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THE LATTICE STRUCTURES OF SOFT MULTISET

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Abstract

Soft set which is a mapping from a set of parameters to a power set of a universe was initiated by Molodtsov with the aim of modelling vague-ness and uncertainty in real life situation. In this paper, we deal with the algebraic structures of soft multisets. In particular, the lattice structures of soft multisets were constructed.

Keywords: Multiset, Soft Set, Soft Multiset, Lattice

1. Introduction

Multiset which is an unordered collection of objects where duplicates of objects are admitted was initiated with the goal of addressing repetition which is significant in real life situations. The theory has applications in many fields such as mathematics, computer science, social sciences and so on [1 - 6].

Soft set which is an approximate description of an object consisting of predicate and approximate value set comes up as a result of the requirements for exact solutions in classical mathematics. In real life, some problems are so complicated such that, we cannot set an exact solution, but an approximate one. In soft set, as the initial description of object, is itself of an approximate nature, we needn't require to introduce the concept of an exact solution. The theory attract applications in various fields such as decision making, medical diagnosis, algebra, data analysis, forecasting, game theory etc., as shown by [7 - 11] in recent years.

Soft multisets was first initiated in [12] using the idea of universes, after which various scholars contributed to the development of the theory using different approaches. Using the idea of [12], in [13], the notion of complement of a soft multiset is reintroduced and further shows that the laws of exclusion and contradiction are satisfied which fails in [12]. Distance and similarity between two soft multisets were studied in [14]. Contributing on the idea of [14], the concept of soft multiset was explored and its applications to decision making problems were further discussed in [15].

In this paper, we deal with the algebraic structures of soft multisets. In particular, the lattice structures of soft multisets were constructed.

2. Soft Set

Definition 2.1 [16]

Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F: A \longrightarrow P(U)$.

In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A,\ F(A)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A). Thus (F, A) is defined as

$(F, A) = \{F(e) \in P(U) | e \in E, F(e) = \emptyset \text{ if } e \notin A\}$.

Definition 2.4 Multisets [17, 18] An mset M drawn from the set X is represented by a function *Count* M or C_M defined as $C_M: X \longrightarrow \mathbb{N}$.

Let M be a multiset from X with x appearing n times in M, this is denoted by $x \in^n M$. $M = \{k_1/x_1, k_2/x_1, \dots, k_n/x_n\}$ where M is a multiset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Definitions 2.5 Let M and N be two msets drawn from a set X. Then

(a) $M \subseteq N$ iff $C_M(x) \leq C_N(x), \forall x \in X$.
(b) $M = N$ if $C_M(x) = C_N(x), \forall x \in X$.
(c) $M \cup N = max\{C_M(x), C_N(x), \forall x \in X$.
(d) $M \cap N = min\{C_M(x), C_N(x)\}, \forall x \in X$.
(e) $M - N = \max\{C_M(x) - C_N(x), 0\}, \forall x \in X$.

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Let M be a multiset drawn from a set X. The support set or root set of M denoted by M^* is a subset of X given by $M^* = \{x \in X | C_M(x) > 0\}$. Note that $M \subseteq N$ iff $M^* \subseteq N^*$.
The power multiset of a given mset M, denoted by $P(M)$ is the multiset of all submultisets of M, and the power set of a multiset M is the root set of $P(M)$, denoted by $P^*(M)$.

3. Soft Multiset (Soft mset, for short)

Definition 3.1 [19, 20] Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then a pair (F, A) or F_A is called a soft multiset where F is a mapping given by $F : A \longrightarrow P^*(U)$. For all $e \in A$, the mset $F(e)$ is represented by a count function $C_{F(e)}: U^* \longrightarrow \mathbb{N}$.
Definition 3.3 Let (F, A) and (G, B) be two soft multisets over U. Then
(i) (F, A) is a soft submultiset of (G, B) written $(F, A) \sqsubset (G, B)$ if
(a) $A \subseteq B$
(b) $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.
$(F, A) = (G, B) \Leftrightarrow (F, A) \sqsubset (G, B)$ and $(G, B) \sqsubset (F, A)$.

(ii) **Union:**
$(F, A) \sqcup (G, B) = (H, C)$ where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.
(iii) **Intersection:**
$(F, A) \sqcap (G, B) = (H, C)$ where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.
(iv) **Difference:** $(F, E) \backslash (G, E) = (H, E)$ where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall x \in U^*$.
(v) **Null:** A soft multiset (F, A) is called a Null soft multiset denoted by Φ if $\forall e \in A, F(e) = \emptyset$.
(vi) **Complement:** The complement of a soft multiset (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \longrightarrow P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e), \forall e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \ \forall x \in U^*$.

