Journal of the Nigerian Association of Mathematical Physics
***Volume 48* (*Sept. & Nov., 2018 Issue*), pp199 –208**

SELECTING THE BEST INITIAL METHOD FOR A TRANSPORTATION PROBLEM

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Abstract

This research paper studies the initial methods of obtaining the basic feasible solution to a transportation problem. Seven initial methods are considered and comparison is done on the basis of the number of iterations required to reach an optimal solution. Using the University of Nigeria, Nsukka as a case study, we look at the transportation of bread and sachet water from three bakeries and three water companies to four hostels on campus.

***Keywords*: Transportation problems, Initial method of solving transportation problems, Iteration, Optimal Solution.**

1.0 Introduction:

Whenever any product is produced, it has to reach it's end users and the sole aim of every manufacturing company is to effectively meet the demands of their customers, in as much as the goal of increasing profit is at heart, which is why transportation (the act of moving something from one location to another) has a crucial role to play in balancing the supply chain of manufacturing companies.

The transportation problem studies how efficiently homogeneous products from different supply origins are transported to the different demand destination in such a way that the total transportation cost is minimum. In other words, the transportation problem minimizes the total cost of shipping a homogeneous commodity from a number of sources to a number of destinations. The cost of transportation is minimized in order to increase the profit [1].

The transportation problem is one of the subclasses of linear programming which is a mathematical method of solving practical problems (such as the allocation of resources) by means of linear functions where the variables involved are subject to constraints. It is part of a very important area of mathematics called Optimization Techniques, the process of finding the conditions that give the minimum or maximum value of a function, where the function represents the effort required or the desired benefit.

It all started in 1930 when A.N. Tolstoi, a pioneer in Operations Research, published an article in a book on transportation planning called "Methods of Finding the Minimal Kilometrage in Cargo-transportation in Space" that was issued by the National Commissariat of Transportation of the Soviet Union, in which he studied the transportation problem and described a number of solution approaches. Tolstoi was one of the first to study the transportation problem mathematically and he straightened out his approach by applications to the transportation of salt, cement and other cargo between sources and destinations along the railway network of the Soviet Union [2].

In 1942, the Russian mathematician and economist Leonid Kantorovich published a paper on the continuous version of the transportation problem and later with Gavurian in 1949 applied the study of the capacitated transportation problem. George B. Dantizig, [3], applied the concept of linear programming in solving the transportation problem. Ever since, the transportation problem has become a common subject in almost every textbook on operation research and mathematical programming. [4]

2.0 Literature Review

Abdul et al [5] in their publication 'A New Method for Finding an Optimal Solution for Transportation Problems' proposed a new method, called the Assigning Shortest Minimax (ASM) Method for directly finding an optimal solution for a wide range of transportation problems. They established a numerical illustration and checked the optimality of the result yielded by this method. The ASM-Method requires very simple arithmetical and logical calculation which makes it very easy even for the layman to understand and use.

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Abdallah et al [6] presented an alternative method to the conventional problem solution methods. Since most of the currently used methods for solving transportation problems are trying to reach the optimal solution, their proposed method obtains the best initial feasible solution to a transportation problem and performs faster than existing methods with a minimal computation time and less complexity. The key idea in their work "Solving Transportation Problems Using the Best Candidates Method" is to minimize the combinations of the solution by choosing the best candidates to reach the optimal solution. The best candidate method (BCM) can be used successfully to solve different business problems of distribution of products that is commonly referred to as a transportation problem.

A study by [7] called the "Optimal Solution of Transportation Problem Using Linear Programming: A Case of a Malaysian Trading Company" highlights the application of linear programming and spreadsheet to facilitate managers in a Malaysian Trading Company in determining the optimum transportation plan that leads to the lowest transportation cost of polymer from four plants or supply origins to four demand destinations. The sensitivity technique in analyzing the impact of uncertainty of unit shipping cost to the total shipping cost of the trading company was also discussed in the study.

Mohammed et al [8] attempted to minimize the total distance and well as the overall breakages of a perishable/deteriorating item and the total transportation time. In their study "Transportation of Eggs: A Case Study Using Multi-Objective Transportation Problem (MOTP)" they assert that time plays a crucial role in transportation problem and that minimum time can only be achieved by keeping high or maximum speed during transportation. However, speed maximization might increase the risks of breakages as more breakages will lead to more losses. A proposed model is applied for the transportation of eggs from various cities of the state Andhra Pradesh to various cities of Maharashtra state so as to minimize total breakages among other objectives.

