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SIMILARITY IN SOLUTIONS OF NONLINEAR STRETCHED BIOMAGNETIC FLOW AND HEAT TRANSFER WITH SIGNUM FUNCTION AND TEMPERATURE POWER LAW GEOMETRY

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Abstract

Biomagnetic fluid dynamics is an interdisciplinary field comprising engineering, medicine, and biologye.t.c.Biofluid dynamics is directed towards finding and developing the solutions to some of the human body related diseases and disorders. This article describes the flow and heat transfer of two dimensional, steady, laminar, viscous and incompressible biomagnetic fluids over a nonlinear stretching sheet in the presence of magnetic dipole. Our model is consistent with blood fluid namely biomagnetic fluid dynamics (BFD). This model isbased on the principles of ferrohydrodynamic (FHD). The temperature at the stretching surface is assumed to follow a power law variation, and stretching velocity is assumed to have a nonlinear form with signum function or sign function.

The governing boundary layer equations with boundary conditions are simplified to couple higher order equations using usual transformations. Numerical solutions for the governing momentum and energy equations are obtained by efficient numerical techniques based on the common finite difference method with central differencing, on a tridiagonal matrix manipulation and on an iterative procedure. Computations are performed for a wide range of the governing parameters such as magnetic field parameter, power law exponent temperature parameter, and other involved parameters and the effect of these parameters on the velocity and temperature field is presented.

It is observed that for different values of the magnetic parameter, the velocity distribution decreases while temperature distribution increases. Besides, the finite difference solutions results for skin-friction coefficient and rate of heat transfer are discussed. This study will have an important bearing on a high targeting efficiency, a high magnetic field is required in the targeted body compartment.

Keywords—Biomagnetic fluid, FHD, nonlinear stretching sheet, slip parameter.

1. INTRODUCTION

Biomagnetic fluid is a fluid which exists in living creatures under the influence of magnetic field. The most common example of biomagnetic fluid is blood. Blood is also a magnetic fluid due to its complex interaction of protein, cell membrane and hemoglobin that's from iron oxides. During the last few decades, many research works have already been done on biomagnetic fluids in the theoretical and experimental approaches due to its numerous applications of biomedical engineering and medical sciences such as magnetic devices development for cell separation, high-gradient magnetic separation, reducing bleeding during surgeries, targeted transport of drugs using magnetic particles as drug carries, as well as Magnetic Resonance imaging for imaging technique of a specific part of a body using strong magnetic field, and treatment of cancer tumors causing magnetic hyperthermia [1,2].

The BFD model was first developed by Haik et al. [3]; this model is based on the principle of FHD. Further, an extended BFD model was developed by Tzirtzilakis [4], and this model is based on the principle of FHD and magneto hydrodynamics (MHD). Under a certain condition, blood exhibits Newtonian and non-Newtonian behavior. When blood flows through a

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large vessel then it behaves like a Newtonian fluid, but actually, blood is a non-Newtonian fluid due to its variable viscosity. Tzirtzilakis and Tanoudis [5] analyzed the study of two dimensional, steady, laminar and incompressible biomagnetic fluid past a stretching sheet with heat transfer. They considered their study, the magnetization of the fluid varied with the magnetic field strength H and the temperature. Misra et al. [6] investigated the biomagnetic fluid flow over a stretching sheet and considered the viscoelastic property of the fluid. The study of biomagnetic fluid flow and heat transfer over a stretching sheet has been presented by many researchers [7-11].

Most of the researchers studied the biomagnetic fluid flow over a linearly stretching sheet. However, to the best of our knowledge, biomagnetic fluid with slip condition was not studied previously over a nonlinear stretching sheet. But in some practical situation, we need to investigate the fluid flow with partial slip condition. Anderson [12] analyzed the flow of a Newtonian fluid with partial slip condition. Bhattacharyya et al. [13] analyzed the fluid flow and heat transfer over a stretching sheet with slip boundary condition and considered variable viscosity.

The main purpose of the present study is the biomagnetic fluid flow and heat transfer over a nonlinear stretching sheet with slip boundary condition. The transform similarity equations are solved numerically by using finite different scheme with central differencing. The influence of the physical controlling parameter on velocity and temperature profiles is elucidated through the graphs. Also, we computed the missing slopes. The hope is that the present analysis will be used in bio-medical and bio-engineering sciences.

