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GENERALIZED DYNAMICAL GRAVITATIONAL SCALAR POTENTIAL FOR STATIC HOMOGENEOUS SPHERICAL DISTRIBUTION OF MASS USING TAYLOR SERIES APPROACH

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Abstract

In this article, the Taylor Series expansion approach was applied to the Schwarzschild metric to obtain generalization of Newton's dynamical equations of motion. The generalized dynamical gravitational equations of motion obtained are applied to the motion of the planets in the solar system. The results are that the generalized dynamical gravitational scalar potential are augmented with additional correction terms of all order of c^{-2} and c^{-4} which are not found in Newton's dynamical gravitational scalar potential or Einstein's geometrical equations of motion. The result obtained in this article was compared with the result obtained by using Great Riemannian operator method.

Keywords: Gravitational Scalar Potential, Taylor Series, Static Homogeneous, Spherical Massive Bodies and Additional Correction terms.

1.0 Introduction

In this article, the gravitational scalar potential for exterior region is evaluated by means of Taylor Series expansion method. According to Newton's dynamical law of gravitation, any two bodies in the universe attracts each other with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance separating the bodies [2-4]. The region surrounding the existence of this force is known as gravitational field. It was observed that this force governs the motion of the moon, planets and galaxies in their various orbits [1, 5]. The significance of laws of motion and gravity explain the experimental facts of the existence of the solar system [1]. The well known Newton's dynamical gravitational scalar potential exterior to a homogeneous spherical body is given by [4, 7]

$$\Phi = \frac{-GM}{r} \tag{1}$$

where
G is the universal gravitational constant
M is the mass of the spherical bodies
r is the mean distance from the centre of the body and negative in equation(1) is for attraction between masses.
Let us consider the result obtained when the great Riemannian operator was applied to the generalized dynamical gravitational scalar potential exterior to a spherical body given by [1, 5, 6]

$$\Phi = \frac{-GM}{r}\left[1 - \frac{GM}{c^2 R}\right] - \frac{G^2 M^2}{c^2 r^2} \tag{2}$$

Where R is the radius of the spherical bodies. Equation (2) above was formulated using a Great Riemannian approach.

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2.0 Theoretical Analysis

The generalized Great Riemannian gravitational scalar potential exterior to the body for a static homogeneous spherical massive body is given by[5]

$$\Phi = \frac{-GM}{r}\left[1-\frac{GM}{c^2R}\right]-\frac{G^2M}{c^2r^2} \tag{3}$$

The metric that describes the spacetime curvature around static massive objects is given by

$$ds^2 = -\left(1-\frac{2GM}{c^2r}\right)(cdt)^2+\left(1-\frac{2GM}{c^2r}\right)dr^2+r^2d\theta^2+r^2Sin^2\theta d\phi^2 \tag{4}$$

The metric in equation (4) can be represented in the exponential form as

$$ds^2 = -e^{\frac{2\Phi}{c^2}}(cdt)^2+dl^2 \tag{5}$$

Where dl is the spatial part of the metric.p

The comparison of equations (4) and (5) of time coordinates if

$$\left(1-\frac{2GM}{c^2r}\right)=e^{\frac{2\Phi}{c^2}} \tag{6}$$

Taking the natural logarithm of equation (6) is given by

$$\ln\left(1-\frac{2GM}{c^2r}\right)=\frac{2\Phi}{c^2} \tag{7}$$

Therefore equation (7) can be described as

$$\Phi=\frac{c^2}{2}\ln\left(1-\frac{2GM}{c^2r}\right) \tag{8}$$

The logarithm function in equation (8) can be expanded in Taylor Series expansion. The natural logarithm expansion is given by (Mungan, 2009).

$$\ln(1+p)=p-\frac{1}{2}p^2+\frac{1}{3}p^3 \tag{9}$$

Substituting $p=-\frac{2GM}{c^2r}$ into equation (9) gives

$$\Phi=\frac{c^2}{2}\left[-\frac{2GM}{c^2r}-\frac{1}{2}\left(\frac{4G^2M^2}{c^4r^2}\right)-\frac{1}{3}\left(\frac{8G^3M^3}{c^8r^3}\right)\right] \tag{10}$$

$$\Phi=\frac{-GM}{r}-\frac{G^2M^2}{c^2r^2}-\frac{4G^3M^3}{3c^4r^3} \tag{11}$$

The veracity of this research work is determined by the comparison of equations (11) and (3) as applied to all the planetary bodies in the solar system. In the table below, the comparison of Generalized Newton dynamical gravitational scalar potential with the Great Riemannian dynamical gravitational scalar potential shows that the result obtained in the ratio is efficacious.

