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NONLINEAR ANALYSIS OF THE DYNAMICS OF HAZARDOUS DRINKING: MATHEMATICAL INSIGHTS FROM LOCAL ALCOHOLIC BEVERAGES IN JOS, NIGERIA

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Abstract

A nonlinear mathematical alcoholism model was formulated to study the dynamics of the Local Alcoholic Beverages (LABs). The expression for the Jos locally brewed beverages' threshold, R_0, was determined using the next generation method and using it, the conditions for the existence of both the drinking-free and drinking-persistence equilibria were established and discussed. The drinking free equilibrium was shown to be locally asymptotically stable if $R_0 < 1$, in which case the density of individuals that consume LABs gradually disappears. The existence of the global stabilities of the drinking-free and drinking-persistence equilibria were also. The model was numerically simulated and findings showed that an effective implementation of rehabilitation programme has the capacity to substantially reduce the menace of problem drinking.

1. Introduction

The characteristic dynamism and complexity of human life [1 – 4], basically influences the formation of human culture and traditions; and is fundamentally the modifier of behavioural patterns. There are many reasons why people identify with associations and groups [2 – 4]. They may include, but not limited to, exuberances, curiosities, sociability, ignorance, imitations and influences from revered personalities.

Depending on the level of consumption, alcohol affects the drinker in a wide range of areas [1, 4, 5, 6, 7]. Alcohol consumption, particularly heavy drinking, is an important risk factor for many health issues [4, 6, 7, 8, 9]. The most common disease categories that are entirely or partly caused by alcohol consumption include infectious diseases, cancer, diabetes, neuropsychiatric diseases (including alcohol used is orders), cardiovascular disease, liver and pancreas disease, and unintentional and intentional injury [7].WHO asserts that harmful alcohol consumption is responsible for about 2.3 million deaths and accounts for 3.8% of global mortality [4].

The poor hygienic conditions under which most African traditional native beers are brewed often leads to their contamination [1, 4, 7, 8]. With a total alcohol per capita (at least 15 years of age)consumption of 13.4 litres of pure alcohol per year, Nigeria leads in Africa while at the global stage trails Czechai, Lithuania and the Republic of Moldova at, respectively, 14.4, 15.0 and 15.2 litres per year [10].The incessant consumption of *Burkutu*, *pito* and *goskolo* in and around Jos has significantly affected the sociocultural ideology and productivity of these communities [3, 4, 11].While both *burkutu* and *pito* are tolerated by law, *goskolo* is a lethal banned liquor secretly consumed across Jos [5].

Mathematical modelling has been used to study a range of human behavioural dynamics including drinking, smoking, and drug use (see [2, 12– 19] and the references contained therein). In particular, the behavioural effect and consequence of accepting or not accepting heavy drinking as a societal problem has studied in [11].

2. Model formulation

The total population of the model, $P(t)$, at time t is categorised into five mutually exclusive alcohol situational compartments, namely, the category of individuals with very strong moral dislike for alcohols and so do not drink, denoted by $P_N(t)$, individuals with compromised moral standpoints about alcoholic beverages (due to alcoholic contacts)and so are contemplating drinking or are drinking only moderately, denoted by $P_M(t)$, the compartment of those who drink heavily at least some of the time but do not consider heavy drinking a problem, denoted by $H_D(t)$, those alcoholics who consider and admit that heavy drinking is a problem, denoted by $H_A(t)$ and the category of temporarily recovered drinkers, denoted by $H_T(t)$. So that the total population at time t is given by

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$$P(t) = P_{\mathrm{N}}(t) + P_{\mathrm{M}}(t) + H_{\mathrm{D}}(t) + H_{\mathrm{A}}(t) + H_{\mathrm{T}}(t). \quad (1)$$

We assume that the contemplation for drinking is influenced by the direct alcoholic contacts between individuals with dislike for alcoholic beverages and either/both moderate or/and heavy (hazardous) drinkers (alcoholics). Individuals with strong dislike for alcohol would hypothetically avoid making contacts with alcoholics because of morality (or religious/health reasons). Heavy drinking on the other hand, is consequent on sustained alcoholic contacts between moderate drinkers and heavy drinkers due to condescending or waning reservations. The population of individuals with dislike for alcoholic beverages is increased by the recruitment of individuals (assumed to have an initial strong dislike for alcoholic beverages) into the population, at a rate Π. An individual from this subpopulation begins to contemplate drinking, following a condescending moral standpoints due to effective initiation contacts with alcoholics (i.e. those in the H_{D}, H_{A} and H_{T} classes) at an initiation rate λ, given by:

