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AN EOQ MODEL FOR DELAYED DETERIORATING ITEMS WITH QUADRATIC TIME DEPENDENT HOLDING COST AND BACKORDERING

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Abstract

In this paper, permissible delay in payment is not considered rather the payment is made instantaneously. The optimal cycle length that gives the minimum total inventory cost was at the end determined and the maximum backorder level determined.

Keywords: Inventory, Delayed Deterioration, Backordering

1.0 Introduction

The classical economic order quantity (EOQ) inventory models were developed under the assumptions of constant demand rate. Later, many researchers developed EOQ models with the assumptions of linearly increasing or decreasing demand and exponentially increasing or decreasing demand.

The study of inventory model comes into force in 1915. Harris [1] was the first mathematician who carried out on inventory problems. He developed the simple but famous EOQ formula that was also derived, apparently independently, by Wilson [2]. Gradually, demand of goods may vary with time or with price or with the instantaneous inventory level displayed in a market. In recent years, inventory modelers are working for finding the economic replenishment policy for an inventory system having time dependent demand pattern. Silver and Meal [3] first developed a heuristic approach to determine EOQ in the general case of a time varying-demand pattern.

Donaldson [4] first constructed a model on come out with a full analytic solution of the inventory replenishment problem with a linear trend in demand pattern over a finite-time horizon. Musa and Sani [5] constructed an inventory model of delayed deteriorating items under permissible delay in payment. Musa and Sani [6] developed an EOQ model for delayed deteriorating items. with linear time dependent holding cost. Khanra, et, al. [7] developed an inventory model considering time-quadratic demand rate. During a delay period (or trade credit period) suppliers usually offer their retailers a certain credit period with interest during the permissible delay period. Goyal [8] first developed the EOQ model under the conditions of permissible delay in payment.

In this paper an inventory model for delayed deteriorating items with quadratic time dependent holding cost and backordering is developed. The retailer in this situation does not allow for backordering. The items backordered are settled first when a new replenishment account is received.

2.0 Assumption and Notation

The following notation and assumptions are considered in developing the mathematical model:

Assumptions

(i) instantaneous Inventory replenishment
(ii) Permissible delay in payment not allowed
(iii) Backordering allowed
(iv) Lead time is zero

Notation

K_1 = The demand rate during the period before deterioration set in
K_2= The demand rate after deterioration sets in
EOQ = Economic Order Quantity
TThe inventory cycle length
C The unit cost of the item
T_1 The time the deterioration sets in

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T_2 The length of time with positive stock of the item
T_3 The length of time for which there is deterioration
A The Ordering cost per cycle
i The inventory carrying charge
θ The rate of deterioration
b_1 The maximum shortage (backorder) level permitted
B_c The backorder cost per unit time
B_B The total backorder cost per cycle
$ND(T_2)$ The number of items that deteriorate during the time T_3
q_1 The quantity sold as at the time T_2
I_0 The initial inventory
$I(t)$ The inventory level at any time t before deterioration begins
I_d The inventory level at the time deterioration begins
$I_d(t)$ The inventory level at any time t after deterioration sets in
T_d The total demand between T_1 and T_2
$C(D(T_2))$ The cost of deteriorated items
$H(t)$ The inventory holding cost, where $H(t) = a + bt + ct^2$
C_H The total inventory holding cost in a cycle

