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EXISTENCE OF EQUILIBRIUM POINTS OF MATHEMATICAL MODEL OF TRANSMISSION AND CONTROL OF ZIKA VIRUS FEVER DYNAMICS

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Abstract

Mathematical model of Zika virus fever (ZIKVF) dynamics was formulated, incorporating control and four medium of the virus transmissions. The model passed through two stages of reformations in order to obtain equilibrium points of the model. We obtained two distinct equilibrium points of the model, the disease free equilibrium (DFE) point and the endemic equilibrium (EE) point respectively.

Keywords: Zika virus fever, Disease free equilibrium point and the endemic equilibrium point.

1. Introduction

Zika virus (ZIKV) derives its name from a forest called zika in Uganda where it was first identified in monkey in 1947, in a female mosquito in 1948 from the same forest and discovered in human in 1952. Zika virus causes zika virus fever (ZIKVF) with no or mild symptoms such as fever, joint pain, red eyes, muscle pain and rash. About 80% of infected individuals are asymptomatic and the infection caused by zika does not kill. The scientific concern focuses on the effect in humans with temporary paralysis and if transmitted from infected pregnant women to their fetuses then it causes abnormalities to the brain of fetuses which can result into miscarriage or microcephaly [1-2]. The virus lives longer in the semen than in the blood or virginal fluid [3]. The outbreaks of the infection occurred on Yap Island in 2007; in Pacific area in 2013 -2014; Cape –Verde in 2015 -2016; South and Central America attained pandemic in 2015 -2016; [4].

We reviewed models of the outbreaks of ZIKVF: on Yap Island and French Polynesia of 2013 -2014 [5-6]. Co –infection of Dengue, Chikungunya and Zika viruses based on the same vector, zika virus as mosquito borne and sexually transmitted disease are also reviewed [7-9]. Our study improved on the above literatures towards better understanding of the transmission and control of ZIKVF dynamics.

2. Material and Methods

Human population in a non enzootic region is split into female and male population respectively. The partitions of humans into sub –populations are as follow: Total female population is N_1, Susceptible female compartment is S_1, Exposed female compartment is E_1, Symptomatic female compartment is I_{11} Asymptomatic female compartment is I_{12} ,Removed female compartment is R_1 and the total population gives $N_1 = S_1 + E_1 + I_{11} + I_{12} + R_1$. The males sub –populations are similarly defined and male population gives $N_2 = S_2 + E_2 + I_{21} + I_{22} + R_2$, and mosquito population is split into mosquitoes without zika virus (S_3) and mosquitoes with virus (I_3) and the total population gives $N_3 = S_3 + I_3$.

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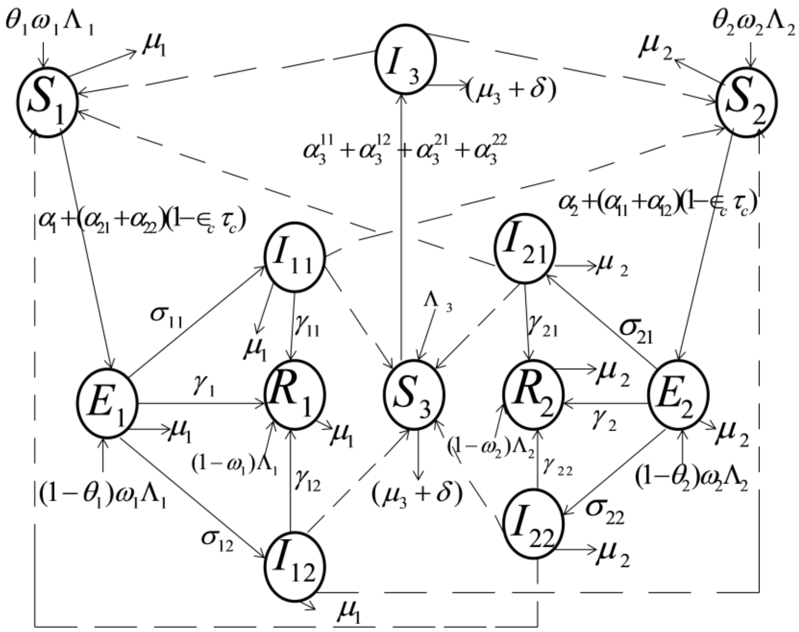