$$\lambda = \beta \frac{\eta_1 H_{\mathrm{D}} + \eta_1 H_{\mathrm{A}} + \eta_3 H_{\mathrm{T}}}{\mathrm{P}}, \quad (2)$$

where β is effective alcoholism contact rate sufficient to for initiation into drinking. The modification parameters η_1, η_2 and η_3 account for the relative initiation effectivity of individuals in the H_{D}, H_{A} and H_{T} classes. Here, we consider alcoholics who refuse accepting heavy drinking as a problem to have the most influence on probability of initiation due, hypothetically, to their unguarded mixing within the population. Further, rehabilitated alcoholics have the least initiation effect in the population (because therapy ultimately improves/restores integrity of rehabilitated alcoholics).Thus, we assume that $\eta_1 > \eta_2 > \eta_3$. The subpopulation of moderately drinking individuals, P_{M}, is increased by initiation and reduced by progression into heavy drinking at the rate ϕ. In categorising the heavy drinking class into individuals who refuse to admit and those who have admitted heavy drinking as a problem, we haveproposed the quantity *d* to modify the fraction of the moderately drinking individuals that progress to the problem admitting alcoholics, H_{A}, and the remaining fraction, $1-d$, move to the problem non-admitting alcoholic class. The status (alcoholism) receding possibility for an individual in the nonadmitting class is modelled by the eventual admittance rate α. The fraction of alcoholics who get enrolled into rehabilitation is modelled by the parameter ψ. We represent the recovery success rate by the parameter γ, and maintain that a fraction, ρ, of these would relapse into drinking to increase the number of problem admitting alcoholics while the remaining fraction, $1-\rho$, move to the class of individuals that dislike alcoholic beverages. It is assumed that individuals who have been exposed to alcoholic beverages and may only drink moderately do not initiate others into drinking. Finally, natural mortality occurs in all classes at a rate μ, while heavy drinkers (individuals in both H_{A} and H_{D} classes) are prone to an alcohol-induced mortality, accounted for by the rate δ.

combing the aforementioned definitions and assumptions, the transfer diagram for the initiation dynamics of locally brewed beverages among the residents of Jos can be described by the following system of differential equations:

$$\begin{aligned}
\frac{dP_{\mathrm{N}}}{dt} &= \Pi + \gamma(1-\rho)H_{\mathrm{T}} - \eta_1 \beta P_{\mathrm{N}} \frac{H_{\mathrm{D}}}{P} - \eta_2 \beta P_{\mathrm{N}} \frac{H_{\mathrm{A}}}{P} - \eta_3 \beta P_{\mathrm{N}} \frac{H_{\mathrm{T}}}{P} - \mu P_{\mathrm{N}}, \\
\frac{dP_{\mathrm{M}}}{dt} &= \eta_1 \beta P_{\mathrm{N}} \frac{H_{\mathrm{D}}}{P} + \eta_2 \beta P_{\mathrm{N}} \frac{H_{\mathrm{A}}}{P} + \eta_3 \beta P_{\mathrm{N}} \frac{H_{\mathrm{T}}}{P} - (\mu + \phi)P_{\mathrm{M}}, \\
\frac{dH_{\mathrm{D}}}{dt} &= \phi(1-d)P_{\mathrm{M}} - (\alpha + \mu + \delta)H_{\mathrm{D}}, \\
\frac{dH_{\mathrm{A}}}{dt} &= \gamma\rho H_{\mathrm{T}} + \phi d P_{\mathrm{M}} + \alpha H_{\mathrm{D}} - (\mu + \psi + \delta)H_{\mathrm{A}}, \\
\frac{dH_{\mathrm{T}}}{dt} &= \psi H_{\mathrm{A}} - (\mu + \gamma)H_{\mathrm{T}}.
\end{aligned} \quad (3)$$

3. Stability analyses

3.1. Local stability analysis of the model

The initiation free equilibrium (IFE), E_0, of the Jos locally brewed alcohol (JLBA) model (3) is obtained by setting all the alcohol related classes to $(P_{\mathrm{M}}, H_{\mathrm{D}}, H_{\mathrm{A}}, H_{\mathrm{T}})$ zero. Thus,

$$E_0 = \left(\frac{\Pi}{\mu}, 0, 0, 0, 0\right).$$

Thus, without initiation into alcoholism, the Jos population will reach a demographic equilibrium, E_0. It is therefore instructive to analyse the stability of this point.