A study by Ocotlan et al [9] named "A Survey of Transportation Problems" aims at being a guide to understand the different types of transportation problems. In the survey, they present mathematical models and algorithms which can be used to solve different types of transportation modes, that is, ship, plane, train, bus, motorcycle, truck et cetera by various means of transportation such as air, water, space, cables, tubes, and road. Some of the problems considered include: bus scheduling problem, delivery problem, combining truck trip problem, open vehicle routing problem, helicopter routing problem, truck loading problem, truck dispatching problem, truck routing problem, truck transportation problem, vehicle routing problem and variants, convoy routing problem, railroad blocking problem (RBP), inventory routing problem (IRP), air traffic flow management problem (TFMP), cash transportation vehicle routing problem and so forth

In conclusion, transportation problems have been studied extensively by many authors and have even found applications in different fields like geometry, fluid mechanics, statistics, economics and meteorology. However, selecting the best initial method for a transportation problem is hardly ever mentioned at all where the transportation problem is treated.

3.0 The Transportation Problem

The common objective of the transportation problem is either to minimize the cost of shipping m units to n destinations or to maximize the profit of shipping m units to n destinations. The source capacities, destination requirements and cost of transportation from each source to each destination are constantly given. The transportation problem can be described using linear programming mathematical model and it is usually shown in a transportation table [10].

Table 1: Transportation Table

Destination (j)

		1	2	...	n	Supply
Source (i)	1	x_{11} C_{11}	x_{12} C_{12}	⋯	x_{1n} C_{1n}	a_1
	2	x_{21} C_{21}	x_{22} C_{22}	⋯	x_{2n} C_{2n}	a_2
	⋮	⋮	⋮		⋮	⋮
	m	x_{m1} C_{m1}	x_{m2} C_{m2}	⋯	x_{mn} C_{mn}	a_m
	Demand	b_1	b_2	...	b_n	

Each of the boxes in table 1 is called a cell. The unit cost C_{ij} of shipment from source i to each destination j is represented at the center of the cell ij (i for row and j for column). The bottom row contains the demands and the rightmost column contains the supplies. If the total supply equals the total demand, we say that it is a Balanced Transportation Problem. That is,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

In most applications, this is not the case. However, dummy sources (rows) and destinations (column) can always be added to the problem to make it balanced. Being a "balanced" problem is an important feature of the transportation model as we will now show it is necessary and sufficient condition for the transportation problem to be feasible. To show the sufficiency of this condition, let S be equal to the total supply (which is also equal to the total demand)

$$S = \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Let $X_{i,j} = \dfrac{a_i b_j}{S}$

For $i = 1, 2 \cdots m$ and $j = 1, 2 \cdots n$.

$$So, \quad \sum_{j=1}^{n} X_{i,j} = \sum_{j=1}^{n} \frac{a_i b_j}{S} = a_i \quad and \quad \sum_{i=1}^{m} X_{i,j} = \sum_{i=1}^{m} \frac{a_i b_j}{S} = b_j$$

Therefore, x is feasible and a feasible solution always exist for a balanced transportation problem. Conversely, if the transportation problem has a feasible solution x, then

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{i,j} = \sum_{j=1}^{n} b_j \quad and \quad \sum_{j=1}^{n}\sum_{i=1}^{m} X_{i,j} = \sum_{i=1}^{m} a_i$$

Therefore, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, which establishes the neccesity of the balance.

To solve the transportation problem, we first formulate the transportation problem mathematically, then determine an initial basic feasible solution and finally, test for optimality. If the initial basic feasible solution is not optimal, iterations are performed to revise the solution until the solution is optimized [11].

4.0 Numerical Illustration of the Transportation Methods

Problem 1:

Below is a transportation table showing the cost per unit [in Naira (₦)] of transporting bread loaves from three bakeries to four hostels in the University of Nigeria, Nsukka, the quantity of bread loaves demanded daily and the quantity available at the three bakeries.

Table II: The first unbalanced table for problem I

	Nkurumah	Bello	Akintola	Isa-kaita	Quantity Supplied
Mum's Bread	180	190	220	250	500
Makau Bread	175	200	200	230	200
Ife Best	145	110	120	130	300
Quantity Demanded	170	70	65	100	

We see that the total quantity available to be supplied is ₦1000 and the total quantity demanded is ₦405. Hence, the transportation problem is unbalanced. We attach a dummy hostel to the transportation table.