4. The lattice structures of soft multisets

Definition 4.1 Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then
(i) (F, E) is called the absolute soft multiset denoted U_E if $F(e) = U, \forall e \in E$.
(ii) (F, A) is called a relative absolute soft multiset with respect to A, denoted U_A if $F(e) = U, \forall e \in A$.
Theorem 4.2 Let $(F, A), (G, B)$ and (H, C) be soft multisets over a universe U. Then,
(i) $(F, A) \sqcup (F, A) = (F, A)$
(ii) $(F, A) \sqcup (G, B) = (G, B) \sqcup (F, A)$
(iii) $\big((F, A) \sqcup (G, B)\big) \sqcup (H, C) = (F, A) \sqcup ((G, B) \sqcup (H, C))$
Proof
(i) and (ii) is trivial.
(iii) Let $\big((F, A) \sqcup (G, B)\big) \sqcup (H, C) = (J, A \cup B \cup C)$,
$(F, A) \sqcup \big((G, B) \sqcup (H, C)\big) = (K, A \cup B \cup C)$.
For all $e \in A \cup B \cup C$, $e \in A$ or $e \in B$ or $e \in C$. Without loss of generality, let $e \in A$.
(a) If $e \notin B$ and $e \notin C$, then $J(e) = F(e) = K(e)$.
(b) If $e \in B$ and $e \notin C$, then $J(e) = F(e) \cup G(e)$;
If $C_{F(e)}(x) < C_{G(e)}(x), \forall x \in U^*$, we have $J(e) = G(e) = K(e)$,
If $C_{F(e)}(x) > C_{G(e)}(x), \forall x \in U^*$, we have $J(e) = F(e) = K(e)$,
If $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in U^*$, we have $J(e) = F(e) = G(e) = K(e)$,
(c) If $e \notin B$ and $e \in C$, then $J(e) = F(e) \cup H(e)$;
If $C_{F(e)}(x) < C_{H(e)}(x), \forall x \in U^*$, we have $J(e) = H(e) = K(e)$,
If $C_{F(e)}(x) > C_{H(e)}(x), \forall x \in U^*$, we have $J(e) = F(e) = K(e)$,
If $C_{F(e)}(x) = C_{H(e)}(x), \forall x \in U^*$, we have $J(e) = F(e) = G(e) = K(e)$,
(d) If $e \in B$ and $e \in C$, then $J(e) = \big(F(e) \cup G(e)\big) \cup H(e) = F(e) \cup \big(G(e) \cup H(e)\big) = K(e)$.
Since J and K are the same mset-valued mappings, we have $\big((F, A) \sqcup (G, B)\big) \sqcup (H, C) = (F, A) \sqcup \big((G, B) \sqcup (H, C)\big)$.
Hence the operation $\sqcup$ is idempotent, associative and commutative.
The following theorem can be proved in a similar way.
Theorem 4.3 Let $(F, A), (G, B)$ and (H, C) be soft multisets over a universe U. Then,
(i) $(F, A) \sqcap (F, A) = (F, A)$
(ii) $(F, A) \sqcap (G, B) = (G, B) \sqcap (F, A)$

(iii) $\big((F,A)\sqcap(G,B)\big)\sqcap(H,C)=(F,A)\sqcap((G,B)\sqcap(H,C))$

Theorem 4.4 Let $(F,A),(G,B)$ and (H,C) be soft multisets over a universe U. Then,

(i) $\big((F,A)\sqcup(G,B)\big)\sqcap(F,A)=(F,A)$

(ii) $\big((F,A)\sqcap(G,B)\big)\sqcup(F,A)=(F,A)$

Proof

(i) Let $(F,A)\sqcup(G,B)=(H,A\cup B)$ and $\big((F,A)\sqcup(G,B)\big)\sqcap(F,A)=(J,(A\cup B)\cap A)$.
For all $e\in A$,
(a) If $e\notin B$, then $J(e)=H(e)\cap F(e)=F(e)\cap F(e)=F(e)$.
(b) If $e\in B$, then $J(e)=H(e)\cap F(e)=\big(F(e)\cup G(e)\big)\cap F(e)$;
If $C_{F(e)}(x)<C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)\cap F(e)=F(e)$,
If $C_{F(e)}(x)>C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)\cap F(e)=F(e)$,
If $C_{F(e)}(x)=C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)\cap F(e)=F(e)\cap F(e)=F(e)$,
It follows that $J\,(e)\ =\ F(e)$.
Hence $\big((F,A)\sqcup(G,B)\big)\sqcap(F,A)=(F,A)$.

(ii) Let $(F,A)\sqcap(G,B)=(H,A\cap B)$ and $\big((F,A)\sqcap(G,B)\big)\sqcup(F,A)=(K,(A\cap B)\cup A)$.
For all $e\in A$,
(a) If $e\notin B$, then $e\notin A\cap B$, thus, $\mathrm{K}(e)=F(e)$.
(b) If $e\in B$, then $K(e)=H(e)\cup F(e)=\big(F(e)\cap G(e)\big)\cup F(e)$;
If $C_{F(e)}(x)<C_{G(e)}(x),\forall x\in U^*$, we have $K(e)=F(e)\cup F(e)=F(e)$,
If $C_{F(e)}(x)>C_{G(e)}(x),\forall x\in U^*$, we have $K(e)=G(e)\cup F(e)=F(e)$,
If $C_{F(e)}(x)=C_{G(e)}(x),\forall x\in U^*$, we have $K(e)=G(e)\cup F(e)=F(e)\cup F(e)=F(e)$,
It follows that $K(e)\ =\ F(e)$.
Hence $\big((F,A)\sqcap(G,B)\big)\sqcup(F,A)=(F,A)$.

