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ACTUARIAL MODELING OF PENSION LIABILITIES AND COMPUTATIONS

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Abstract

Pension valuation exercise for a defined benefit (DB) scheme requires assessment of both the scheme's assets and its liabilities in different circumstances. In any circumstance, there is no specific requirement for consistency in the valuation assumptions used. This study developed models based on the sponsor's criteria for eligibility. The developed models were used for the computations of accruing liabilities for the current and past employees. Data in respect of the categories of employees were collected from a going concern establishment located in Lagos, Nigeria. We assumed that the mortality rates of active members will be in accordance with A67/70 Tables (Ultimate) published by the Institute and Faculty of Actuaries in UK while a(55) Tables (for male and female) published by the same bodies were assumed for the pensioners. These assumptions are based on our experience of similar schemes. The valuation result revealed that the scheme is deficit to the sum of ₦63,160,521.76. Based on this result, we recommend among other things that the Board of Trustee of the scheme to pay adequate attention to scheme's investment put in place by the fund managers with a view to negotiating interest rate from time to time. We also recommend that funding policy should be revised and audited yearly to check if the funding policy has actually met funding objectives since the actuarial costing methods used may fall short of policy objectives.

Keywords: Actuarial modeling, DB, asset valuation, pension liabilities Mathematical Subject classification: 65C20, 91B30

1. INTRODUCTION

The primary intent of pension systems is to protect retired employees from poverty and to enable them to live decent living standards and economic independence when ageing [1,2]. Right from the inception of any scheme, there are targets set which must be met as and when due. At any particular point in the life of a pension scheme, its ability to meet its targeted pension liabilities can, (and is required by regulation to) be assessed, although this can only be a best estimate given that the future is always uncertain [3,4]. To meet the scheme's target, there is need to regularly carryout out valuation of liabilities if the scheme were defined benefit scheme.

A valuation exercise for a defined benefit (DB) scheme requires assessment of both the scheme's assets and its liabilities. While there may be some subjectivity in the valuation of certain types of assets where for example there is no ready market (more prevalent during recessionary times) or where it is considered appropriate to use a smoothed value, the main area of estimation arises in relation to the valuation of liabilities [3,4,5]. The valuation of a defined benefit pension scheme's assets and liabilities is required in at least three different circumstances. It may be required for the purposes of determining whether the fund satisfies the minimum funding standard valuation set down by the regulatory authority. The fund trustees may also require a valuation to review contribution rates and for the purposes of their annual trust report to the members of the pension scheme. Finally in the case of a defined benefit scheme, a valuation may be required for the purposes of the financial statements of the sponsoring company, to recognize the "fair value" of the surplus or deficit in the pension scheme. What is interesting is that there is no specific requirement for consistency in the valuation assumptions used in each of the three valuation processes.

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At any given valuation date therefore, a defined benefit scheme may have three different valuation results, required for three different purposes, each of which would be regarded as fully acceptable for its specific purpose and to its specific target audience. Indeed, the prescribed guidelines to be followed in each of the three valuation processes in themselves necessitate differing assumptions and calculation bases and different emphasis in the produced results [4].

In the current Pension Reform Act [6], it is required for the pension sponsors of defined benefit scheme to set aside funds to pay benefits of employees. Although there are qualifying rule for employees qualifying for claim, the valuation of gratuity after this qualifying rule is met must be done retrospectively as all employees have already survived. For defined benefit plan which has long term liabilities, the valuation approach should include risks such as investment risk and volatility risks which are significantly associated with both claims as well as with investment returns that affect the sufficiency of the capital reserved for covering the claims [7]. Due to significant uncertainties of the incomplete financial markets, DB insurance liabilities cannot be fully hedged since full hedging may result to unreasonable costs. Hence, the inclusion of mortality for purpose of valuation is key to measuring replacement ratio of future funds. For the purpose of calculation involved in the valuation exercise, the measurement of liabilities already earned requires accumulation as opposed to pension that deals with discounting a stream of promised future benefits to the present [8].