II. MODEL ANALYSIS

Let us consider the two dimensional flow of steady, laminar and incompressible viscous fluids past a nonlinear stretching sheet. Two equal and opposite steady uniform stresses are applied along the x axis, so that the sheet is stretched where the origin is fixed. It is assumed that the velocity at a point of the sheet is varying non-linearly of its distance from the slit and the sheet is maintained at the prescribed sheet temperature, i.e., $U = csin(x)|x|^n$, $-\infty \le x \le \infty$ and $T_w = T_c - Ax^m$ where n is a nonlinear stretching parameter and m is the temperature power index. A magnetic dipole generated a magnetic field of strength which is located below the sheet at a distance d . The temperature of the sheet T_w is kept fixed and T_c is far away from the sheet, whereas $Tw < Tc$.

The steady governing equations of the flow field can be written in the dimensional form as:

$$\frac{du}{dx} + \frac{dv}{dy} = 0 \quad (1.1)$$

$$\rho(u\frac{du}{dx} + v\frac{du}{dy}) = u\frac{d^2u}{dy^2} + uM\frac{dH}{dx} \quad (1.2)$$

$$\rho C_p\left(u\frac{dT}{dx} + v\frac{dT}{dy}\right) + u_0 M\frac{dH}{dT}\left(u\frac{dH}{dx} + v\frac{dH}{dy}\right) = K\frac{d^2T}{dy^2} \quad (1.3)$$

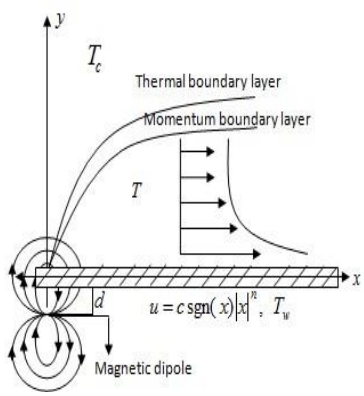

Fig. 1 Coordinate system of the physical model

With appropriate boundary conditions are [12]:

$$U = csin(x)|x|^n + Nv\frac{dv}{dy},\ -\infty \le x \le \infty,\ v=0,\ T=T_w=T_c - Ax^m \text{ at } y=0 \quad (1.4)$$

$$u \to 0, T \to T_c \text{ as } y \to \infty \quad (1.5)$$

where (u,v) is the velocity component along x and y axis respectively, ρ and k are the fluid density and thermal conductivity, respectively. T is the temperature of the fluid and c_p is the specific heat at constant pressure, μ is the fluid viscosity, u_0 is the magnetic permeability, $N = N_1|x|^{\frac{-(n-1)}{2}}$is the velocity slip parameter and $H = (H_x, H_y)$ is the magnetic field strength, $B$is the magnetic induction where$\beta = u_0 H$. We considered that the magnetization M varies with temperature T according to the following relation $M = K(T_c - T)$. Akyildiz et al. [14], considered the stretching velocity $u = cx^n$, which was employed for any positive odd integer values of n for $0 < x < \infty$, but in the case of $-\infty < x < 0$, the profile will be a wrong direction for any even integer values of *n* by Gorder and Vajravelu [15]. So, we considered the modified stretching velocity $u = c\sin(x)x^{n}$, $-\infty < x < \infty$ [15], where *n* can be any positive real number, which is a more general nonlinear power law stretching velocity of the sheet. The magnetic dipole is located at a distance *d* below the sheet, which generates magnetic field whose components are:

$$H_x(x,y) = -\frac{y(y+d)}{2\pi x^2+(y+d)^2} \text{ and, } H_y(x,y) = \frac{yx}{2\pi x^2+(y+d)^2} \quad (1.6)$$

Therefore, the magnitude |H| = H of the magnetic field is given by:

$$H_x(x,y) = [H_x^2 + H_y^2]^{\frac{1}{2}} = \frac{y}{2\pi}\frac{1}{\sqrt{x^2+(y+d)^2}} \quad (1.7)$$

$$\approx \frac{y}{2\pi}\left[\frac{1}{(y+d)^2} - \frac{1}{2}\frac{x^2}{(y+d)^4}\right] \quad (1.8)$$