3.2. The basic reproduction number (BRN)

The BRN is here, implied from epidemiological sense, noted for its significance in the estimation of the average number of secondary alcoholics initiated by a “chronic” drunk in an entirely nondrinking population. It is fundamentally the threshold parameter that determines the stability of the disease free equilibrium in epidemiology. We therefore use the next generation operator approach to obtain the expression for R_0 as

$$\frac{\phi\beta\{dQ_2(\psi\eta_3+\eta_2Q_4)+(1-d)[\alpha(\psi\eta_3+\eta_2Q_4)+\eta_1(Q_3Q_4-\gamma\rho\psi)]\}}{Q_1Q_2(Q_3Q_4-\gamma\psi\rho)},$$

where

$Q_1=\mu+\phi, Q_2=\alpha+\mu+\delta, Q_3=\mu+\psi+\delta, Q_4=\mu+\gamma.$

We can now state the following result.

Lemma 3.1 *The drinking free equilibrium (DFE)* E_0, *of the model (3) is locally asymptotically stable (LAS) if* $R_0<1$ *and unstable if* $R_0>1$.

Thus, Lemma 3.1 implies that when R_0 is less than unity, then the introduction of a small number of alcoholics into Jos would not be sufficient to result in large initiation into problem drinking, and as such problem alcoholism would eventually die out in time.

3.3. Global Stability of DFE

We now investigate the global asymptotic stability (GAS) of the DFE of the model (3)

To achieve this, we first define the invariant region

$\tilde{D}=\{(P_N,P_M,H_D,H_A,H_T)\in\Re_+^5 \mid P_N^*\le P\}$

Theorem 3.2 *The DFE, of the model (3) is GAS in* $\tilde{D}$ *whenever* $R_0\le 1$.

We follow the procedure in [20] to establish the present claim. To achieve this, we reproduce the procedure below for emphasis.

We rewrite the system 1 in the form

$$\frac{dX}{dt}=F(x,\tilde{D}),$$
$$\frac{d\tilde{D}}{dt}=G(x,\tilde{D}), G(x,0)=0, \quad (4)$$

where he components of $x\in\Re^m$ denote the number of nondrinking individuals and $\tilde{D}\in\Re^n$ represents (its component) the number of drinkers including moderate and heavy drinkers. As previously, $E_0=(x^*,0)$ represents the alcohol-free equilibrium of the system (3). The conditions (H1) and (H2) below musts be met to guarantee the LAS of the system

(H1) For $\frac{dx}{dt}=F(x,0), x^*$ is globally asymptotically stable (GAS)

(H2) $G(x,D)=AD-\tilde{G}(x,D), \tilde{G}(x,D)\ge 0$ for $(x,D)\in\tilde{D},$ where A is an M-matrix with nonnegative off diagonal elements and D is the region where the model makes sociological sense.

If the system (4) satisfies the two conditions above, then the following theorem holds:

Theorem *The fixed point* $E_0=(x^*,0)$ *is a GAS equilibrium of (4) if* $R_0<1$ *is LAS and the assumptions (H1) and (H1) are satisfied.*

Proof

It can be noted that H1 follows immediate since $E_0=(x^*,0)$ is LAS for $R_0<1$.

We are now left to establish H2.

It can be noted that the required computational forms are, the nondrinking class $x=(P_N)$ and the drinking class $D=(P_M,H_D,H_A,H_T)$, $F(x,0)=\begin{pmatrix}\Pi-\mu P_N\\0\end{pmatrix}$, so that

$$A=\begin{pmatrix}-(\phi+\mu) & \eta_1\beta & \eta_2\beta & \eta_3\beta\\ \phi(1-d) & -(\alpha+\mu+\delta) & 0 & 0\\ \phi d & \alpha & -(\mu+\psi+\delta) & \gamma\rho\\ 0 & 0 & \psi & -(\mu+\gamma)\end{pmatrix},$$

and

$$\tilde{G}(x,D)=\begin{pmatrix}\beta(\eta_1 H_{\mathrm{D}}+\eta_2 H_{\mathrm{A}}+\eta_3 H_{\mathrm{T}})\left(1-\dfrac{P_{\mathrm{N}}}{P}\right)\\ 0\\ 0\\ 0\end{pmatrix}.$$

Thus, since $0\le P_{\mathrm{N}}\le P,$ then $\tilde{G}(x,D)\ge 0.$ The GAS of $x^*=\left(\frac{\Pi}{\mu},0\right)$of the system $\frac{dx}{dt}=F(x,0)$ follows from [20]. Hence, by the above theorem E_0 is GAS and we conclude that the drinking problem in Jos can be eradicated once the sociological threshold, R_0, can be brought to a value less than unity.