Financial economists argue that the discount rate should reflect the risk associated with the liabilities, and given that benefits are guaranteed under most occupational pensions, the appropriate discount factor is a riskless rate, roughly 5 percent, as discussed below. Thus, the economists' model would produce much higher liabilities than those currently reported in the books of states and localities. The intensity of the debate is exacerbated by the assumption that the magnitude of the liabilities dictates the size of the funding contribution and even how the pension fund assets should be invested. This brief attempts to separate the question of valuing liabilities from the questions of funding and investment. As background, it explains the current approach to valuing liabilities in the private and public sectors [8]. Second, it explains reasons why, given their guaranteed status, occupational pension liabilities should be discounted at a riskless rate and shows how much measured liabilities would increase by applying such a rate. Third, it argues that valuing liabilities is only a factor entering the funding calculation and that using a riskless discount rate does not necessarily mean that contributions should increase immediately [8].

Policymakers should also consider the impact of the volatility created by market-based valuations on companies' balance sheets and income statement. More recently, advocates of financial economics have turned their attention to conforming public pensions to their worldview. Proponents of these two worldviews disagree on the way pension liabilities should be measured and accounted for. The conventional approach to accounting and funding is based on a long term funding view of the pension obligation, while the financial economics approach is based on a snapshot benefit accrual view [4].

2. PENSION VALUATION MODELS AND PENSION LIABILITIES

The existence of a pension plan enhances the optimization of in welfare income in old age. The concept of optimization in the light of labour income was pioneered by Merton [9, 10]. Bodie and colleague [11] later introduced flexible labour supply of a life cycle investor under a realistic calibration of the labour income process. Retirement plans are commonly referred to as pension plans or schemes through which funds are set aside for retirement purpose [12], which has to do with a long term financial promise to obtain old age income for retirees. Also, Ross [13] also noted that term pension synonymous to a benefit paid to an employee who retires from work after reaching a prescribed age. Retirement plans are commonly referred to as pension plans or schemes through which funds are set aside for retirement purpose [12], which has to do with a long term financial promise to obtain old age income for retirees. It was also noted that the term pension is synonymous to a benefit paid to an employee who retires from work after reaching a prescribed age [13]. The current salaries for individual employees (representing final salary) as of valuation date were supplied. These salaries were discounted in order to get employees' starting salaries. These amounts were then accumulated to obtain pension for individual employees. Details of the formula used are as follow:

Valuation of the liabilities- An actuarial valuation deals with determination of assets worth and compared with liabilities of a pension plan. The purpose of actuarial valuations is to assess the sustainability of defined benefit plan in the long run for plan sponsors. Defined benefit plan deals with financial commitments promised filled for the years ahead. The models below are developed in line with the rule of defined benefit scheme available in public sector.

Following the retention of defined benefit scheme for the military and secret services in the federation of Pension Reform Act 2004 (amended 2014), the government and those employers that subscribe to DB scheme are duty bound to carry out valuation of liabilities so as to protect the scheme's members from financial inadequacy at retirement. Hence, the rule of formal pension scheme is still required. One of the uncertainties that await the DB members is how to come up with actuarial models that will lead to fair computation of the retirement benefits. The major challenge that many employers of DB scheme likely going to face is how to develop actuarial models in line with the eligibility requirement of DB. In this study, we noted that when a pension is based on final salary, it means that the pension for each year of service will be in a fraction of the salary near retirement, in general 1/Kth. While it is possible for the salary rate at that time of retirement to be used, it is usual for the rules of the scheme to provide that the salary be averaged over the last few years, such as the last three or five years of service. According to Neil [14], the salaries taken may be the salary over a year or the salary rate at the yearly anniversary of the commencement of the scheme. The models used in this paper assumed a final average salary based on the average of the last m years before retirement using the salary over each year. The average of the final salaries on which the pension for a person retiring at age x is based is given the symbol z_x. That is,

$$z_x = \frac{1}{m}\left(s_{x-m} + s_{x-m+1}s_{x-m+2} + \ldots + s_{x-1}\right) \quad (1)$$