Figure 1: Model graph

2.2 Model Equations

$$\frac{dS_1}{dt}=\theta_1\omega_1\Lambda_1-\frac{S_1}{N_1}\{\alpha_1\phi_3 I_3+\alpha_{21}\phi_{21}I_{21}(1-\in_c\tau_c)+\alpha_{22}\phi_{22}I_{22}(1-\in_c\tau_c)\}-\mu_1 S_1 \quad (2.1)$$

$$\frac{dE_1}{dt}=(1-\theta_1)\omega_1\Lambda_1+\frac{S_1}{N_1}\{\alpha_1\phi_3 I_3+\alpha_{21}\phi_{21}I_{21}(1-\in_c\tau_c)+\alpha_{22}\phi_{22}I_{22}(1-\in_c\tau_c)\}-(\gamma_1+\sigma_{11}+\sigma_{12}+\mu_1)E_1 \quad (2.2)$$

$$\frac{dI_{11}}{dt}=\sigma_{11}E_1-(\gamma_{11}+\mu_1)I_{11} \quad (2.3)$$

$$\frac{dI_{12}}{dt}=\sigma_{12}E_1-(\gamma_{12}+\mu_1)I_{12} \quad (2.4)$$

$$\frac{dR_1}{dt}=(1-\omega_1)\Lambda_1+\gamma_1 E_1+\gamma_{11}I_{11}+\gamma_{12}I_{12}-\mu_1 R_1 \quad (2.5)$$

$$\frac{dS_3}{dt}=\Lambda_3-\frac{S_3}{N_3}(\alpha_3^{11}\phi_{11}I_{11}+\alpha_3^{12}\phi_{12}I_{12}+\alpha_3^{21}\phi_{21}I_{21}+\alpha_3^{22}\phi_{22}I_{22})-(\mu_3+\delta)S_3 \quad (2.6)$$

$$\frac{dI_3}{dt}=\frac{S_3}{N_3}(\alpha_3^{11}\phi_{11}I_{11}+\alpha_3^{12}\phi_{12}I_{12}+\alpha_3^{21}\phi_{21}I_{21}+\alpha_3^{22}\phi_{22}I_{22})-(\mu_3+\delta)S_3 \quad (2.7)$$

$$\frac{dS_2}{dt}=\theta_2\omega_2\Lambda_2-\frac{S_2}{N_2}\{\alpha_2\phi_3 I_3+\alpha_{11}\phi_{11}I_{11}(1-\in_c\tau_c)+\alpha_{12}\phi_{12}I_{12}(1-\in_c\tau_c)\}-\mu_2 S_2 \quad (2.8)$$

$$\frac{dE_2}{dt}=(1-\theta_2)\omega_2\Lambda_2+\frac{S_2}{N_2}\{\alpha_2\phi_3 I_3+\alpha_{11}\phi_{11}I_{11}(1-\in_c\tau_c)+\alpha_{12}\phi_{12}I_{12}(1-\in_c\tau_c)\}-(\gamma_2+\sigma_{21}+\sigma_{22}+\mu_2)E_2 \quad (2.9)$$

$$\frac{dI_{21}}{dt}=\sigma_{21}E_2-(\gamma_{21}+\mu_2)I_{21} \quad (2.10)$$

$$\frac{dI_{22}}{dt}=\sigma_{22}E_2-(\gamma_{22}+\mu_2)I_{22} \quad (2.11)$$

$$\frac{dR_2}{dt}=(1-\omega_2)\Lambda_2+\gamma_2 E_2+\gamma_{21}I_{21}+\gamma_{22}I_{22}-\mu_2 R_2 \quad (2.12)$$