3.3.1. Existence and local stability of Persistent Initiation Equilibrium (PIE)

Existence To find conditions for the existence of an equilibrium for which initiation into problem drinking is persistent in the population (that is a situation where at least one of $P_{\mathrm{M}}^{**}, H_{\mathrm{D}}^{**}, H_{\mathrm{A}}^{**}$ and H_{T}^{**} is non vanishing), denoted by $E_1=\left(P_{\mathrm{N}}^{**},P_{\mathrm{M}}^{**},H_{\mathrm{D}}^{**},H_{\mathrm{A}}^{**},H_{\mathrm{T}}^{**}\right),$ we solve the equations in (3) in terms of the force of initiation at steady state,

$$\lambda^{**}=\frac{\beta\left(\eta_1 H_{\mathrm{D}}^{**}+\eta_2 H_{\mathrm{A}}^{**}+\eta_3 H_{\mathrm{T}}^{**}\right)}{P^{**}}, \tag{5}$$

gives

$$P_{\mathrm{N}}^{**}=\frac{a_1\Pi}{a_1\mu+a_2\lambda^{**}},\ P_{\mathrm{M}}^{**}=\frac{a_1\Pi\lambda^{**}}{Q_1\left(a_1\mu+a_2\lambda^{**}\right)},\ H_{\mathrm{D}}^{**}=\frac{\phi a_1\Pi(1-d)\lambda^{**}}{Q_1Q_2\left(a_1\mu+a_2\lambda^{**}\right)}, \tag{6}$$

$$H_{\mathrm{A}}^{**}=\frac{\phi a_0\Pi Q_4\lambda^{**}}{a_1\mu+a_2\lambda^{**}},\ H_{\mathrm{T}}^{**}=\frac{\phi\psi a_0\Pi\lambda^{**}}{a_1\mu+a_2\lambda^{**}},$$

where

$$a_0=\alpha+d(\mu+\delta),\ a_1=Q_1Q_2(Q_3Q_4-\gamma\rho\psi),\ a_2=a_1-\gamma\phi\psi a_0(1-\rho).$$

Substituting (6) into (5) gives

$$\lambda^{**}=\frac{\beta\lambda^{**}\left[\phi a_1\eta_1(1-d)+\phi a_0Q_1Q_2(\psi\eta_3+\eta_2Q_4)\right]}{a_1Q_1Q_2+\left\{a_1Q_2+\phi\left[a_1(1-d)+\phi a_0Q_1Q_2(\psi+Q_4)\right]\right\}\lambda^{**}}.$$

Multiplying and collecting like terms shows that the nonzero persistence equilibrium of the model satisfies the linear system

$$a_{11}\lambda^{**}-a_{12}=0, \tag{7}$$

where

$$a_{11}=(Q_3Q_4-\gamma\rho\psi)\left[Q_2+\phi(1-d)\right]+\phi(\psi+Q_1)\left[\alpha+d(\mu+\delta)\right]$$

and

$$a_{12}=(Q_3Q_4-\gamma\rho\psi)(R_0-1).$$

It can be easily verified that $a_{11}>0$ and since $Q_3Q_4-\gamma\rho\psi>0,$ then also $a_{12}>0,$ provided $R_0>0.$ Thus, it is obvious that the linear system (7) has only one positive root. Further, for $R_0<1,$ the model would have no positive equilibria as the force of initiation would be negative and sociologically meaningless. These results are summarised below:

Lemma 3.3*The model for the dynamics of initiation into problem drinking of locally brewed beverages in Jos has a unique persistence equilibrium whenever* $R_0>1$ *and no positive persistence equilibrium whenever* $R_0<1.$

3.3.2. Local stability analysis of the Persistent Drinking Equilibrium (PDE)

Theorem 3.5*The unique persistence equilibrium of the dynamics of system (3) is LAS if* $R_0>1$

Proof.

To establish the proof, we equivalently approach it through transforming the problem of analysis to that of analysing the stability of a fixed point. It can be understood that the linear system (7) gives a fixed point problem of the form

$$f\left(\lambda^{**}\right)=\frac{\beta\lambda^{**}\left[\phi a_1\eta_1(1-d)+\phi a_0Q_1Q_2(\psi\eta_3+\eta_2Q_4)\right]}{a_1Q_1Q_2+\left\{a_1Q_2+\phi\left[a_1(1-d)+\phi a_0Q_1Q_2(\psi+Q_4)\right]\right\}\lambda^{**}}. \tag{8}$$