Diagrammatic representation of the model

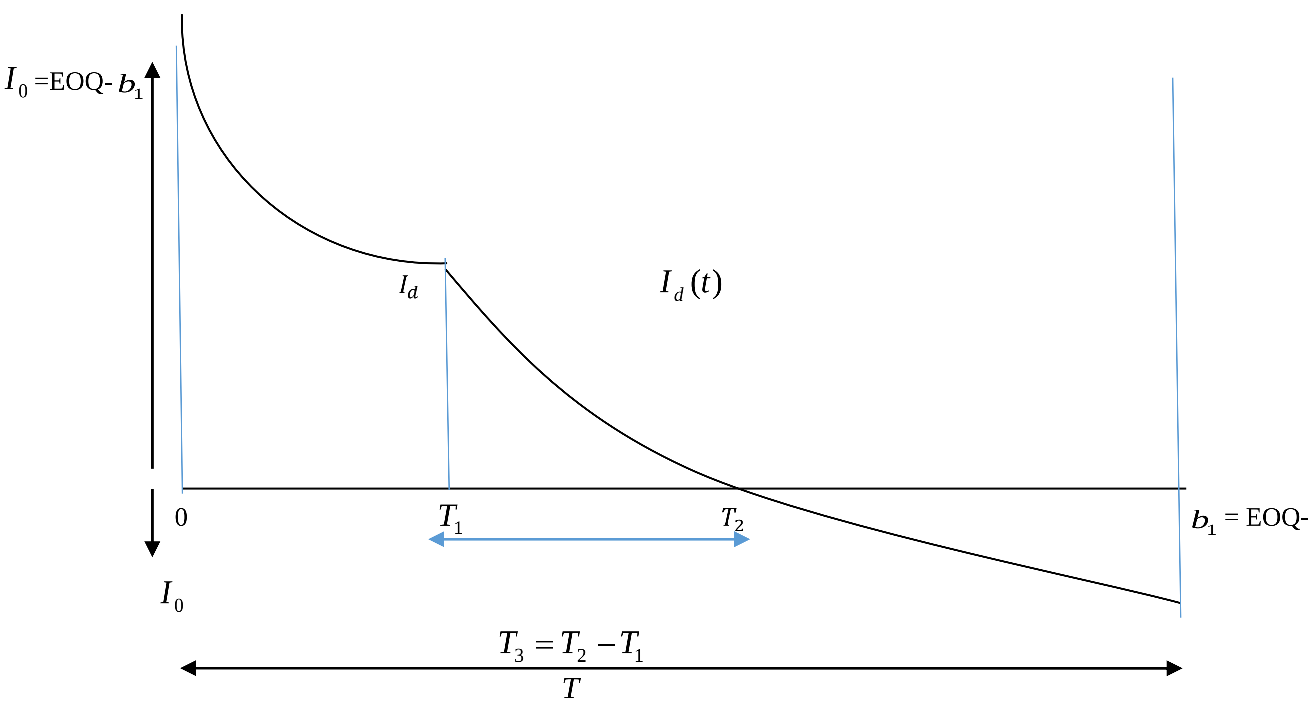

Figure 1: Inventory depletion in a delayed deterioration situation with shortages

3.0 The Mathematical Model

The differential equation that represents the depletion of inventory due to demand only before deterioration sets in is given by:

$$\frac{dI(t)}{dt} = -K_1, \qquad 0 \le t \le T_1 \qquad (1)$$

Separating the variables and solving (1) gives:

$$I(t) = -K_{1,}t + \delta_1 \qquad (2)$$

Whereδ_1 is an arbitrary constant? Now, at $t = 0, I(t) = I_0$, equqtion (2) becomes $I_0 = \delta_1$, so that From (2), we get:

$$I(t) = I_d\, t + I_0 \qquad (3)$$

Also at $t = T_1, I(t) = I_d$, we obtain from equation (3)

$$I_0 = I_d + K_1 T_1 \qquad (4)$$

Substituting equation (4) into equation (3), we have

$$I(t) = I_d + (T_1 - t)\, K_1 \qquad (5)$$

The differential equation that represents the depletion of inventory after deterioration sets in which depends on both demand and deterioration is given by:

$$\frac{dI_d(t)}{dt} + \theta I_d\,(t) = K_2, \qquad T_1 \le t \le T \qquad (6)$$

The solution of equation (6) is given by:

$$I_d\,(t) = \frac{K_2}{\theta} + \delta_2\, e^{-\theta t} \qquad (7)$$

Where δ_2 is an arbitrary constant, applying the conditions at $t = T_1,\ I_d\,(t) = I_d$, we have from Equation (7),$I_d = -\frac{K_2}{\theta} + \delta_2\, e^{-\theta T_1}$

$$\therefore \delta_2 = \left(I_d + \frac{K_2}{\theta}\right) e^{\theta T_1} \qquad (8)$$