Table 1: Parameters of the Model

Parameter	Description
Λ_1	Number of recruitment into the population of females
ω_1	Proportion of births without microcephaly into the population of females
$(1-\omega_1)$	Proportion of female births with microcephaly
θ_1 $(1-\theta_1)$	Proportion of female susceptible births without microcephaly Proportion of exposed female births without microcephaly
μ_1	Natural death rate of females
Λ_2	Number of recruitment into the population of males
ω_2	Proportion of male births without microcephaly
$(1-\omega_2)$	Proportion of male births with microcephaly
θ_2	Proportion of susceptible male births without microcephaly
$(1-\theta_2)$	Proportion exposed male of births without microcephaly
μ_2	Natural death rate of males
Λ_3	Number of recruitment into the population of mosquitoes without ZIKV
μ_3	Natural death rate of mosquitoes
δ	Death rate of mosquitoes due to spraying of insecticides
α_1	Transmission rate of infection through mosquito bite to the susceptible females
α_2	Transmission rate of infection through mosquito bite to the susceptible males
α_{21}	Transmission rate of infection through sex from symptomatic infectious males to susceptible females
α_{22}	Transmission rate of infection through sex from asymptomatic infectious males to susceptible females
α_{11}	Transmission rate of infection through sex from symptomatic infectious females to susceptible males
α_{12}	Transmission rate of infection through sex from asymptomatic infectious females to susceptible males
α_3^{11}	Transmission rate of ZIKV from symptomatic infectious females to mosquitoes without virus through mosquito bite
α_3^{11}	Transmission rate of ZIKV from symptomatic infectious females to mosquitoes without virus through mosquito bite
α_3^{12}	Transmission rate of ZIKV from asymptomatic infectious females to mosquitoes without virus through mosquitoes bite
α_3^{21}	Transmission rate of ZIKV from symptomatic infectious males to mosquitoes without virus through mosquitoes bite
α_3^{22}	Transmission rate of ZIKV from asymptomatic infectious males to mosquitoes without virus through mosquitoes bite

σ_{11}	Progression rate of exposed females to the symptomatic infectious compartment
σ_{12}	Progression rate of exposed females to the asymptomatic infectious compartment
σ_{21}	Progression rate of exposed males to the symptomatic infectious compartment
σ_{22}	Progression rate of exposed males to the asymptomatic infectious compartment
γ_1	Rate of recovery from the compartment of exposed females to the removed compartment
γ_2	Rate of recovery from the compartment of exposed males to the removed compartment
γ_{11}	Rate of recovery from the compartment of symptomatic, infectious, females to the removed compartment
γ_{12}	Rate of recovery from the compartment of asymptomatic, infectious, females to the removed compartment
γ_{21}	Rate of recovery from the compartment of symptomatic, infectious, males to the removed compartment
γ_{22}	Rate of recovery from the compartment of asymptomatic, infectious, males to the removed compartment
ϕ_3	Is measuring the reduction in effectiveness of mosquito activities in transmitting ZIKV by creating non conducive environment for the mosquitoes through the use of air conditioner
ϕ_{11}	Is measuring the reduction in effectiveness of sexual transmission through adherence to the preventive instructions
ϕ_{12}	Is measuring the reduction in effectiveness of sexual transmission through adherence to the preventive instructions
ϕ_{21}	Is measuring the reduction in effectiveness of sexual transmission through adherence to the preventive instructions
ϕ_{22}	Is measuring the reduction in effectiveness of sexual transmission through adherence to the preventive instructions
$(1-\in_c \tau_c)$	Reflects the impact of condom usage which is enhanced by public campaign (efficacy and compliance) on sexual transmission where $0<\in_c,\tau_c<1$