The salary on which the pension is based for a member age x whose present salary is S if he retires at age last birthday $x+t$ $\left(\text{assumed to be } x+t+\frac{1}{2}\right)$ will be

$$S\frac{z_{x+t+\frac{1}{2}}}{s_x}$$

The present value of pension on ill-health retirement during the year from age $x+t$ if past service is n years will be

$$\frac{n}{K}\times S\times\frac{z_{x+t+\frac{1}{2}}}{s_x}\times\frac{C^{ia}_{x+t}}{D_x} = \frac{n}{K}\times S\times\frac{{}^{z}C^{ia}_{x+t}}{D_x} \quad (2a)$$

$${}^{z}C^{ia}_{x} = z_{x+t+\frac{1}{2}}C^{ia}_{x} \quad (2b)$$

Summing over the future years of possible retirement, the total value of the past service pension is

$$\frac{n}{K}\times\frac{S}{{}^{s}D_x}\times\sum_{t=0}^{64-x}{}^{z}C^{ia}_{x+t} = \frac{n}{K}\times\frac{S}{{}^{s}D_x}\times{}^{z}M^{ia}_{x+t} \quad (3a)$$

$${}^{z}M^{ia}_{x} = \sum_{t=0}^{64-x}{}^{z}C^{ia}_{x+t} = \sum_{y=x}^{64}{}^{z}C^{ia}_{y} \quad (3b)$$

The total value of past service pension for members at present age x nearest birthday is then

$$\frac{(nS)_x}{{}^{s}D_x}\times\frac{{}^{z}M^{ia}_{x}}{{}^{s}D_x} \quad (4)$$

For future service pension the amount of pension accruing in respect of service in the year from age x+t to x+t+1 will be 1/K of the final salary on retirement after x+t+1 and half of this pension is then

$$\frac{S}{K{}^{s}D_x}\left[\frac{1}{2}{}^{z}C^{ia}_{x}+{}^{z}C^{ia}_{x+t+1}+{}^{z}C^{ia}_{x+t+2}+\ldots\right] = \frac{S}{K{}^{s}D_x}\left[{}^{z}M^{ia}_{x}-\frac{1}{2}{}^{z}C^{ia}_{x+t}\right]$$

$$=\frac{S}{K{}^{s}D_x}{}^{z}\overline{M}^{ia}_{x} \quad (5a)$$

where $${}^{z}\overline{M}^{ia}_{x} = {}^{z}M^{ia}_{x+t} - \frac{1}{2}{}^{z}C^{ia}_{x+t} \quad (5b)$$

Summing this expression for all future years of service gives the value of the total future service pension as

$$\frac{S}{K{}^{s}D_x}\sum_{t=0}^{64-x}{}^{z}\overline{M}^{ia}_{x+t} = \frac{S}{K{}^{s}D_x}{}^{z}R^{ia}_{x+t}; \quad (6a)$$

where $${}^{z}R^{ia}_{x+t} = \sum_{t=0}^{64-x}{}^{z}\overline{M}^{ia}_{x+t} \quad (6b)$$

The total value of future service pension for members at present age x nearest birthday is then

$$\frac{1}{K}(AS)_x\frac{{}^{z}R^{ia}_{x+t}}{K{}^{s}D_x} \quad (7)$$

The computations for pension funds involve assumptions on the rate of mortality, rates of retirement due to age and ill-health, withdrawals rates and salary scales (Benjamin & Pollard, 1980). There is likely to be considerably less data for any individual fund from which to obtain the necessary tables compared to life office valuations so there is not much point in introducing too many refinements into the calculation when so much judgment may be required from the actuary in determining the bases.