In mathematical analysis, the following non-dimensional variables are introduced:

$$\xi = \sqrt{\frac{c(n+1)}{2v}}|x|^{\frac{n+1}{2}}, \quad \eta = y\sqrt{\frac{c(n+1)}{2v}}|x|^{\frac{n-1}{2}}, \quad (1.9)$$

$$u = csin(x)|x|^{n} f^{1}(\eta).$$

$$V = -\sin(x)\sqrt{\frac{c(n+1)v}{2v}}|x|^{\frac{n-1}{2}}\left(f(\eta)\right) + \frac{n-!}{n+1}\eta f^{1}(\eta)) \quad (2.0)$$

$$\theta(\xi,\eta) = \frac{T_\infty - T}{T_\infty - T_w} \quad (2.1)$$

Using the similarity variable and dimensionless temperature, the governing equations reduced to the following ordinary differential equations:

$$f''' + ff'' - \frac{2n}{n+1}f'^2 - \frac{2\beta\theta}{(n+4)^4} = 0 \quad (2.2)$$

$$\theta^{11} - p_r\left(\frac{2m}{n+1}f^1\theta - f\theta^1\right) + \frac{2\lambda\beta(\theta-\varepsilon)}{(n+\alpha)^3}(f + \frac{n-1}{n+1}\eta f^1) = 0 \quad (2.3)$$

Transformed boundary conditions are

$$f^1(0) = 1 + bf^{11}(0),\ f(0) = 0, \theta(0) = 1 \quad (2.4)$$

$$f^1(\infty) \to 0, \theta(\infty) \to 0$$

Where $\beta = \frac{\gamma}{2\pi}\frac{\mu_{0K(T_c - T_w\rho}}{\mu^2}, \lambda = \frac{C\mu^2}{\rho K(T_c - T_w)}$, $\mathrm{Pr} = \frac{\mu c_p}{k}$

$$b = {}^{N_1}\sqrt{\frac{CV(n+1)}{2}}$$

III. RESULTS AND DISCUSSION

The transform governing equations with associated boundary conditions are solved numerically using finite difference solution [16]. The effect of different physical parameters such as ferromagnetic parameter, velocity slip parameter, nonlinear stretching parameter, thermal exponent parameter on velocity and temperature distributions, as well as skin friction coefficient and rate of heat transfer coefficient are illustrated through the graphs.

The accuracy of the numerical scheme is justified with those tabulated by Cortell [17] and the results are in very good agreement.

TABLE I : COMPARISON OF SKIN FRICTION COEFFICIENT FOR DIFFERENT VALUES OF *n* WITH β=0, b=0

n	-*f* ''(0)	
	Cortell [17]	Present
0	0.627547	0.63004
0.1	0.705900	0.70814
0.2	0.766758	0.76886
0.3	0.81570	0.81758
0.5	0.889477	0.8912
0.6	0.91817	0.9197
1	1.00000	1.0012
3	1.14859	1.1496
10	1.23488	1.2357
100	1.276768	1.2775

Figs. 2 and 3 depict the effect of ferromagnetic parameters on velocity and temperature profiles. It can be observed that velocity of the fluid decreases with an increase of magnetic number, whereas temperature distribution increases in this case. This is because the ferromagnetic number is directly related to Kelvin force, which is also known as the resist force or drag force.

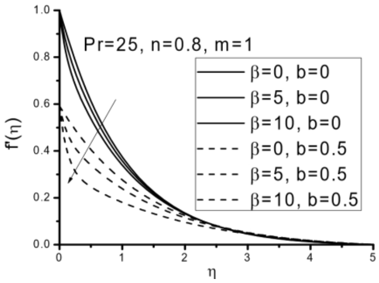

Fig. 2 Variation of $f'(\eta)$ with β

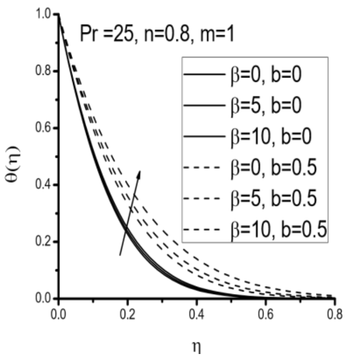

Fig. 3 Variation of Θ with β

Figs. 4 and 5 show the nature of velocity and temperature profiles for various values of the nonlinear stretching parameter. We found that the velocity profiles decrease with increasing the nonlinear stretching parameter, while the temperature distributions increase. Physically, the impact of nonlinear stretching parameters on the velocity profile is more significant when *n* is smaller. This happens because of the large value of *n* , then the value of $\frac{2n}{n+1}$ approaches 2. Therefore, the larger value of *n* is not the interest of the present study.

