This work further reveals that efficiency of SSUR estimator over the OLS is more of a function of contemporaneous correlation and the sample size rather than the presence of multicollinearity alone which is the most commonly cited scenario. The various collinearity levels considered for the two covariates *X11* and *X12* of equation *Y1* only affected the regression estimates for equation Y_1 while the estimates of equations *Y2, Y3, Y4* remain the same under specific cases considered. This is because the two covariates that are collinear belong to equation *Y1* only. Therefore, the changes in the levels of their collinearity can only affect the regression estimates of equation *Y1*.

7. Conclusion

Therefore, this study found the TNCP to fall within the two ends -0.3 and +0.3. That is, $TNCP \leq 0.3$. This is the admissible correlation range of values that could exist between any pair of predictors in SSUR system of equations at which the efficiency of SSUR estimators would still be preserved at large sample sizes. This is specifically true for SSUR models that satisfied the conditions established for the simulation studies in this work.

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