Table III: A balanced table for problem I

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0	500
Makau Bread	175	200	200	230	0	200
Ife Best	145	110	120	130	0	300
Quantity Demanded	170	70	65	100	595	1000

1. **The North-West Corner Method:**

Obtaining the Initial Basic Feasible Solution

Table IV: A balanced table for problem I in regards to NW corner method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180 170	190	220	250	0	$500 - 170$ 330
Makau Bread	175	200	200	230	0	200
Ife Best	145	110	120	130	0	300
Quantity Demanded	170	70	65	100	595	1000

$= 180 \times 170 + 190 \times 70 + 220 \times 65 + 250 \times 100 + 0 \times 95 + 0 \times 200 + 0 \times 300$
$= ₦(30{,}600 + 13{,}300 + 14{,}300 + 25{,}000 + 0)$
$= ₦83{,}200$

Obtaining the Optimal Solution

We see that the number of allocated cells equals $m + n - 1$ (where m is the number of rows and n is the number of columns) that is, $3 + 5 - 1 = 7$ allocated cells. Hence, the solution is non-degenerate.

We carried out five iterations, and hence, obtained the optimal solution as
$= 175 \times 105 + 145 \times 65 + 110 \times 70 + 120 \times 65 + 130 \times 100 + 0 \times 95 + 0 \times 500$
$= ₦(18{,}375 + 9{,}425 + 7{,}700 + 7{,}800 + 1{,}300 + 0)$
$= ₦56{,}300$

2. **The Least Cost Method**

Table V: A balanced table for problem I in regards to LC method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0 500	$500 - 500$ 0
Makau Bread	175	200	200	230	0	200
Ife Bread	145	110	120	130	0	300
Quantity Demanded	170	70	65	100	$595 - 500$ 95	1000

$= 175 \times 170 + 230 \times 30 + 110 \times 70 + 120 \times 65 + 130 \times 70 + 0 \times 95 + 0 \times 500$
$= ₦(29{,}750 + 6{,}900 + 7{,}700 + 7{,}800 + 9{,}100 + 0)$
$= ₦61{,}250$

Obtaining the Optimal Solution
Here also, there are 7 allocated cells which equals $3 + 5 - 1$. Hence, the solution is non-degenerate. In this case we carried out two iterations and hence obtained the optimal solution as
$= 175 \times 105 + 145 \times 65 + 110 \times 70 + 120 \times 65 + 130 \times 100 + 0 \times 95 + 0 \times 500$
$= ₦(18{,}375 + 9{,}425 + 7{,}700 + 7{,}800 + 1{,}300 + 0)$
$= ₦56{,}300$

3. **Vogel's Approximation Method**
Table VI: A balanced table for problem I in regards to VA method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0 500	500 − 500 0
Makau Bread	175	200	200	230	0	200
Ife Bread	145	110	120	130	0	300
Quantity Demanded	170	70	65	100	595 – 500 95	1000

$= 175 \times 105 + 145 \times 65 + 110 \times 70 + 120 \times 65 + 130 \times 100 + 0 \times 95 + 0 \times 500$
$= ₦(18{,}375 + 9{,}425 + 7{,}700 + 7{,}800 + 1{,}300 + 0)$
$= ₦56{,}300$
Obtaining the Optimal Solution
Here also, there are 7 allocated cells which equals $3 + 5 - 1$. Hence, the solution is non-degenerate.
Hence, ₦56,300 is the optimal solution.
4. **Column Minimum Method**
Table VII: A balanced table for problem I in regards to CM method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0	500
Makau Bread	175	200	200	230	0	200
Ife Bread	145 170	110	120	130	0	300 − 170 130
Quantity Demanded	170	70	65	100	595	1000

$= 200 \times 5 + 230 \times 100 + 145 \times 170 + 110 \times 70 + 120 \times 60 + 0 \times 95 + 0 \times 500$
$= ₦(1{,}000 + 23{,}000 + 24{,}650 + 7{,}700 + 7{,}200 + 0)$
$= ₦63{,}550$