It can be confirmed that

$$f'\left(\lambda^{**}\right)=\frac{\phi\beta(Q_3Q_4-\gamma\rho\psi)\left\{dQ_2(\psi\eta_3+\eta_2Q_4)+(1-d)\left[\alpha(\psi\eta_3+\eta_2Q_4)+\eta_1(Q_3Q_4-\gamma\rho\psi)\right]\right\}}{\left(a_{11}\lambda^{**}+Q_3Q_4-\gamma\rho\psi\right)^2},$$

which can further be simplified to give

$$f'\left(\tfrac{a_{21}}{a_{11}}\right)=\frac{\phi\beta\left(Q_3Q_4-\gamma\rho\psi\right)^2 R_0}{\left(a_{12}+Q_3Q_4-\gamma\rho\psi\right)^2}=\frac{1}{R_0}.$$

It is obvious that

$$f'\left(\tfrac{a_{21}}{a_{11}}\right)=\frac{1}{R_0}<1,$$

whenever $R_0>1.$ Thus, the unique persistence equilibrium is LAS if $R_0>1$ and the prove follows.

Theorem 3.6*The PDE of the model of the dynamics of initiation into problem drinking of locally brewed beverages in Jos (3) is globally-asymptotically stable (GAS) whenever* $R_0>1$

Proof

Consider the Lyapunov function

$$U=P_{\mathrm{N}}-P_{\mathrm{N}}^{**}\left(1+\ln\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\right)+P_{\mathrm{M}}-P_{\mathrm{M}}^{**}\left(1+\ln\frac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\right)+q_1\left[H_{\mathrm{D}}-H_{\mathrm{D}}^{**}\left(1+\ln\frac{H_{\mathrm{D}}}{H_{\mathrm{D}}^{**}}\right)\right]$$
$$+q_2\left[H_{\mathrm{A}}-H_{\mathrm{A}}^{**}\left(1+\ln\frac{H_{\mathrm{A}}}{P_{\mathrm{A}}^{**}}\right)\right]+q_3\left[H_{\mathrm{T}}-H_{\mathrm{T}}^{**}\left(1+\ln\frac{H_{\mathrm{N}}}{H_{\mathrm{T}}^{**}}\right)\right],$$

where $q_1=\dfrac{\eta_1\tilde{\beta}P_{\mathrm{N}}^{**}}{\mu+\delta},\quad q_2=\dfrac{\tilde{\beta}(\mu\eta_2+\psi\eta_3)P_{\mathrm{N}}^{**}}{\mu(\mu+\psi+\delta)},\quad q_3=\dfrac{\eta_3\tilde{\beta}P_{\mathrm{N}}^{**}}{\mu}.$

Then the derivative of the *U* with respect to time is

$$\dot{U}=\dot{P}_{\mathrm{N}}\left(1-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}\right)+\dot{P}_{\mathrm{M}}\left(1-\frac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\right)+q_1\dot{H}_{\mathrm{D}}\left(1-\frac{H_{\mathrm{D}}^{**}}{H_{\mathrm{D}}}\right)+q_2\dot{H}_{\mathrm{A}}\left(1-\frac{H_{\mathrm{A}}^{**}}{H_{\mathrm{A}}}\right)+q_3\dot{H}_{\mathrm{T}}\left(1-\frac{H_{\mathrm{T}}^{**}}{H_{\mathrm{T}}}\right).$$

Substituting the corresponding expressions of the derivatives from the equations in (3) with $\tilde{\beta}=\mu\beta/\Pi,$ and simplifying a little at the persistence state, gives,

$$\dot{U}=\mu P_{\mathrm{N}}^{**}\left(2-\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}\right)+\eta_1\tilde{\beta}P_{\mathrm{N}}^{**}H_{\mathrm{D}}^{**}\left(3-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}-\frac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\frac{H_{\mathrm{D}}^{**}}{H_{\mathrm{D}}}-\frac{H_{\mathrm{D}}}{H_{\mathrm{D}}^{**}}\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\frac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\right)$$
$$+\eta_2\tilde{\beta}P_{\mathrm{N}}^{**}H_{\mathrm{A}}^{**}\left(3-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}-\frac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\frac{H_{\mathrm{A}}^{**}}{H_{\mathrm{A}}}-\frac{H_{\mathrm{A}}}{H_{\mathrm{A}}^{**}}\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\frac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\right)$$
$$+\eta_3\tilde{\beta}P_{\mathrm{N}}^{**}H_{\mathrm{T}}^{**}\left(4-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}-\frac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\frac{H_{\mathrm{A}}^{**}}{H_{\mathrm{A}}}-\frac{H_{\mathrm{A}}}{H_{\mathrm{A}}^{**}}\frac{H_{\mathrm{T}}^{**}}{H_{\mathrm{T}}}-\frac{H_{\mathrm{T}}}{H_{\mathrm{T}}^{**}}\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\frac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\right).$$