Table1:Comparison of Generalized Newton Dynamical Gravitational Scalar Potential (GNDGSP) with the Great Riemannian Dynamical Gravitational Scalar Potential (NDGSP)

Body	Mass (kg)	Radius (m)	Mean Distance from the sun (Km)	NDGSP $\frac{-GM}{r}\left\{1-\frac{GM}{c^2R}\right\}-\frac{G^2M^2}{c^2r^2}$ (GREAT RIEMANNIAN)	GNDGSP $\frac{-GM}{r}-\frac{G^2M^2}{c^2r^2}-\frac{4G^3M^3}{3c^4r^3}$ RESEARCH RESULT	Ratio Between GNDGSP to NDGSP (GREAT RIEMANNIAN)
MERCURY	3.30×10^{23}	2.44×10^{6}	57.9×10^{6}	-380.16	-380.16	1.0000000000
VENUS	4.87×10^{24}	6.051×10^{6}	108.2×10^{6}	-3002.12	-3002.12	1.0000000000
EARTH	5.97×10^{24}	$6.378\ 10^{6}$	149.6×10^{6}	-2661.76	-2661.76	1.0000000000
MARS	6.42×10^{23}	$3.397\ 10^{6}$	227.9×10^{6}	-187.90	-187.90	1.0000000000
JUPITER	1.90×10^{27}	71.492×10^{6}	778.6×10^{6}	-162766.50	-16266.50	1.0000000000
SATURN	5.69×10^{26}	60.268×10^{6}	1427.0×10^{6}	-26595.87	-26595.87	1.0000000000
URANUS	8.66×10^{25}	25.559×10^{6}	2871×10^{6}	-2011.92	-2011.92	1.0000000000
NEPTUNE	1.03×10^{26}	24.764×10^{6}	4497.0×10^{6}	-1527.71	-1527.71	1.0000000000
PLUTO	1.31×10^{22}	1.160×10^{6}	7376×10^{6}	-0.1185	-0.1185	1.0000000000

$c = 3 \times 10^{8}$ $G = 6.67 \times 10^{-11}$

3.0 Remarks and Conclusion

We have been able to show in this research work how to obtain a generalized dynamical gravitational scalar potential exterior to static homogeneous spherical massive bodies using a Taylor Series expansion approach. The immediate consequences are: In equation (3), the author used great Riemannian approach to formulate a generalized dynamical gravitational scalar potential. The equation is the generalized dynamical gravitational scalar potential for static homogenous spherical massive bodies [5]. In the same vein, equation(11), the researcher used Taylor Series expansion approach to obtain the generalized dynamical gravitational scalar potential for static homogenous spherical massive bodies. In equations (3) and (11), the leading term on the right hand side is the well-known Newtonian dynamical gravitational scalar potential to the body. These equations reduce in the limit C^0, to the corresponding pure Newton's dynamical gravitational field equation showing that it agrees with the well known equivalence principles in physics. These equations consist of post-Newton and Post-Einstein correction terms of all order of C^{-2} which are open for theoretical development and applications. It is interesting to note that equation (11) contains correction terms of the order C^{-4} which is not found in equation (3) and in reference [5]. It must be noted that equations (3) and (11) contain, post-Newton and post-Einstein correction terms of all order of C^{-2} to the gravitational field equation and gravitational scalar potential. The result obtained in this research work shows that it is possible to obtain similar result predicted by Einstein's geometrical gravitational theory of gravitation.

4.0. References

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