Obtaining the Optimal Solution
Here also, there are 7 allocated cells which equals $3 + 5 - 1$. Hence, the solution is non-degenerate.
We carried out three iteration on this method to obtain our optimal solution as ₦56,300

5. Row Minimum Method

Table VIII: A balanced table for problem I in regards to RM method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0 500	500 − 500 0
Makau Bread	175	200	200	230	0	200
Ife Bread	145	110	120	130	0	300
Quantity	170	70	65	100	595 – 500	1000

$= 175 \times 105 + 145 \times 65 + 110 \times 70 + 120 \times 65 + 130 \times 100 + 0 \times 95 + 0 \times 500$
$= ₦(18{,}375 + 9{,}425 + 7{,}700 + 7{,}800 + 1{,}300 + 0)$
$= ₦56{,}300$

This is the same solution obtained by Vogel's Approximation Method. Hence, we repeat the same procedure to obtain the optimal solution.

6. **The Row- Column Minimum Method**

Table IX: A balanced table for problem I in regards to RCM method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0 500	500 − 500 0
Makau Bread	175	200	200	230	0	200
Ife Bread	145 170	110	120	130	0	300 − 170 130
Quantity Demanded	170	70	65	100	595 – 500 95	1000

$= 200 \times 5 + 230 \times 100 + 145 \times 170 + 110 \times 70 + 120 \times 60 + 0 \times 95 + 0 \times 500$
$= ₦(1{,}000 + 23{,}000 + 24{,}650 + 7{,}700 + 7{,}200 + 0)$
$= ₦63{,}550$

This is the same solution obtained by Column Minimum Method. Hence, we repeat the same procedure to obtain the optimal solution.

7. **Russell's Approximation Method**

Table X: A balanced table for problem I in regards to RA method

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Mum's Bread	180	190	220	250	0	500
Makau Bread	175	200	200	230	0	200
Ife Bread	145	110	120	130 100	0	300 − 100 200
Quantity	170	70	65	100	595	1000

$= 175 \times 105 + 145 \times 65 + 110 \times 70 + 120 \times 65 + 130 \times 100 + 0 \times 95 + 0 \times 500$
$= ₦(18{,}375 + 9{,}425 + 7{,}700 + 7{,}800 + 13{,}000 + 0)$
$= ₦56{,}300$

This is the same solution obtained by the Row Minimum Method and Vogel's Approximation Method. Hence, we repeat the same procedure to obtain the optimal solution.

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Problem 2:

Here is a second transportation table showing the cost per unit [in Naira (₦)] of transporting bags of sachet water from three factories to four hostels in the University of Nigeria, Nsukka, the quantity of bags demanded daily and the quantity available at the three factories.

Table XI: An unbalanced table for problem II

	Nkurumah	Bello	Akintola	Isa-kaita	Quantity Supplied
Jives Table Water	75	90	120	145	600
Lion Table Water	50	40	70	110	500
Classic Table Water	170	110	130	145	700
Quantity Demanded	145	110	120	90	1800

In this transportation problem, the total quantity available to be supplied is ₦1,800 and the total quantity demanded is ₦465. Hence, we attach a dummy hostel to balance the transportation table.

Table XII: A balanced table for problem II

	Nkurumah	Bello	Akintola	Isa-kaita	Dummy	Quantity Supplied
Jives Table Water	75	90	120	145	0	600
Lion Table Water	50	40	70	110	0	500
Classic Table Water	170	110	130	145	0	700
Quantity Demanded	145	110	120	90	1335	1800

1. **The North-West Corner Method:**

Following the same procedure of the north –west Corner method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 75 \times 145 + 90 \times 110 + 120 \times 120 + 145 \times 90 + 0 \times 135 + 0 \times 500 + 0 \times 700$
$= ₦(10{,}875 + 9{,}900 + 14{,}400 + 13{,}050 + 0)$
$= ₦48{,}225$

2. **The Least Cost Method:**

Following the same procedure of the least cost method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 50 \times 145\ + 40 \times 110\ + 70\ \times 120 + 110\ \times 90\ + 0 \times 35 + 0\ \times 600 + 0\ \times 700$
$= ₦(7{,}250\ + 4{,}400\ + 8{,}400\ + 9{,}900\ + 0)$
$= ₦29{,}950$