Since the arithmetic mean exceeds the geometric mean, the following inequalities hold:

$$2-\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}\le 0,\;\; 3-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}-\frac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\frac{H_{\mathrm{D}}^{**}}{H_{\mathrm{D}}}-\frac{H_{\mathrm{D}}}{H_{\mathrm{D}}^{**}}\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\frac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\le 0,\;\; 3-\frac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}-\frac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\frac{H_{\mathrm{A}}^{**}}{H_{\mathrm{A}}}-\frac{H_{\mathrm{A}}}{H_{\mathrm{A}}^{**}}\frac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\frac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\le 0$$

and $4-\dfrac{P_{\mathrm{N}}^{**}}{P_{\mathrm{N}}}-\dfrac{P_{\mathrm{M}}}{P_{\mathrm{M}}^{**}}\dfrac{H_{\mathrm{A}}^{**}}{H_{\mathrm{A}}}-\dfrac{H_{\mathrm{A}}}{H_{\mathrm{A}}^{**}}\dfrac{H_{\mathrm{T}}^{**}}{H_{\mathrm{T}}}-\dfrac{H_{\mathrm{T}}}{H_{\mathrm{T}}^{**}}\dfrac{P_{\mathrm{N}}}{P_{\mathrm{N}}^{**}}\dfrac{P_{\mathrm{M}}^{**}}{P_{\mathrm{M}}}\le 0.$

Further, following from the nonnegativity of the parameters of the model, it follows that $\dot{U}\le 0$ $\dot{F}$ £ 0 for $R_0>1.$ Therefore, *U*is a Lyapunov function on *D*.

Thus, we have that

$\lim\limits_{t\to\infty}P_{\mathrm{N}}(t)=P_{\mathrm{N}}^{**},\ \lim\limits_{t\to\infty}P_{\mathrm{M}}(t)=P_{\mathrm{M}}^{**},\ \lim\limits_{t\to\infty}H_{\mathrm{D}}(t)=H_{\mathrm{D}}^{**},\ \lim\limits_{t\to\infty}H_{\mathrm{A}}(t)=H_{\mathrm{A}}^{**}$ and $\lim\limits_{t\to\infty}H_{\mathrm{T}}(t)=H_{\mathrm{T}}^{**}.$

Consequently, the largest invariant set in $\left\{(P_{\mathrm{N}},P_{\mathrm{M}},H_{\mathrm{D}},H_{\mathrm{A}},H_{\mathrm{T}})\in D\middle|\dot{U}=0\right\}$ is the singleton $\{E_1\}$.Thus, from LaSalle's invariance principle [21],

$$\left(P_{\mathrm{N}}(t),P_{\mathrm{M}}(t),H_{\mathrm{D}}(t),H_{\mathrm{A}}(t),H_{\mathrm{T}}(t)\right)\to\left(P_{\mathrm{N}}^{**},P_{\mathrm{M}}^{**},H_{\mathrm{D}}^{**},H_{\mathrm{A}}^{**},H_{\mathrm{T}}^{**}\right),$$

as $t\to\infty.$ Thus, every solution of the equations of the model (3) with initial conditions in D approaches E_1 as $t\to\infty$ (whenever $R_0>1$), implying that E_1 is GAS in D when $R_0>1.$

the sociological implication of the above theorem is that once R_0 is greater than unity, then the influx of alcoholics into Jos would escalate the consumption of locally brewed beverages to becoming a societal menace.

4. Numerical Simulations

In this section, we run numerical analyses on H_D and H_A of the model of the dynamics of initiation into problem drinking of locally brewed beverages in Jos (3) using MATLAB to substantiate further on the results found earlier. To achieve this, we adopt the following parameter estimates

Table 1: Parameter for the drinking model and their various descriptions

Parameter	Description	Baseline value	Reference
Π	Recruitment rate of non-drinkers in the population	5000	[4]
γ	Recuperation rate to nondrinking class	0.6	[16]
μ	Natural death rate	0.02	
δ	Alcohol induced mortality rate	0.038	[4]
y	Recovery rate of problem admitting drinkers	0.65	
ρ	Probability that a heavy drinker recuperating will relapse to problem admitting heavy drinking	Variable	Assumed
$1-\rho$	Probability that a heavy drinker recuperating will relapse to problem nonadmitting heavy drinking	Variable	Assumed
η_1, η_2	Transmission coefficients for non-admitting, and admitting drinkers	0.65. 0.25	[18]
η_3	Transmission coefficients for drinkers rehabilitation therapy	0.05	
β	Sufficient alcoholism initiation contact rate	0.336	[4]
α	Problem admittance rate of non-admitting drinkers	0.001	
ϕ	Alcoholism recruitment due to sufficient encounters between non-drinkers and heavy drinkers	0.25	Assumed
d	Probability that a moderate drinker become a problem admitting heavy drinker	Variable	
$1-d$	Probability that a moderate drinker become a problem nonadmitting heavy drinker	Variable	