Theorem 4.5 Let $(F,A),(G,B)$ and (H,C) be soft multisets over a universe U. Then,

(i) $(F,A)\sqcup\big((G,B)\sqcap(H,C)\big)=\big((F,A)\sqcup(G,B)\big)\sqcap((F,A)\sqcup(H,C))$

(ii) $(F,A)\sqcap\big((G,B)\sqcup(H,C)\big)=\big((F,A)\sqcap(G,B)\big)\sqcup((F,A)\sqcap(H,C))$

Proof

(i) Let $(F,A)\sqcup\big((G,B)\sqcap(H,C)\big)=(J,A\cup(B\cap C))$,
$\big((F,A)\sqcup(G,B)\big)\sqcap((F,A)\sqcup(H,C))=(K,(A\cup B)\cap(A\cup C))=(K,A\cup(B\cap C))$.
For all $e\in A\cup(B\cap C)$,
(a) If $e\in A, e\notin B$ and $e\notin C$, then $J(e)=F(e)\cap F(e)=F(e)=K(e)$.
(b) If $e\notin A, e\in B$ and $e\in C$, then $J(e)=G(e)\cap H(e)$;
If $C_{G(e)}(x)<C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)=K(e)$,
If $C_{G(e)}(x)>C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=H(e)=K(e)$,
If $C_{G(e)}(x)=C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=H(e)=G(e)=K(e)$,
(c) If $e\in A, e\in B$ and $e\notin C$, then $J(e)=(F(e)\cup G(e))\cap F(e)$;
If $C_{F(e)}(x)<C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)\cap F(e)=F(e)=K(e)$,
If $C_{F(e)}(x)>C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)\cap F(e)=F(e)=K(e)$,
If $C_{F(e)}(x)=C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)=K(e)$,
(d) If $e\in A, e\notin B$ and $e\in C$, then $J(e)=F(e)\cap\big(F(e)\cup H(e)\big)$;
If $C_{F(e)}(x)<C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)\cap H(e)=F(e)=K(e)$,
If $C_{F(e)}(x)>C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)\cap F(e)=F(e)=K(e)$,
If $C_{F(e)}(x)=C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)=K(e)$,
(e) If $e\in A, e\in B$ and $e\in C$, then $J(e)=F(e)\cup\big(G(e)\cap H(e)\big)=\big(F(e)\cup G(e)\big)\cap\big(F(e)\cup H(e)\big)=K(e)$;
If $C_{F(e)}(x)<C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)\cap\big(F(e)\cup H(e)\big)$;
If $C_{F(e)}(x)<C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)\cap H(e)=K(e)$,
If $C_{F(e)}(x)>C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=G(e)\cap F(e)=K(e)$,
If $C_{F(e)}(x)=C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=K(e)$,
Now, If $C_{F(e)}(x)>C_{G(e)}(x),\forall x\in U^*$, we have $J(e)=F(e)\cap(F(e)\cup H(e))=K(e)$,
If $C_{F(e)}(x)=C_{G(e)}(x)=C_{H(e)}(x),\forall x\in U^*$, we have $J(e)=K(e)$.

It follows that $J\,(e) = K(e)$.
Hence $(F,A) \sqcup \big((G,B) \sqcap (H,C)\big) = \big((F,A) \sqcup (G,B)\big) \sqcap \big((F,A) \sqcup (H,C)\big)$.
(ii) could be proved in a similar way.
This theorem indicates that $\sqcup$ distributes over $\sqcap$, and $\sqcap$ distributes over $\sqcup$.

Theorem 4.6 Let $S(U,E)$ be the set of all soft multisets over the universe U and the parameter set E, that is, $S(U,E) = \{F: A \longrightarrow P^*(U), A \subseteq E\}$, then $(S(U,E),\ \sqcup,\ \sqcap)$ is a bounded distributive lattice.

Proof

By theorems 4.2, 4.3, 4.4 and 4.5, $(S(U,E),\ \sqcup,\ \sqcap)$ is a distributive lattice with U_E and Φ as the upper bound and lower bound respectively.

Theorem 4.7 Let $S(U,A)$ be the set of all soft multisets over the universe U and
the parameter set $A \subseteq E$, that is, $S(U,A) = \{F: A \longrightarrow P^*(U)\}$, then $(S(U,A),\ \sqcup,\ \sqcap)$ is a sub lattice of $(S(U,E),\ \sqcup,\ \sqcap)$. In $(S(U,A),\ \sqcup,\ \sqcap)$, U_A and Φ are the upper bound and lower bound respectively.

Conclusion

In this work, after reviewing some concepts related to soft set, multiset and soft multisets, some lattice structures of soft multisets were presented.

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