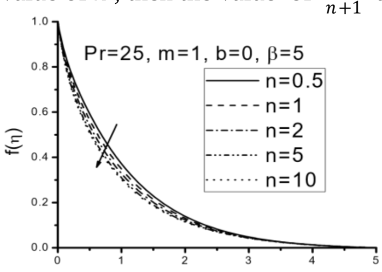

Fig. 4 Variation of $f'(\eta)$ with *n*

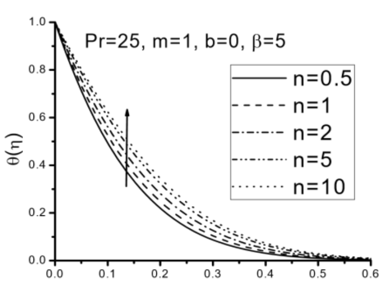

Fig. 5 Variation of Θwith *n*

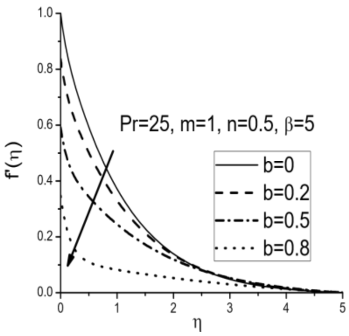

Fig. 6 Variation of f '(η) with$b$

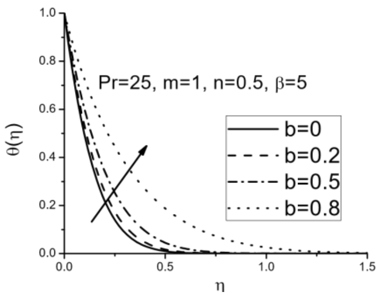

Fig. 7 Variation of ϴ with b

Figs. 6 and 7 show the effect of slip velocity parameters on dimensionless velocity and temperature profiles. It is observed that the fluid velocity decreases and temperature profile increases with the increase of slip parameter. This happens because increasing the slip parameter causes a reduction in the penetration of the stagnant surface through the boundary layer. We also observe that the increasing value of slip parameters results in enhancing the thermal boundary layer.

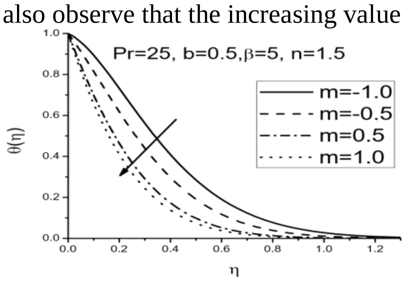

Fig. 8 Variation of ϴ with m

Fig. 8 depicts the temperature distribution for various values of temperature exponent parameter m. It is noticed that an increase in the value of m leads to a decrease of the temperature profile. This happens because when $m > 0$, heat transfers from the stretching sheet into fluid, and when $m < 0$, the heat flows from the fluid into the stretching sheet.

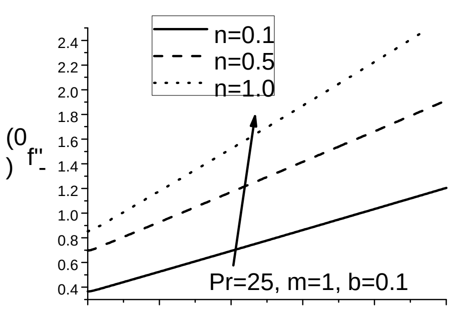

Fig. 9 Variation of f ''(0) with β for different value of n

Figs. 9-16 demonstrate the variation of the skin friction coefficient f ''(0) and rate of heat transfer ϴ'(0) with the ferromagnetic parameter, nonlinear stretching parameter and velocity slip parameter. From Figs. 9 and 10, we observe that the skin friction coefficient increases with ferromagnetic number and stretching parameter; whereas, the rate of heat transfer decreases.