Substituting equation (8) into equation (7) gives,

$$I_d\,(t) = -\frac{K_2}{\theta} + \left(I_d + \frac{K_2}{\theta}\, e^{\theta T_1}\right) e^{-\theta t} = -\frac{K_2}{\theta} + \left(I_d + \frac{K_2}{\theta}\right) e^{(T_1 - t)\theta}$$

$$\therefore I_d\,(t) = \frac{K_2}{\theta}\left(e^{(T_1 - t)\theta} - 1\right) + I_d\, e^{(T_1 - t)\theta} \qquad (9)$$

Now at $t = T_2 I_d\,(t) = 0$, equation (9) then becomes

$$I_d = -\frac{K_2}{\theta}\left(e^{(T_1 - T_2)\theta - (T_1 - T_2)\theta} - e^{(T_2 - T_1)\theta}\right) = -\frac{K_2}{\theta}\left(1 - e^{(T_2 - T_1)\theta}\right) \qquad (10)$$

Substituting equation (10) into (9) yields

$$I_d\,(t) = \frac{K_2}{\theta}\left(e^{(T_1 - t)\theta} - 1\right) - \frac{K_2}{\theta}\left(1 - e^{(T_2 - T_1)\theta}\right) e^{(T_1 - t)\theta} = \frac{K_2}{\theta}\left(e^{(T_1 - t)\theta} - 1\right) \qquad (11)$$

Now, substituting equation (10) into (5) yields:

$$I(t) = -\frac{K_2}{\theta}\left(1 - e^{(T_2 - T_1)\theta}\right) + (T_1 - t)\, K_1 \qquad (12)$$

4.0 Computation of the Total Inventory Costs

The total inventory or variable cost is the sum of the inventory ordering cost, cost due to deterioration of inventory items, the total inventory carrying cost and the total backorder cost. The costs as computed individually before they are added together are given below:

(a) The inventory ordering cost is given as A

(b) To compute the cost due to deterioration of inventory, we take into cognizance that:

The total demand between T_1 and $T_2 =$ the demand rate at the beginning of deterioration x the time period during which the item deteriorates. This is given as:

$$T_d = K_2 T_3 = K_2\,(T_2 - T_1)$$

The number of items that deteriorate during the interval, $[T_1, T_2]$ is given as:

$$N(\,d_1\,) = I_d - K_2 T_3 = I_d - K_2\,(T_2 - T_1) \qquad (13)$$

Substituting equation (10) into (13) to have

$$N(\,d_1\,) = -\frac{K_2}{\theta}\left(1 - e^{(T_2 - T_1)\theta}\right) - K_2\,(T_2 - T_1) = -\frac{K_2}{\theta}\left(1 - e^{(T_2 - T_1)\theta} + \theta(T_2 - T_1)\right) \qquad (14)$$

And the total cost due to deterioration of inventory items is given as:

$$CN(\,d_1\,) = -\frac{CK_2}{\theta}\left(1 - e^{(T_2 - T_1)\theta} + \theta(T_2 - T_1)\right) \qquad (15)$$

(c) Inventory Carrying Cost (Or Holding Cost)

The total inventory carrying is given as:

$$C_H = i\int_0^{T_1} H(t)I(t)dt + i\int_{T_1}^{T_2} H(t)\ I_d(t)dt$$

$$= i\int_0^{T_1}(a+bt+ct^2)\left(-\frac{K_2}{\theta}\left(1-e^{(T_2-T_1)\theta}\right)+\ K_1\,(T_1-t)\right)dt + i\int_{T_1}^{T_2}(a+bt+ct^2)\left(\frac{K_2}{\theta}\left(e^{(T_2-t)\theta}-1\right)\right)dt \qquad (16)$$

$$\frac{-iK_2\,a}{\theta}\int_0^{T_1}dt + \frac{iK_2\,ae^{(T_2-T_1)\theta}}{\theta}\int_0^{T_1}dt + iK_1\,aT_1\int_0^{T_1}dt - iK_1\,a\int_0^{T_1}tdt - \frac{-iK_2\,b}{\theta}\int_0^{T_1}tdt + \frac{iK_2\,be^{(T_2-T_1)\theta}}{\theta}\int_0^{T_1}tdt + iK_1\,bT_1\int_0^{T_1}tdt$$