Models for Liabilities Repayment: Amortisation Schedules - If a loan is being repaid by the amortization method, each payment is partially repayment of principal and partially payment of interest. Determining the amount of principal and interest contained in each payment is important to both the borrower and lender. An amortization schedule is a table which shows the division of each payment into principal and interest, together with the outstanding principal after each payment is made [15]. Consider a loan of $a_{\overline{n}|}$ being repaid with payments of 1 at the end of each year for n years. Consider the first year of the loan. At the end of the first year the interest due the balance at the beginning of the year is $ia_{\overline{n}|} = 1 - v^n$ (see Table 1). The rest of the total payment of 1, i.e. v^n, must be principal repaid.

The outstanding principal at the end of the year equals the outstanding principal at the beginning of the year less the principal repaid, i.e., $a_{\overline{n}|} - v^n = a_{\overline{n-t}|}$. The same reasoning applies for each successive year of the schedule.

Further insight into the nature of the amortization schedule can be gained by the following argument. The original principal of a $a_{\overline{n}|}$ will accumulate to $a_{\overline{n}|}(1+i) = \ddot{a}_{\overline{n}|}$ at the end of the first year. However, $\ddot{a}_{\overline{n}|} = 1 + a_{\overline{n-1}|}$, $\ddot{a}_{\overline{n}|}$ is sufficient to make the annuity payment of 1 and leave an outstanding balance of $a_{\overline{n-1}|}$ at the end of the first year [15]. The same reasoning applies for each successive year of the schedule.

Table 1: Summarized amortization models for loan repayment

Duration	Payment Amount	Interest Paid	Principal Repaid	Outstanding Principal
0				$a_{\overline{n}|}$
1	1	$ia_{\overline{n}|} = 1 - v^n$	v^n	$a_{\overline{n}|} - v^n = a_{\overline{n-1}|}$
2 . . . t . . . n - 1 n	1 . . . 1 . . . 1 1	$ia_{\overline{n-1}|} = 1 - v^{n-1}$ . . $ia_{\overline{n-t+1}|} = 1 - v^{n-t+1}$. . . 1 1	v^{n-1} . . v^{n-t+1}	$a_{\overline{n-1}|} - v^{n-1} = a_{\overline{n-2}|}$ . . $a_{\overline{n-t+1}|} - v^{n-t+1} = a_{\overline{n-t}|}$ . . . $a_{\overline{3}|} - v^2 = a_{\overline{\ }|}$ $a_{\overline{1}|} - v = 0$
Total	N	n - $a_{\overline{n}|}$	$a_{\overline{n}|}$	

It should be noted that Table 1 is based on an original principal of $a_{\overline{n}|}$.

3. METHODOLOGY

Valuation assumptions and model building

(a) Mortality rates: it has been assumed that the mortality rates of active members will be in accordance with A 67/70 Tables (Ultimate) published by the Institute and Faculty of Actuaries in UK. With regards to pensioner, the mortality rates will be in accordance with the a(55) Tables (for male and female) published by the same bodies. These assumptions are based on our experience of similar schemes.

Expenses: We have not provided for expenses as it was presumed that this would be born separately by the company.

The fund: We are supplied with ₦30,000,000 worth of assets but have no information that there were any other assets besides this.

Relaxing models (1) to (7) models and applying them based on the rule of scheme for the purpose of qualification, many criteria were given on how employees become entitle to 100% of their pay salary. The conditions attached to benefit any employee may be entitled to depend on years of service is contained in the employer's scheme. In order to determine assets of a going concern business, the Actuary uses both economic and demographic statistics which may be guided partly by information supplied by the plan sponsor in question. Such assumptions include but are not limited to average retirement age, rate of salary increment, and expectation of life at retirement and investment return. In the subsequent models, we modified the model (1) to (7) based on how employees will qualify for both gratuities and pensions. The valuation of liabilities was carried out on Past Service Liabilities (PSL). Future Service Liabilities (FSL) consists of benefits already accrued to the existing active members. Also, included are the benefits to current pensioners and deferred pensioners as these are liabilities that had already accrued to past staff of the Company.