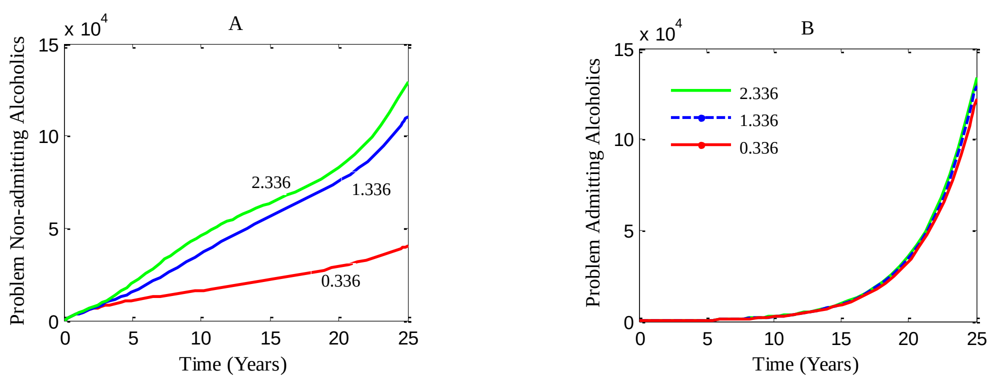

Fig. 1. Plot of the cumulative density of Problem (A) Non-admitting and (B) Admitting alcoholics for different values of β while other parameter values are as in Table 1

In Fig. 1 it can be noted that while both densities show signs of increases with increasing number of initiation contacts, the cumulative density of nonadmitting heavy drinkers (A) records the highest impact of this phenomenon. This could be as a result of the fact that at the onset of drinking, recruits often have the illusion that alcoholism is a world of new wonderful experience and so very little of them, if at all any know of the problems associated with drinking. Fig. 2 show the obvious consequence of the eventual change in perception about drinking. It is noted that the density nonadmitters continue to decrease (A) as members began to admit to the problems of drinking, thereby increasing the density of admitters (B). It is noted from Fig. 4 that rehabilitation has the capacity to completely rehabilitate hazardous drinkers if it will succeed in effectively addressing any possibility of relapse. This presents an enormous potentiality for the eradication of alcoholism. It is therefore a call for political commitment in increasing the capacity of rehabilitation centres as well as an appeal to all relevant pressure groups to engage on massive information based advocacy campaigns.

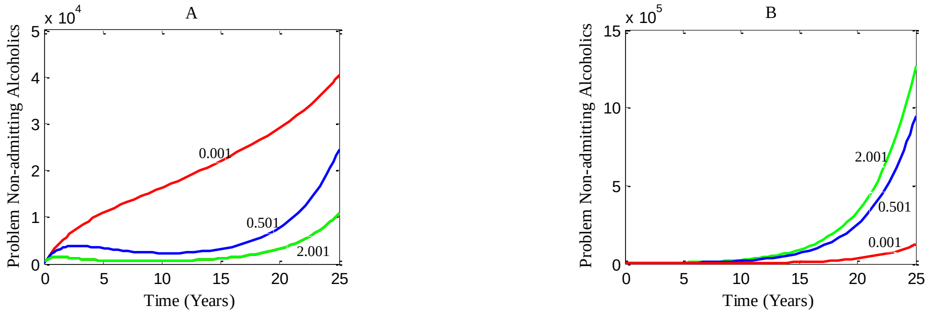

Fig. 2. Plot of the cumulative density of Problem (A) Non-admitting and (B) Admitting alcoholics for different values of α while other parameter values are as in Table 1