$$-iK_1\,b\int_0^{T_1}t^2\,dt - \frac{iK_2\,c}{\theta}\int_0^{T_1}t^2\,dt$$

$$+\frac{iK_2\,ce^{(T_2-t)\theta}}{\theta}\int_0^{T_1}t^2dt + iK_1\,cT_1\int_0^{T_1}t^2dt - iK_1\,c\int_0^{T_1}t^3dt + \frac{iaK_2}{\theta}\int_{T_1}^{T_2}e^{(T_2-t)\theta}dt - \frac{iaK_2}{\theta}\int_{T_1}^{T_2}dt$$

$$+\frac{ibK_2}{\theta}\int_{T_1}^{T_2}te^{(T_2-t)\theta}dt - \frac{ibK_2}{\theta}\int_{T_1}^{T_2}tdt + \frac{icK_2}{\theta}\int_{T_1}^{T_2}t^2\,e^{(T_2-t)\theta}dt - \frac{iK_2\,c}{\theta}\int_{T_1}^{T_2}t^2dt$$

$$= \frac{-iaK_2}{\theta}[t]_0^{T_1} + \frac{iK_2\,ae^{(T_2-T_1)\theta}}{\theta}[t]_0^{T_1} + iK_1\,aT_1[t]_0^{T_1} - \frac{iK_1\,a}{2}[t^2]_0^{T_1} - \frac{iK_2\,b}{2\theta}[t^2]_0^{T_1} + \frac{iK_2\,be^{(T_2-T_1)\theta}}{2\theta}[t^2]_0^{T_1} + \frac{iK_1\,bT_1}{2}[t^2]_0^{T_1}$$
$$-\frac{iK_1\,b}{3}[t^3]_0^{T_1} - \frac{icK_2}{3\theta}[t^3]_0^{T_1} + \frac{icK_2\,e^{(T_2-T_1)\theta}}{3\theta}[t^3]_0^{T_1} + \frac{icK_1\,T_1}{3}[t^3]_0^{T_1} - \frac{icK_1}{4}[t^4]_0^{T_1} - \frac{iK_2\,a}{\theta^2}\left[e^{(T_2-t)\theta}\right]_{T_1}^{T_2} - \frac{iK_2\,a}{\theta}[t]_{T_1}^{T_2}$$
$$-\frac{iK_2\,b}{\theta^2}\left[te^{(T_2-t)\theta}\right]_{T_1}^{T_2} - \frac{iK_2\,b}{\theta^3}\left[e^{(T_2-t)\theta}\right]_{T_1}^{T_2} - \frac{ibK_2}{2\theta}[t^2]_{T_1}^{T_2} - \frac{iK_2\,c}{\theta^2}\left[t^2e^{(T_2-t)\theta}\right]_{T_1}^{T_2} - \frac{2iK_2\,c}{\theta^3}\left[te^{(T_2-t)\theta}\right]_{T_1}^{T_2}$$
$$-\frac{2iK_2\,c}{\theta^4}\left[e^{(T_2-t)\theta}\right]_{T_1}^{T_2} - \frac{icK_2}{3\theta}[t^3]_{T_1}^{T_2}$$