$$\text{Pass Service Gratuity}[PSG] = \begin{cases} (\mathrm{AS})_x \times 0.2n \times \dfrac{{}^z\mathrm{M}_{x-n+5}}{D_x} \text{ if } n < 5 \\ \left[(0.6+0.08n)^z\mathrm{M}_x - 0.08(n-5)^z\mathrm{M}_{x-n+10}\right]\dfrac{(\mathrm{AS})_x}{3D_x};\ 5 \le n < 10 \\ (0.2+0.08n)(\mathrm{AS})_x\dfrac{{}^z\mathrm{M}_x}{3D_x}; 10 \le n < 35 \\ 3\times(\mathrm{AS})_x \times \dfrac{{}^z\mathrm{M}_x}{3D_x};\quad n \ge 35 \end{cases} \quad \ldots(8)$$

$(\mathrm{AS})_x$ = The total salaries expected to be recieved in te following year for members at present aged x nearest birthday

$$\text{Future Service Gratuity}[FSG = \begin{cases} \dfrac{(\mathrm{AS})_x}{{}^sD_x}\times\left[0.2(5-n)^z\mathrm{M}_{x-n+5} + 0.08({}^z\mathrm{R}_{x-n+5} - \mathrm{M}_{x-n+10} - {}^z\mathrm{R}_{x-n+35})\right\} \\ \qquad \text{if } n < 5 \\ \dfrac{(\mathrm{AS})_x}{{}^sD_x}\times\left[0.8(5-n)^z\mathrm{R}_x - (10-n)({}^z\mathrm{M}_{x-n+10} - \mathrm{R}_{x-n+10} - {}^z\mathrm{R}_{x-n+35})\right\} \\ \qquad \text{if } 5 \le n < 10 \\ \dfrac{(\mathrm{AS})_x}{{}^sD_x}\times\left[0.8({}^z\mathrm{R}_x - {}^z\mathrm{R}_{x-n+35})\right\}; 10 \le n < 35 \\ 0;\quad n \ge 35 \end{cases} \quad \ldots(9)$$

Guaranteed pensions for active members: We assumed a guarantee of 5 years for members. The nominated beneficiaries or estates of members who die before 5 years elapse at the commencement of pension's payment would receive the remaining outstanding balance.

$$\text{Pass service pension}[PSP] = \begin{cases} \dfrac{(\mathrm{AS})_x}{{}^sD_x}[0.03n^z\mathrm{M}^a_{x-n+5}];\ n < 10 \\ \dfrac{(\mathrm{AS})_x}{{}^sD_x}[0.1+0.02n^z\mathrm{M}^a_{x-n+35}]; 10 \le n < 35 \\ \dfrac{(\mathrm{AS})_x}{{}^sD_x}[0.8^z\mathrm{M}^a_x];\ n \ge 35 \end{cases} \quad \ldots.(10)$$

In most pensions scheme special provision is made for retirement due to ill-health, and as indicated a separate decrement is normally aged for these compared to age retirement.

$$\text{Future Service Pesnion}\,(FSP) = \begin{cases} \dfrac{(\mathrm{AS})_x}{{}^sD_x}[0.03(10-n)^z\mathrm{M}^a_{x-n+10} + 0.02(\mathrm{R}^a_{x-n+10} - \mathrm{R}^a_{x-n+35})];\ n < 10 \\ \dfrac{(\mathrm{AS})_x}{{}^sD_x}[0.02(\mathrm{R}^a_x + \mathrm{R}^a_{x-n+35})]; 10 \le n < 35 \\ 0;\ n \ge 35 \end{cases} \quad \ldots(11)$$

The Projected Unit Credit Method of valuation was adopted whereby the valuation of the current assets of the scheme is compared with the past service liabilities taking into consideration future salary increases. The anticipated contribution for the year immediately following this valuation also considers one year salary increment.