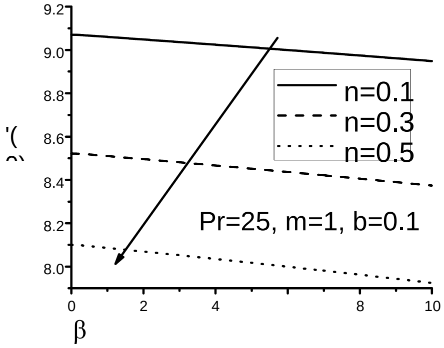

Fig. 10 Variation of Θ '(0) with β for different value of *n*

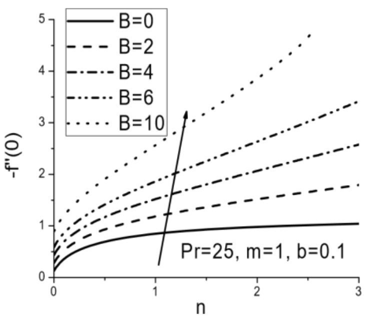

Fig. 11 Variation of *f* ''(0) with *n* for different value of β

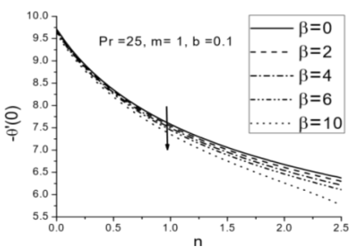

Fig. 12 Variation of rate Θ'(0) with *n* for different value of β

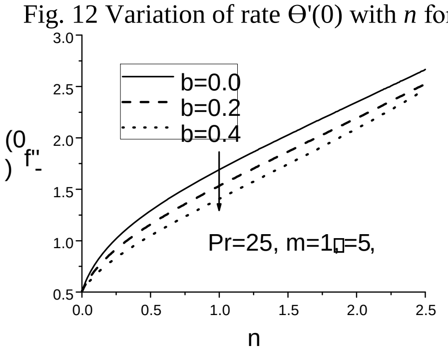

Fig. 13 Variation of *f* ''(0) with *n* for different value of *b*

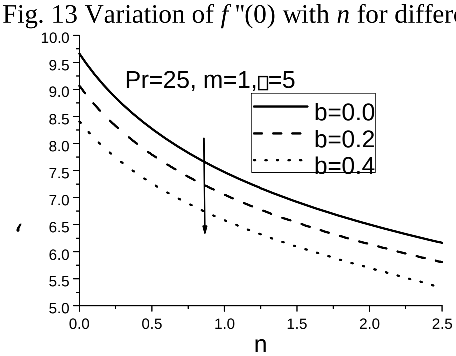

Fig. 14 Variation of Θ'(0) with *n* for different value of *b*

Figs. 11-14 show how the skin friction coefficient increases with increasing nonlinear stretching parameter and ferromagnetic parameter but decreases with the velocity slip parameter, while the rate of heat transfer decreases in all cases. It is noticed that when the skin friction coefficient and rate of heat transfer are a function of the stretching parameter, then its increment/decrement is nonlinear. Figs. 15 and 16 demonstrate the skin friction coefficient and heat transfer rate with respect to the velocity slip parameter. From these figures we notice that $f''(0)$increases as b increases, whereas $\Theta'(0)$ decreases with b and n .

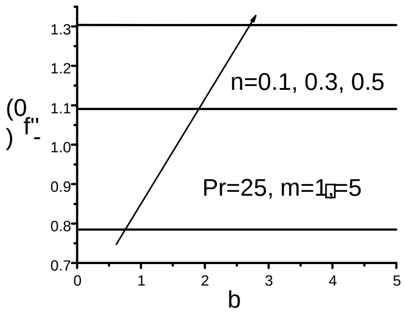

Fig. 15 Variation of $f''(0)$ with b for different value of n

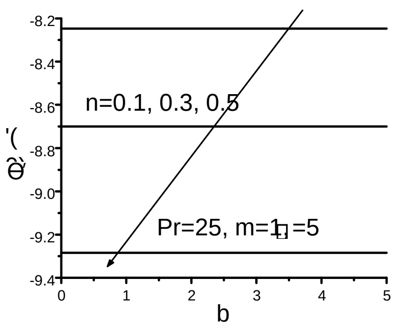

Fig. 16 Variation of $\Theta'(0)$ with b for different value of n

IV. CONCLUSION

The main findings from this analysis are summarized as follows:

i) Fluid velocity is found to be reduced with ferromagnetic number, velocity slip parameter and nonlinear stretching parameter; whereas, temperature increases in all cases.
ii) Local skin friction coefficient and rate of heat transfer linearly decreases when the nonlinear stretching parameter increases with the variation of the ferromagnetic parameter.
iii) Local skin friction and rate of heat transfer increased with the ferromagnetic parameter and velocity slip parameter with the variation of nonlinear stretching parameter. It is noticed that for the nonlinear stretching parameter, its decrement is nonlinear.
iv) There is similarity in the solutions of skin friction Coefficients obtained by Cortell[17] using nonlinear stretched Biomagnetic flow and heat transfer with signum function and those obtained in this work using Temperature power law as seen in Table 1. Hence, this study shows high degree of importance as high magnetic field is needed for good body comportment

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