$$=\frac{-iaK_{2T_1}}{\theta} + \frac{iK_2\,aT_1e^{(T_2-T_1)\theta}}{\theta} + iK_1\,aT_1^2 - \frac{iK_1\,aT_1^2}{2} - \frac{iK_1\,aT_1^2}{2\theta} + \frac{iK_2\,bT_1^2e^{(T_2-T_1)\theta}}{2\theta} + \frac{iK_{1b}\,T_1^3}{2} - \frac{iK_1\,bT_1^3}{3} - \frac{icK_2\,T_1^3}{3\theta} + \frac{icK_2\,T_1^3e^{(T_2-T_1)\theta}}{3\theta} + \frac{icK_1\,T_1^4}{3} - \frac{icK_1\,T_1^4}{4} - \frac{iK_2\,a}{\theta^2} + \frac{iK_2\,ae^{(T_2-T_1)\theta}}{\theta^2} - \frac{iK_2\,aT_2}{\theta} + \frac{iK_2\,aT_1}{\theta} + \frac{iK_2\,b}{\theta^2}\left\{\frac{-T_2}{\theta} + \frac{T_1e^{(T_2-T_1)\theta}}{\theta} - \frac{1}{\theta^2} + \frac{e^{(T_2-T_1)\theta}}{\theta^2}\right\} - \frac{ibK_2\,T_2^2}{2\theta} + \frac{ibK_2\,T_1^2}{2\theta} - \frac{icK_2\,T_2^2}{\theta^2} + \frac{icK_2\,T_1^2e^{(T_2-T_1)\theta}}{\theta^2} - \frac{2icT_2K_2}{\theta^3} + \frac{2iT_1K_2\,ce^{(T_2-T_1)\theta}}{\theta^3} - \frac{2icK_2}{\theta^4} + \frac{2icK_2\,e^{(T_2-T_1)\theta}}{\theta^4} - \frac{icK_2\,T_2^3}{3\theta} + \frac{icK_2\,T_1^3}{3\theta}$$

$$=\frac{iK_2\,aT_1e^{(T_2-T_1)\theta}}{\theta} + \frac{iK_1\,aT_1^2}{2} + \frac{iK_2\,bT_1^2e^{(T_2-T_1)\theta}}{2\theta} + \frac{iK_{1b}\,T_1^3}{2} - \frac{iK_1\,bT_1^3}{3} + \frac{icK_2\,T_1^3e^{(T_2-T_1)\theta}}{3\theta} + \frac{icK_1\,T_1^4}{3} - \frac{icK_1T_1^4}{4} - \frac{iK_2\,a}{\theta^2} + \frac{iK_2\,ae^{(T_2-T_1)\theta}}{\theta^2} - \frac{iK_2\,aT_2}{\theta} - \frac{iK_2\,bT_2}{\theta^2} + \frac{iK_2\,bT_1e^{(T_2-T_1)\theta}}{\theta^2} - \frac{iK_2\,b}{\theta^3} + \frac{iK_2\,be^{(T_2-T_1)\theta}}{\theta^3} - \frac{ibK_2\,T_2^2}{2\theta} - \frac{icK_2\,T_2^2}{\theta^2} + \frac{icK_2\,T_1^2e^{(T_2-T_1)\theta}}{\theta^2} - \frac{2icT_2K_2}{\theta^3} + \frac{2iT_1K_2\,ce^{(T_2-T_1)\theta}}{\theta^3} - \frac{2icT_2K_2}{\theta^3} + \frac{2icK_2\,e^{(T_2-T_1)\theta}}{\theta^4} - \frac{icK_2\,T_2^3}{3\theta}$$

$$\therefore \quad C_H = \left\{e^{(T_2-T_1)\theta} + \frac{K_1\,T_1\theta}{2K_2} + \frac{bT_1e^{(T_2-T_1)\theta}}{2a} + \frac{K_1\,bT_1}{2aK_2} - \frac{K_1\,bT_1^2\theta}{3aK_2} + \frac{icT_1^2e^{(T_2-T_1)\theta}}{3a} + \frac{bT_2^2\theta}{3aK_2} - \frac{cK_1\,T_1^3\,\theta}{4} - \frac{1}{T_1} + \frac{e^{(T_2-T_1)\theta}}{T_1\theta} - \frac{T_2}{T_1} - \frac{bT_2}{T_1a\theta} + \frac{be^{(T_2-T_1)\theta}}{\theta a} - \frac{bT_2}{T_1a\theta^2} + \frac{be^{(T_2-T_1)\theta}}{T_1a\theta^2} - \frac{bT_2^2}{2T_1a} - \frac{cT_2^2}{T_1a\theta} + \frac{cT_1e^{(T_2-T_1)\theta}}{\theta a} - \frac{2cT_2}{T_1a\theta^2} + \frac{2ce^{(T_2-T_1)\theta}}{\theta^2a} - \frac{2c}{\theta^{2T_1a}} + \frac{2Ce^{(T_2-T_1)\theta}}{\theta^3T_1a} - \frac{cT_2^3}{3aT_1}\right\}\frac{iK_2\,aT_1}{\theta} \qquad (16)$$