$$\text{Projected Unit Method Gratuity}(PUMG) = \begin{cases} \frac{1}{5}^{z} M^{a}_{x-n+5} \frac{(AS)_x}{{}^{s}D_x};\ n<5 \\ 0.08({}^{z}M_{x+1} - {}^{z}M_{x+n+10}) \frac{(AS)_x}{{}^{s}D_x};\ 5 \le n < 10 \\ 0.08^{z} M_{x+1} \frac{(AS)_x}{{}^{s}D_x};\ 10 \le n < 35 \\ 0;\ n \ge 35 \end{cases} \quad .....(12)$$

Deferred Pensioners

According to [16], the value of pension is given by

$$\text{Value of Pension} = \left\langle \begin{array}{l} A\left[\bar{a}_{\overline{5}|} v^{45-x} + \frac{\bar{N}_{50}}{D_x}\right]; x < 45 \\ A\left[\bar{a}_{\overline{5}|} + \frac{\bar{N}_{x+5}}{D_x}\right]; x \ge 4 \end{array} \right\} \rightarrow \left(\begin{array}{l} \text{This assumes guaranteed pension deferred} \\ \text{until age 45is attained even when death occurs} \\ \text{before age 45} \end{array} \right) \quad ....(13a)$$

Similar result is obtained using [14] as follow:

$$\text{Value of Pension} = \left\langle \begin{array}{l} A\left[\bar{a}_{\overline{5}|} + \frac{\bar{N}_{50}}{D_{45}}\right]\frac{D_{45}}{D_x} + \bar{a}_{\overline{5}|}\left[\frac{\bar{M}_x - \bar{M}_{45}}{D_x}\right]; x < 45 \\ A\bar{a}_{\overline{5}|}\bar{A}_{x:\overline{45-x}|} + A\frac{\bar{N}_{50}}{D_x}; x < 45 \\ A\left\{\bar{a}_{\overline{5}|} + A\frac{\bar{N}_{x+5}}{D_x}\right\}; x \ge 45 \\ A\left[\bar{a}_{\overline{5}|} + \frac{\bar{N}_{x+5}}{D_x}\right]; x \ge 45 \end{array} \right\} \rightarrow \left(\begin{array}{l} \text{This assumes guaranteed pension P} \\ \text{and immediately in the event of death} \\ \text{even if death occurs before age 45} \\ \text{whilst pension is deferred until age} \\ \text{45 for survivours} \end{array} \right) ...(13b)$$

PENSIONERS-Using the fundamental basis of life contingence already established [16], the modified version for the purpose of valuation for active employees is given by:

$$\text{Value of Pension} = \begin{cases} A\left[\bar{a}_{\overline{5-n}|} + \frac{\bar{N}_{x+5-n}}{D_{45}}\right]; n < 5 \\ A\frac{\bar{N}_x}{D_x}; x \ge 5 \end{cases} \quad ...(14)$$

$$\text{n = number of annual pensioners already collected}$$

Membership - A list of active population A(t) of the scheme showing dates of birth and entry into service and salaries were provided. Also provided were list of current pensioners R(t) and the deferred pensioners. For the two categories, dates of birth, dates of commencement of pension and the annual pensioners were given. Following the analysis of the lists, we set below a summary of the members who were included in the valuation.

Table 2: Categories of pensioners with associated benefits

Pensioners	Number	Pensionable salary
Active members	10	390,503.50
Current pensioners	14	6,176,770.00
Deferred pensioner	4	1,121,650.75
Total	28	7,688,924.25

Source: Previous valuation results.

Table 2 reveals previous valuation results for active members, current pensioners and deferred pensioners amounting to ₦7,688,924.25 in the distribution of 10 active members, ₦390,503.50; 14 current pensioners, ₦6,176,770.00; and 4 deferred pensioners, ₦1,121,650.75.