2.3 First Model Reformation

To reform the model we assumed the following:

$$\left.\begin{array}{l}\beta_1=\alpha_1\phi_3,\beta_2=\alpha_{21}\phi_{21}(1-\in_c \tau_c),\beta_3=\alpha_{22}\phi_{22}(1-\in_c \tau_c),\beta_4=\alpha_3^{11}\phi_{11},\beta_5=\alpha_3^{12}\phi_{12},k_6=\gamma_{21}+\mu_2\\ \beta_6=\alpha_3^{21}\phi_{21},\beta_7=\alpha_3^{22}\phi_{22},\beta_8=\alpha_2\phi_3,\beta_9=\alpha_{11}\phi_{11}(1-\in_c \tau_c),\beta_{10}=\alpha_{12}\phi_{12}(1-\in_c \tau_c),k_7=\gamma_{22}+\mu_2\\ k_1=\gamma_1+\sigma_{11}+\sigma_{12}+\mu_1,k_2=\gamma_{11}+\mu_1,k_3=\gamma_{12}+\mu_1,k_4=\mu_3+\delta,k_5=\gamma_2+\sigma_{21}+\sigma_{22}+\mu_2\end{array}\right] \tag{2.13}$$

1.

Equations (2.1) to (2.12) become

$$\frac{dS_1}{dt}=\theta_1\omega_1\Lambda_1-\frac{S_1}{N_1}\{\beta_1 I_3+\beta_2 I_{21}+\beta_3 I_{22}\}-\mu_1 S_1 \tag{2.14}$$

$$\frac{dE_1}{dt}=(1-\theta_1)\omega_1\Lambda_1+\frac{S_1}{N_1}\{\beta_1 I_3+\beta_2 I_{21}+\beta_3 I_{22}\}-k_1 E_1 \tag{2.15}$$

$$\frac{dI_{11}}{dt}=\sigma_{11}E_1-k_2 I_{11} \tag{2.16}$$

$$\frac{dI_{12}}{dt}=\sigma_{12}E_1-k_3 I_{12} \tag{2.17}$$

$$\frac{dR_1}{dt}=(1-\omega_1)\Lambda_1+\gamma_1 E_1+\gamma_{11}I_{11}+\gamma_{12}I_{12}-\mu_1 R_1 \tag{2.18}$$

$$\frac{dS_3}{dt}=\Lambda_3-\frac{S_3}{N_3}(\beta_4 I_{11}+\beta_5 I_{12}+\beta_6 I_{21}+\beta_7 I_{22})-k_4 S_3 \tag{2.19}$$

$$\frac{dI_3}{dt}=\frac{S_3}{N_3}(\beta_4 I_{11}+\beta_5 I_{12}+\beta_6 I_{21}+\beta_7 I_{22})-k_4 I_3 \tag{2.20}$$

$$\frac{dS_2}{dt}=\theta_2\omega_2\Lambda_2-\frac{S_2}{N_2}\{\beta_8 I_3+\beta_9 I_{11}+\beta_{10} I_{12}\}-\mu_2 S_2 \tag{2.21}$$

$$\frac{dE_2}{dt}=(1-\theta_2)\omega_2\Lambda_2+\frac{S_2}{N_2}\{\beta_8 I_3+\beta_9 I_{11}+\beta_{10} I_{12}\}-k_5 E_2 \tag{2.22}$$

$$\frac{dI_{21}}{dt}=\sigma_{21}E_2-k_6 I_{21} \tag{2.23}$$

$$\frac{dI_{22}}{dt}=\sigma_{22}E_2-k_7 I_{21} \tag{2.24}$$

$$\frac{dR_2}{dt}=(1-\omega_2)\Lambda_2+\gamma_2 E_2+\sigma_{21}I_{21}+\sigma_{22}I_{22}-\mu_2 R_2 \tag{2.25}$$