(d) Total backorder cost

The total backorder cost per cycle is given as: $B_b = B_c\int_0^{T-T_2}K_2\,tdt = \frac{B_cK_2}{2}(T-T_2)^2$

The Total Variable (Inventory) cost per unit time T is given as

$TC(T) = \frac{1}{T}$(Inventory ordering cost + cost due to deterioration of inventory items + Total Inventory holding cost + Total backorder cost)

$\therefore TC(T) = \frac{1}{T}\ (A + CN(d_1) + C_H + B_b$

$$=\frac{A}{T} - \frac{CK_2}{\theta}\left(1 - e^{(T_2-T_1)\theta} + \theta(T_2-T_1)\right) + \left\{e^{(T_2-T_1)\theta} + \frac{K_1\,T_1\theta}{2K_2} + \frac{bT_1e^{(T_2-T_1)\theta}}{2a} + \frac{K_1\,bT_1}{2aK_2} - \frac{K_1\,bT_1^2\theta}{3aK_2} + \frac{icT_1^2e^{(T_2-T_1)\theta}}{3a} + \frac{bT_2^2\theta}{3aK_2} - \frac{cK_1\,T_1^3\,\theta}{4} - \frac{1}{T_1} + \frac{e^{(T_2-T_1)\theta}}{T_1\theta} - \frac{T_2}{T_1} - \frac{bT_2}{T_1a\theta} + \frac{be^{(T_2-T_1)\theta}}{\theta a} - \frac{bT_2}{T_1a\theta^2} + \frac{be^{(T_2-T_1)\theta}}{T_1a\theta^2} - \frac{bT_2^2}{2T_1a} - \frac{cT_2^2}{T_1a\theta} + \frac{cT_1e^{(T_2-T_1)\theta}}{\theta a} - \frac{2cT_2}{T_1a\theta^2} + \frac{2ce^{(T_2-T_1)\theta}}{\theta^2a} - \frac{2c}{\theta^{2T_1a}} + \frac{2Ce^{(T_2-T_1)\theta}}{\theta^3T_1a} - \frac{cT_2^3}{3aT_1}\right\}\frac{iK_2\,aT_1}{\theta T} + \frac{B_cK_2}{2}(T-T_2)^2 \qquad (17)$$

Equation (17) is differentiated to determine the value of T which minimize the total variable cost per unit time as follows:

$$\frac{dTC(T)}{T} = -\frac{A}{T^2} - \frac{CK_2}{\theta}\left\{-\frac{1}{T^2} + \frac{e^{(T_2-T_1)\theta}}{T^2} - \frac{\theta(T_2-T_1)}{T^2}\right\}$$
$$+\left\{\frac{-e^{(T_2-T_1)\theta}}{T^2} - \frac{K_1T_1\theta}{2K_2T^2} - \frac{bT_1e^{(T_2-T_1)\theta}}{2aT^2} - \frac{K_1bT_1}{2aK_2T^2} + \frac{K_1bT_1^2\theta}{3aK_2T^2} - \frac{icT_1^2e^{(T_2-T_1)\theta}}{3aT^2} - \frac{bT_2^2\theta}{3aK_2T^2} + \frac{cK_1T_1^3\theta}{4T^2} + \frac{1}{T_1T^2}\right.$$
$$- \frac{e^{(T_2-T_1)\theta}}{T_1\theta T^2} + \frac{T_2}{T_1T^2} + \frac{bT_2}{T_1a\theta T^2} - \frac{be^{(T_2-T_1)\theta}}{\theta aT^2} + \frac{bT_2}{T_1a\theta^2T^2} - \frac{be^{(T_2-T_1)\theta}}{T_1a\theta^2T^2} + \frac{bT_2^2}{2T_1aT^2} + \frac{cT_2^2}{T_1a\theta T^2} - \frac{cT_1e^{(T_2-T_1)\theta}}{\theta aT^2}$$
$$\left.+\frac{2cT_2}{T_1a\theta^2T^2} - \frac{2ce^{(T_2-T_1)\theta}}{\theta^2aT^2} + \frac{2c}{\theta^2T_1aT^2} - \frac{2Ce^{(T_2-T_1)\theta}}{\theta^3T_1aT^2} + \frac{cT_2^3}{3aT_1T^2}\right\}\frac{iK_2aT_1}{\theta} + \frac{B_cK_2\left(2T(T-T_2)-(T-T_2)^2\right)}{2T^2}$$
$$= 0 \qquad (18)$$