4. RESULT OF VALUATION

We set out how the summary of the results of the valuation as at 31st January, 2018.

(i)	Actuarial deficit in respect of past services (see table below)	₦63,160,521.76
(ii)	Add allowable administrative expenses @ 1%	₦631,605.22
		₦67,792,126.98

Table 3: Summary results of the past service liability of the fund

Capital valuation of assets		Capital valuation of liabilities	
	₦		₦
Fund as at 31/01/2018	30000000	Current members past service pension	57,372,645.27
		current members' past gratuity	6,688,098.62
		Pensioners' benefits	1834899
Past service deficit	67,792,126.98	Deferred pensioners benefits	27264878.87
	93,160,521.76		93,160,521.76

Source: Authors' Computation.

Table 3 reveals past service liabilities of the Company being assessed and the liabilities of pension and gratuities for current members are ₦57,372,645.27 and ₦6,688,098.62 respectively. Pension benefits in respect of pensioners and deferred pensioners amount to ₦1834899 and ₦27264878.87 respectively and all of this represents capital valuation of liabilities as at 31/01/2018. The actuarial deficit in respect of past service for the portion of capital valuation of asset with inclusion of ₦631,605.22 allowable expenses @1% is ₦67,792,126.98.

Table 4: Ammortisation result for the funding gap

Duration	Payment Amount	Interest Paid	Principal Repaid	Outstanding Principal
2018				67792126.98
2019	16,093,606.92	4,067,527.62	12,026,079.30	55,766,047.68
2020	16,093,606.92	3,345,962.86	12,747,644.06	43,018,403.61
2021	16,093,606.92	2,581,104.22	13,512,502.71	29,505,900.91
2022	16093606.92	1,770,354.05	14,323,252.87	15,182,648.04
2023	16,093,606.92	910,958.88	15,182,648.04	0.00

Source: Authors' Computation.

Based the actuarial deficit for the year being investigated, amortization method was suggested on how the liabilities can be paid for. Table 4 is the amortization method employed whereby the Company with commitment to each year payment will be able to liquidate all its liabilities by the year 2023 representing five years from now. If ₦16,093,606.92 is set aside every year consisting of interest paid and principal repaid as indicated in the table at interest rate of 4% is expected to liquidate all the liabilities.

Table 5: Demographic profile of the fund

S/No.	**Pensioners ID**	**Age**	**Years in service**	**Salary** $(AS)_x$	${}^{z}M_x^a$	${}^{s}D_x$	**Past service pension**	**Past service gratuity**
1	P1	57	20	15010.75	88345	11586	57,229.62	206026.62
2	P2	55	19	31374.25	88345	12549	106,019.91	379904.69
3	P3	64	22	97216.5	40199	4279	493,181.42	1790065.9
4	P4	50	16	17304.75	88345	14693	43,700.47	153992.14
5	P5	64	31	58190.5	40199	4279	393,602.23	1465075
6	P6	55	25	9125.75	88345	12549	38,547.19	141339.68
7	P7	64	31	42545.25	40199	4279	287,777.30	1071171.1
8	P8	55	21	48000.5	88345	12549	175,720.31	635296.51
9	P9	56	17	56724.5	88345	12068	182,713.24	647801.5
10	P10	64	15	15010.75	40199	4279	56,407.30	197425.57
Total							**1,834,899.00**	**6688099**

Source: Authors' Computation.

Table 5 shows the computation of pension benefits for 10 pensioners including past gratuity. The total liabilities for the past pension and past service gratuity are ₦1,834,899.00 and ₦6,688,098.622.

Table 6: Deferred pensioners of the fund

S/No.	Pensioners ID	Age	Dx	Pension (p/a)	$\ddot{a}_5 * v^{45-x}(4\%)$	$\overline{N}_x / D_x$	Total
1	P1	30	10997	1,890,271.00	2.521056	2.82995	10,114,858.92
2	P2	31	10032	158,361.00	2.621899	3.10217	906,469.71
3	P3	32	9188	1,765,811.00	2.726775	3.38714	10,796,009.36
4	P4	34	7802	785,160.00	2.949279	3.98885	5,447,540.83
Total							**27,264,878.82**

Source: Authors' Computation.