3 Results and Discussion

3.1 Equilibrium States of the Model

At equilibrium

$$\frac{dS_1}{dt}=\frac{dE_1}{dt}=\frac{dI_{11}}{dt}=\frac{dI_{12}}{dt}=\frac{dR_1}{dt}=\frac{dS_3}{dt}=\frac{dI_3}{dt}=\frac{dS_2}{dt}=\frac{dE_2}{dt}=\frac{dI_{21}}{dt}=\frac{dI_{22}}{dt}=\frac{dR_2}{dt}=0 \tag{2.26}$$

If $(S_1^*,E_1^*,I_{11}^*,I_{12}^*,R_1^*,S_3^*,I_3^*,S_2^*,E_2^*,I_{21}^*,I_{22}^*,R_2^*)$ be the equilibrium points and let $C_1=\frac{S_1^*}{N_1^*},C_2=\frac{S_2^*}{N_2^*},C_3=\frac{S_3^*}{N_3^*},k_8=\gamma_1+\sigma_1+\mu_1,k_9=\gamma_{R_1}+\mu_1,k_{10}=\gamma_2+\sigma_2+\mu_2$, $k_{11}=\gamma_{R_2}+\mu_2,I_1^*=I_{11}^*+I_{12}^*,I_2^*=I_{21}^*+I_{22}^*$ then at equilibrium points, we obtain the following equations:

$$\theta_1\omega_1\Lambda_1-C_1(\beta_1 I_3^*+\beta_{21}I_2^*)-\mu_1 S_1^*=0 \tag{2.27}$$

$$(1-\theta_1)\omega_1\Lambda_1+C_1(\beta_1 I_3^*+\beta_{21}I_2^*)-k_8 E_1^*=0 \tag{2.28}$$

$$\sigma_1 E_1^*-k_9 I_1^*=0 \tag{2.29}$$

$$(1-\omega_1)\Lambda_1+\gamma_1 E_1^*+\gamma_{R_1}I_1^*-\mu_1 R_1^*=0 \tag{2.30}$$

$$\Lambda_3-C_3(\beta_{13}I_1^*+\beta_{23}I_2^*)-k_4 S_3^*=0 \tag{2.31}$$

$$C_3(\beta_{13}I_1^*+\beta_{23}I_2^*)-k_4 I_3^*=0 \tag{2.32}$$

$$\theta_2\omega_2\Lambda_2-C_2(\beta_8 I_3^*+\beta_{12}I_1^*)-\mu_2 S_2^*=0 \tag{2.33}$$

$$(1-\theta_2)\omega_2\Lambda_2+C_2(\beta_8 I_3^*+\beta_{12}I_1^*)-k_{10}E_2^*=0 \tag{2.34}$$

$$\sigma_2 E_2^*-k_{11}I_2^*=0 \tag{2.35}$$

$$(1-\omega_2)\Lambda_2+\gamma_2 E_2^*+\gamma_{R_2}I_2^*-\mu_2 R_2^*=0 \tag{2.36}$$

From equation (2.29) $E_1^*=\dfrac{k_9}{\sigma_1}I_1^*$ (2.37)

From equation (2.35) $E_2^*=\dfrac{k_{11}}{\sigma_2}I_2^*$ (2.38)

From equation (2.32) we obtain $I_3^*=\dfrac{C_3\beta_{13}}{k_4}I_1+\dfrac{C_3\beta_{23}}{k_4}I_2$ (2.39)

Put equation (2.29) in equation (2.28) and simplify we then obtain

$$I_3^*=\frac{k_8k_9}{C_1\beta_1\sigma_1}I_1-\frac{\beta_{21}}{\beta_1}I_2-\frac{(1-\theta_1)\omega_1\Lambda_1}{C_1\beta_1} \tag{2.40}$$