Simplifying further and multiplying equation (18) by T^2 yields:

$$-A - \frac{CK_2}{\theta}\left[-1 + e^{(T_2-T_1)\theta} - \theta(T_2-T_1)\right]$$
$$+\left\{-e^{(T_2-T_1)\theta} - \frac{K_1T_1\theta}{2K_2} - \frac{bT_1e^{(T_2-T_1)\theta}}{2a} - \frac{K_1bT_1}{2aK_2} + \frac{K_1bT_1^2\theta}{3aK_2} - \frac{icT_1^2e^{(T_2-T_1)\theta}}{3a} - \frac{bT_2^2\theta}{3aK_2} + \frac{cK_1T_1^3\theta}{4} + \frac{1}{T_1}\right.$$
$$- \frac{e^{(T_2-T_1)\theta}}{T_1\theta} + \frac{T_2}{T_1} + \frac{bT_2}{T_1a\theta} - \frac{be^{(T_2-T_1)\theta}}{\theta a} + \frac{bT_2}{T_1a\theta^2} - \frac{be^{(T_2-T_1)\theta}}{T_1a\theta^2} + \frac{bT_2^2}{2T_1a} + \frac{cT_2^2}{T_1a\theta} - \frac{cT_1e^{(T_2-T_1)\theta}}{\theta a} + \frac{2cT_2}{T_1a\theta^2}$$
$$\left.- \frac{2ce^{(T_2-T_1)\theta}}{\theta^2a} + \frac{2c}{\theta^{2T_1a}} - \frac{2Ce^{(T_2-T_1)\theta}}{\theta^3T_1a} + \frac{cT_2^3}{3aT_1}\right\}\frac{iK_2aT_1}{\theta} + \frac{B_cK_2}{2}(T^2-T_2^2)$$
$$= 0 \qquad (19)$$

We can use equation (19) with other parameters provided to determine the best cycle length T which minimizes the total variable cost per unit time.

5.0Computation of the Economic Order Quantity (EOQ)

The EOQ corresponding to the best cycle length T can be obtained thus:

EOQ $=K_1T_1 + K_2T_3 + N(d_1) + b_1$

$= K_1T_1 + K_2(T_2-T_1) - \frac{K_2}{\theta}\left[\left(1-e^{(T_2-T_1)\theta}\right) + \theta(T_2-T_1)\right] + K_2(T-T_2)$

$= K_1T_1 - \frac{K_2}{\theta}\left(1-e^{(T_2-T_1)\theta}\right) + K_2(T-T_2)$ (20)

6.0Numerical Example

In this section, we provide a numerical example to illustrate the above model.

Example 1: For the numerical illustration of the developed model, the values of various parameters in proper units can be taken as follows:

$$A = 50, \quad C = 10, \quad a = 0.02, b = 8.00, c = 0.05\ K_2 = 100\ and\ i = 0.04.$$

Solving Equation (19) with the above parameters, we obtain the cycle length$T^* = 0.2244$. On substitution of the optimal value T^* in Equations (17) and (20), we obtain the minimum total cost per unit time $TC^* = 4863.51$ and the economic order quantity EOQ = 211 respectively. We now study the effect of changes in the values of the system parameters and A, C, a, b, c, K_2, ion the optimal cost and number of reorder. The sensitivity analysis is performed by changing each of the parameters by 50%, 25%, -25% -50% and taking one parameter at a time and keeping the remaining parameters un-changed.

The analysis is based on the Example -1 and the results are shown in the **Table 1**. The following points are observed.