Table 6 shows the pension benefits of those who have attained retirement age but deferred their pension benefits. The total amount deferred is ₦27,264,878.82. The addition of total computation results in Table 7 and 8 make up the current pensioners for the retired employees and the total value is shown in Table 4. They computations were done separately to give a clear direction to the plan's sponsor in respect of separate tables submitted.

5. CONCLUSION AND RECOMMENDATIONS

This study deals with actuarial modeling of pension liabilities under the defined benefit scheme with the aim of assisting the plan sponsors in ascertaining the accruing pension liabilities and to provide guidelines on how to defray the total liabilities. Using various actuarial assumptions based on the eligibility rules of the scheme submitted by the plan sponsor, valuation result as at 31 November, 2018 was determined. On the basis of the accruing liabilities the following recommendations were made.

(i) The Board of Trustee is advised to pay adequate attention to the management of an investment of the scheme fund by the fund managers. The yield earned on the fund is a major contributor to the solvency of the scheme.

(ii) It is further recommended that the next actuarial valuation should be carried out not later than 31st January, 2023 by which time it is hoped that the above findings and recommendations would have been fully address.

(iii) The funding policy should be amended to include the actuarial cost asset method applied, and amortization and amortization technique used.

(iv) The Trustees should communicate funding policy to employees and plan sponsors based on the advice of the pension actuaries.

(v) The trustees should enforce strict compliance in the funding policy provide for in the actuarial valuation.

(vi) The funding policy must be audited yearly to check if the funding policy has actually met funding objectives since the actuarial costing methods used may fall short of policy objectives.

APPENDIX

Table 7: Pensioner's annuity profile

S/No.	Pensioners ID	Age	$\overline{a}_x$	Pension (A)	$A * \overline{a}_x$
1	P1	57	13.3493	60,043.00	801,532.58
2	P2	54	14.5634	125,497.00	1,827,663.60
3	P3	67	9.39	388,866.00	3,651,450.32
4	P4	53	14.9654	69,219.00	1,035,890.16
5	P5	65	10.15276	232,762.00	2,363,177.75
6	P6	59	12.53867	36,503.00	457,699.17
7	P7	75	6.64839	170,181.00	1,131,429.65
8	P8	55	14.15976	192,002.00	2,718,701.93
9	P9	54	14.5634	226,898.00	3,304,407.40
10	P10	76	8.804782	60,043.00	528,665.51
Total				1,562,014.00	**17,820,618.06**

Source: Authors' Computation.

Table 8: Pensioner's annuity profile

S/No.	Pensioners ID	Age	$\bar{a}_x$	Pension (A)	$A \times a_x$
1	P1	71	7.95727	1,890,271.00	15,041,398.14
2	P2	50	16.1577	153,361.00	2,477,959.01
3	P3	70	8.30212	1,765,811.00	14,659,973.62
4	P4	67	9.39	785,165.00	7,372,696.48
				4,594,608.00	**39,552,027.25**

Source: Authors' Computation.

Table 9:Computation of retirement benefits for calculation of pensions and gratuity in respect of defined benefit scheme

Year of Qualifying Service	Gratuity as percentage of Final pay	Pension as Percentage of final pay
5	100	
6	108	
7	116	
8	124	
9	132	
10	100	30
11	108	32
12	116	34
13	124	36
14	132	38
15	140	40
16	148	42
17	156	44
18	164	46
19	172	48
20	180	50
21	188	52
22	196	54
23	204	56
24	212	58
25	220	60
26	228	62
27	236	64
28	244	66
29	252	68
30	260	70
31	268	72
32	276	74
33	284	76
34	292	78
35	300	80

The above figures are based on models (8) to (14)

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