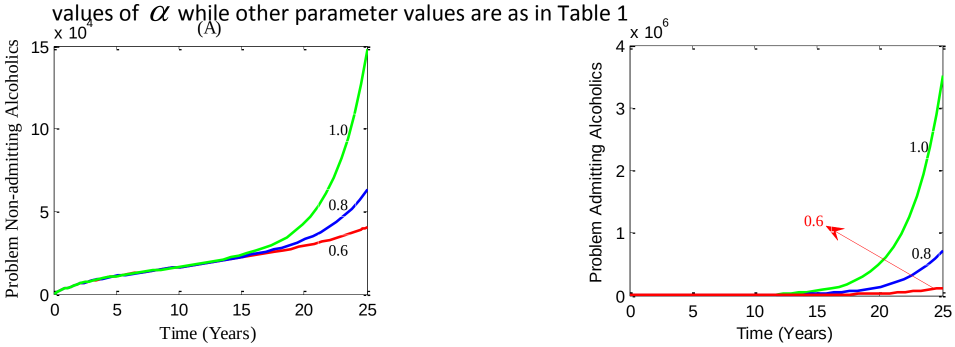

Fig. 3. Plot of the cumulative density of Problem (A) Non-admitting and (B) Admitting alcoholics for different values of γ while other parameter values are as in Table 1

Fig. 3 is seen to suggest the consequence of a defective rehabilitation problem. It can reason out that such a programme would be counterproductive if it fails to adequately facilitate inmates with an informed standpoint against relapse.

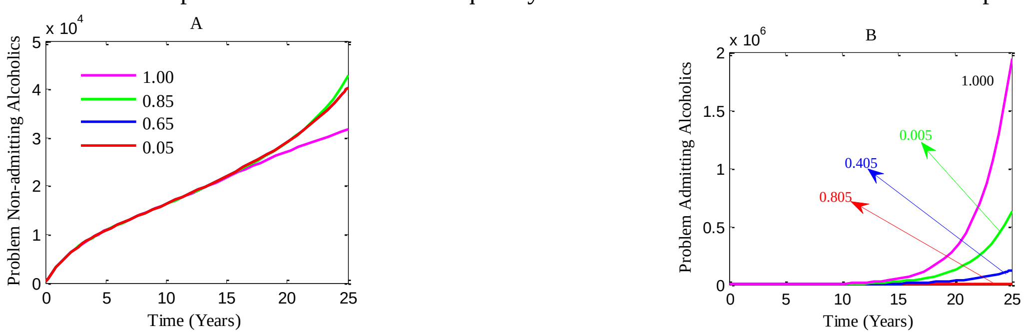

Fig. 4. Plot of the cumulative density of Problem (A) Non-admitting and (B) Admitting alcoholics for different values of ρ while other parameter values are as in Table 1

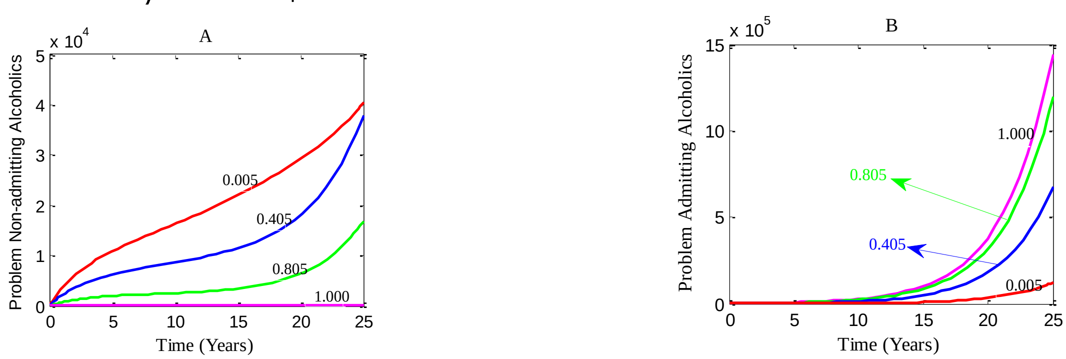

Fig. 5. Plot of the cumulative density of Problem (A) Non-admitting and (B) Admitting alcoholics for different values of d while other parameter values are as in Table 1

As the case scenario in Fig. 4, it can also be understood from Fig. 5 that if the awareness of all moderate drinkers who will eventually become heavy drinkers is raised concerning the eminent problems involved in heavy drinking, then the there could no heavy drinker who will be oblivious of the problems of heavy drinking. This could have a very benificial since admitters are easily enrolled for rehabilitation and with the rajectory in Fig 4, prblem drinking could in time be eradicated in the community.

5. Conclusion

In this paper, a mathematical model has been proposed to study the dynamics of the population of Jos, Nigeria as a result of a characteristic two-fold attitudinal hazardous drinking of locally brewed alcoholic beverages. The basic reproduction number (BRN) for the hazardous drinking model has been obtained and using it, the two equilibria for the model were found. It was shown that both the hazardous drinking free equilibrium and the hazardous persistence equilibrium are locally asymptotically stable when, respectively, the BRN is less than and greater than unity. Computer simulation has equally been carried out to assess the effect of some key parameters on the locally brewed drinking population of the city.

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