Table 1 Sensitivities Analysis

parameter	% change in parameter	T^*	% change in parameter	$TC^*(T)$	% change in parameter	EOQ (unit)	% change in parameter
A	+50	0.2457	9.492	4993.66	2.6761	111	-47.3934
	+25	0.2353	4.857	4930.56	1.3786	111	-47.3934
	-25	0.2130	-5.0802	4791.41	-1.4835	111	-47.3934
	-50	0.2009	-10.4724	4713.59	-3.0825	111	-47.3934
C	+50	0.2018	-23.7211	48.7141	-99.012	111	-47.3934
	+25	0.2013	-10.2941	2504.3230	-48.5079	111	-47.3934
	-25	0.5891	162.5223	6796.0291	39.7351	111	-47.3934
	-50	0.9677	331.2389	9126.2189	87.6468	111	-47.3934
a	+50	0.3331	48.4403	5520.23	13.503	111	-47.3934
	+25	0.2840	26.5597	5233.00	7.5972	111	-47.3934
	-25	0.1416	-36.8984	4261.05	-12.3873	111	-47.3934
	-50	0.1012	-54.902	2823.11	-41.9532	111	-47.3934

b	+50 +25 -25 -50	0.5347 0.4042 0.2596 0.4303	138.2799 80.1248 15.6863 91.7558	5991.300 5515.04 3081.71 2447.18	23.1888 13.3963 -36.6361 -46.6843	111 111 111 111	-47.3934 -47.3934 -47.3934 -47.3934
c	+50 +25 -25 -50	0.6239 0.4672 0.3428 0.5343	178.0303 108.1996 52.7629 138.1016	6318.07 5744.87 2770.87 2062.54	29.9076 18.1219 -43.0274 -57.5915	111 111 111 111	-47.3934 -47.3934 -47.3934 -47.3934
K_2	+50 +25 -25 -50	0.2183 0.2208 0.2299 0.2413	-2.7184 -1.6043 2.451 7.5312	7216.03 6039.93 3686.90 2505.88	48.3708 24.1887 -24.1926 -48.4759	162 212 085 060	-23.2227 0.4739 -59.7156 -71.564
I	+50 +25 -25 -50	6.9001 4.8817 4.8713 6.8928	2974.9109 2075.4456 2070.811 2971.6577	29379.32 21960.89 13886.19 -21315.88	504.0765 351.5441 185.5179 -538.2819	111 111 111 111	-47.3934 -47.3934 -47.3934 -47.3934

7.0 Sensitivities Analysis

Discussion on the results of sensitivity analysis

The results obtained from the sensitivity analysis as presented in Table 1 using Maple software (2017) are discussed below:

1. EOQ decrease while T^*&TC^* increase with increase in value of parameter A. HereT^*,TC^*& EOQ are moderately sensitive to change in A.
2. T^*, TC^*& EOQ decrease with increase in value of parameter C. HereT^*, TC^*& EOQ are moderately sensitive to change in C.
3. T^*, TC^* increase while EOQ decrease with increase in value of parameter a. HereT^*, TC^*& EOQ are moderately sensitive to change in a.
4. T^*, TC^* increase while EOQ decrease with increase in value of parameter b. HereT^*, TC^*& EOQ are moderately sensitive to change in b.
5. EOQ decrease while T^*&TC^* increase with increase in value of parameter c. Here T^*, TC^*& EOQ are moderately sensitive to change in c.
6. EOQ decrease while TC^* increase with increase in value of parameterK_2 HereT^*, TC^*& EOQ are moderately sensitive to change in K_2
7. EOQ decrease while T^*&TC^* increase with increase in value of parameter i. HereT^*, TC^*& EOQ are moderately sensitive to change in i.

8.0 Conclusion

In this paper, we present a mathematical model on the inventory of delayed deteriorating items with backordering. The model is built on the assumption that the holding cost for the inventory items is a quadratic time dependent function. The model considers a situation where the customer is expected to pay for the items as soon as they are received in the inventory which means that the retailer's capital is not constrained.

The optimal cycle length T that gives the minimum total inventory or variable cost, the maximum backorder level allowed and the backorder cost were determined in each of the five examples given in table 1.

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