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A DERIVED METHOD FOR SOLVING FIRST-ORDER LINEAR ORDINARY DIFFERENTIAL EQUATION WITH DERIVATIVE OF THE FORM $ax + by + c$

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Abstract

In this article, we consider the first-order linear ordinary differential equation $y' = -P(x)y + Q(x)$ where P and Q are functions of x (or constants). Though, it is easy to solve foryusing any preferred differential method of solutions yet none of the methods can be used without direct application of integration. We propose a method where the need for differentiation or integration is absent during the computation of the general solution. The derived method is easier and faster than the conventional methods, however it is only applicable when $-P(x)y + Q(x)$ is linear of the form $ax + by + c$.

Keywords: *First-order ordinary differential equation, Derivative, Linear equation.*

1. Introduction

Over half of a century, differential equations are necessity for a mathematical explanation of nature such as Newton's and Lagrange equations for classical mechanics, Maxwell's equations for electromagnetism, Schrodinger's equation for quantum mechanics and Einstein's equation for the general theory of gravitation [1]. A *differential equation* is an equation that involves an unknown function and its derivative [2]. While an *ordinary differential equation* is a differential equation involving ordinary derivatives with respect to a single independent variable [3]. The highest derivative in differential equation is the *order of differential equation* while the highest power of the highest derivative is called degree of differential equation[4].An ordinary differential equation with first derivative is called *first-order ordinary differential equation* if it is in the form [5]

$y' + P(x)y = Q(x).$

Thus, first-order ordinary differential equation involves the dependent variable and its first derivative which can be written in standard form by algebraically solving for y'. The *general solution* of ordinary differential equation is the solution obtained after the integration of the differential equation [6]. The general solution of first-order ordinary differential equation can be obtained from different methods of approach [7, 8, 9, 10, 11].

In this study, we are interested in general solution but not particular solution of first-order ordinary differential equation. Thus, in Section 2, we consider the solution of linear first-order ordinary differential equations whose derivative equals the equation of a straight line. Then we prove a proposition and give some examples to support the proof.

2. First-order ordinary differential equation with derivative of a straight-line equation

If a new proof of an old fact or a new approach to several facts at the same time establishes some previously unsuspected connections between two ideas, then it sometimes leads to a generalization [12]. This research follows the above view by providing an innovative approach to a known method in solving linear first-order ordinary differential equation. Because, sometimes the understanding of a whole field of science is suddenly advanced by the discovery of a single basic equation [13]. From this perception, the proposed method ensures differentiation nor integration do not play role in the computation. However, the derivation of the method is a result of differentiation and integration. The method considers and solves first-order ordinary differential equation of the form$y' = -P(x)y + Q(x)$, such that $Q(x) - P(x)y$ is linear of the form $ax + by + c$.

If the given differential equation, $y' = -P(x)y + Q(x)$, can be rewritten so that the derivative equals equation of a straight line $ax + by + c$, then

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$y' = -P(x)y + Q(x) = ax + by + c$ (2.1)
where a,b and c are known constants. The linear substitution comes from setting
$ax + by + c = u$
and then solve. We proof Equation (2.1) using integrating factor method of solving first-order ordinary differential equation, though other methods can equally work for the solution of y', yet knowledge of differentiation and/or integration is needed

Proposition 2.1 Let the first-order ordinary differential equation be given as $y' = -P(x)y + Q(x)$ such that $y' = -P(x)y + Q(x) = ax + by + c$form a straight-line equation. The general solution of y' from the linear system $ax + by + c$ is
$y = \frac{e^{bx}e^{bk} - a(1-bx) - bc}{b^2}$. (2.2)
Where a,b and c are known constants and$k$is an unknown constant.

Proof
Since $y' = \frac{dy}{dx} = ax + by + c$, let $u = \frac{dy}{dx}$
Thus,
$u = ax + by + c$ (2.3)
Therefore,
$$\frac{du}{dx} = a + b\frac{dy}{dx}$$
This implies that
$$\frac{du}{dx} = a + bu$$
That is
$$\frac{du}{a + bu} = dx$$
Integrating both sides,
$$\int \frac{du}{a + bu} = \int dx$$

That is
$$\frac{1}{b} In(a + bu) = x + k$$
or
$a + bu = e^{b(x+k)}$ (2.4)
Substituting Equation (2.3) in Equation (2.4) to have
$$a + b(ax + by + c) = e^{bx}e^{bk}$$
$$y = \frac{e^{bx}e^{bk} - a - bax - bc}{b^2}$$
Thus,
$$y = \frac{e^{bx}e^{bk} - a(1 - bx) - bc}{b^2}$$

2. Numerical Examples
Example 3.1
Solve the differential equation $y' - 3y = 6$.
Solution
We rewrite the equation in the form
$y' = 3y + 6$.
Thus,$3y + 6$ is an equation of a straight line of the form $ax + by + c$; where $a = 0,\ b = 3,\ c = 6$
Substituting the value of a, b, cin Equation (2.2)
$$y = \frac{e^{bk}e^{bx} - (ax \times b) - (a + bc)}{b^2}$$
Then,

$$y = \frac{e^{3k}e^{3x} - (0(x)\ \times\ 3) - (0\ +\ 3(6))}{3^2}$$

$$y = \frac{e^{3k}e^{3x} - 18}{9}$$

Thus,

$$y = \frac{e^{3k}e^{3x}}{9} - 2$$

Example 3.2

Solve the differential equation $y' + 4y = 3x + 1$.

Solution

We rewrite the equation in the form

$y' = 3x - 4y + 1$

Thus $3x - 4y + 1$ is an equation of straight line of the form $ax + by + c$; where $a = 3,\ b = -4,\ c = 1$

Substituting the value of a, b, c in Equation (2.2)

$$y = \frac{e^{bk}e^{bx} - (ax\ \times\ b) - (a\ +\ bc)}{b^2}$$

Then,

$$y = \frac{e^{-4k}e^{-4x} - \big(3(x) \times (-4)\big) - (3\ + (-4)(1))}{(-4)^2}$$

$$y = \frac{e^{-4k}e^{-4x} - (-12x) - (-1)}{16}$$

$$y = \frac{e^{-4k}e^{-4x} + 12x + 1}{16}$$

Therefore,

$$y = \frac{e^{-4x}e^{-4k} + 12x + 1}{16}$$

Conclusion

The method discussed in this paper can only be used to solve first-order ordinary differential equation of the form$y' = -P(x)y + Q(x)$, where $-P(x)y + Q(x)$ is linear of the form $ax + by + c$. Further investigation may reveal a general form to solve first-order linear ordinary differential equation whose derivative is not linear of the form $ax + by + c$, without recourse to integrating factor.

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