3. **Vogel's Approximation Method:**

Following the same procedure of Vogel's approximation method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 50 \times 145\ + 40 \times 110\ + 70\ \times 120 + 110\ \times 90\ + 0 \times 35 + 0\ \times 600 + 0\ \times 700$
$= ₦(7{,}250\ + 4{,}400\ + 8{,}400\ + 9{,}900\ + 0)$
$= ₦29{,}950$

4. **Column Minimum Method:**

Following the same procedure of column minimum method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 50 \times 145\ + 40 \times 110\ + 70\ \times 120 + 110\ \times 90\ + 0 \times 35 + 0\ \times 600 + 0\ \times 700$
$= ₦(7{,}250\ + 4{,}400\ + 8{,}400\ + 9{,}900\ + 0)$
$= ₦29{,}950$

5. **Row Minimum Method:**

Following the same procedure of row minimum method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 170 \times 145\ + 110 \times 110\ + 130\ \times 120 + 145\ \times 90\ + 0 \times 235 + 0\ \times 600 + 0\ \times 700$
$= ₦(24{,}650\ + 12{,}100\ + 15{,}600\ + 13{,}050\ + 0)$
$= ₦65{,}400$

6. **Row-Column Minimum Method:**

Following the same procedure of row-column minimum method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 50 \times 145\ + 110 \times 110\ + 130\ \times 120 + 145\ \times 90\ + 0 \times 355 + 0\ \times 600 + 0\ \times 700$
$= ₦(7{,}250\ + 12{,}100\ + 15{,}600\ + 13{,}050\ + 0)$
$= ₦48{,}000$

7. **Russell's Approximation Method:**

Following the same procedure of Russell's approximation method used in the previous transportation table, we obtain the Initial Basic Feasible Solution

$= 75 \times 145\ + 40 \times 110\ + 70\ \times 120 + 110\ \times 90\ + 0 \times 455 + 0\ \times 180 + 0\ \times 700$
$= ₦(10{,}875\ + 4{,}400\ + 8{,}400\ + 9{,}900\ + 0)$
$= ₦33{,}575$

Checking for the optimality of the solution obtained by the Least Cost Method, Vogel's Approximation Method and the Column Minimum Method, since they all gave the least initial
basic feasible solution of ₦29,950.
The optimal solution is ₦29,950 since the values of $d_{ij}\ \nless 0$

5.0 Analysis of the Results Obtained

From the results obtained in the previous chapter, the following analysis is made:

In the first transportation problem of transporting bread loaves from three bakeries to four hostels in the University of Nigeria, Nsukka, the North West Corner Method is significantly different from the rest of the methods yielding the worst initial basic feasible solution of ₦83,200 and taking five (5) iterations to reach an optimal solution.
This is followed by the Column Minimum Method and the Row column Minimum Method yielding an initial feasible solution of ₦63,550 and taking three (3) iterations to reach an optimal solution. The Least cost method gives an initial basic feasible solution of ₦61,250 and takes two (2) iterations to reach the optimal solution while the Row Minimum Method, Vogel's Approximation Method and Russell's Approximation Method yields the least initial basic feasible solution of ₦56,300 which is the optimal solution to the transportation problem.
In the second transportation problem of transporting bags of sachet water from three factories to four hostels in the University of Nigeria, Nsukka, the Row Minimum Method yields the worst initial basic feasible solution of ₦65,400 and taking four (4) iterations to reach an optimal solution. The North West Corner Method and the Row Column Minimum Method yield almost the same result of ₦48,225 and ₦48,000 respectively taking three (3) iterations each to reach an optimal solution.
Russell's Approximation Method gives an initial basic feasible solution of ₦33,575 and takes one (1) iteration to reach the optimal solution while the Least Cost Method, the Column Minimum Method and Vogel's Approximation Method yields the least initial basic feasible solution of ₦29,950 which is the optimal solution to the transportation problem.

6.0 Conclusion

From the analysis above, we observe that in both transportation problems, the North West Corner Method and the Row Column Minimum Method yield results far from the optimal solution, that is, multiple iterations are required to get to the optimal solution.
On the other hand, Vogel's Approximation Method yields the best initial basic solution for both transportation problems by giving initial solutions that are optimal. However, computation is slow because it takes longer time to implement the steps just like the Russell's Approximation Method.
The remaining three methods, that is, the Column Minimum Method, the Row Minimum Method and the Least Cost Method are relatively easy to compute but they are not as good as the Vogel's Approximation Method.

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