Put equation (2.35) in equation (2.34) and simplify we then obtain

$$I_3^* = \frac{k_{10}k_{11}}{C_2\beta_8\sigma_2} I_2 - \frac{\beta_{12}}{\beta_8} I_1 - \frac{(1-\theta_2)\omega_2\Lambda_2}{C_2\beta_8} \tag{2.41}$$

Equate equation (2.39) and equation (2.40) then simplifying gives

$$I_1^* = \frac{(C_3\beta_1\beta_{23} + k_4\beta_{21})C_1\sigma_1}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1} I_2^* + \frac{(1-\theta_1)\omega_1\Lambda_1\sigma_1 k_4}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1} \tag{2.42}$$

Equate equation (2.39) and equation (2.41) then simplifying gives

$$I_1^* = \frac{k_4k_{10}k_{11} - C_2C_3\beta_8\beta_{23}\sigma_2}{(C_3\beta_8\beta_{13} + k_4\beta_{12})C_2\sigma_2} I_2^* + \frac{(1-\theta_2)\omega_2\Lambda_2 k_4}{(C_3\beta_8\beta_{13} + k_4\beta_{12})C_2} \tag{2.43}$$

Equating equation (2.42) and equation (2.43) implies

$$I_2^* = \frac{[k_4(1-\theta_2)\sigma_2\omega_2\Lambda_2(k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1) + C_2k_4(1-\theta_1)\omega_1\Lambda_1\sigma_1\sigma_2(C_3\beta_8\beta_{13} + k_4\beta_{12})]}{[(k_4k_{10}k_{11} - C_2C_3\beta_8\beta_{23}\sigma_2)(k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1) - C_1C_2\sigma_1\sigma_2(C_3\beta_8\beta_{13} + k_4\beta_{12})(C_3\beta_1\beta_{23} + k_4\beta_{21})]} \tag{2.44}$$

(2.44) implies $I_2^* = \frac{a}{b}$ where a equals the numerator and b equals the denominator of (2.44), that is

$$I_2^* = \frac{a}{b} \tag{2.45}$$

If ξ proportion of I_2^* gives I_{21}^* then $(1-\xi)$ proportion of I_2^* results in I_{22}^*, imply

$$I_{21}^* = \frac{a\xi}{b} \tag{2.46}$$

$$I_{22}^* = \frac{a(1-\xi)}{b} \tag{2.47}$$

Put (2.45) in (2.43) implies $I_1^* = \frac{(C_3\beta_1\beta_{23} + k_4\beta_{21})C_1\sigma_1}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1}\frac{a}{b} + \frac{(1-\theta_1)\omega_1\Lambda_1\sigma_1 k_4}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1}$ (2.48)

Let $I_1^* = d$ (2.49)

If ρ proportion of I_1^* gives I_{11}^* then $(1-\rho)$ proportion of I_1^* results in I_{12}^*, imply

$$I_{11}^* = \rho d \tag{2.50}$$

$$I_{12}^* = (1-\rho)d \tag{2.51}$$

Put I_1^* in (2.37) gives $E_1^* = \frac{k_9}{\sigma_1}\{\frac{(C_3\beta_1\beta_{23} + k_4\beta_{21})C_1\sigma_1}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1}\frac{a}{b} + \frac{(1-\theta_1)\omega_1\Lambda_1\sigma_1 k_4}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1}\}$ (2.52)

Let $E_1^* = e$ (2.53)

Put I_2^* in (2.38) gives $E_2^* = \frac{k_{11}}{\sigma_2}\frac{a}{b}$ (2.54)

Substitute for I_1^* and I_2^* in (2.39) gives

$$I_3^* = \frac{C_3\beta_{13}}{k_4}\{\frac{(C_3\beta_1\beta_{23} + k_4\beta_{21})C_1\sigma_1}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1}\frac{a}{b} + \frac{(1-\theta_1)\omega_1\Lambda_1\sigma_1 k_4}{k_4k_8k_9 - C_1C_3\beta_1\beta_{13}\sigma_1}\} + \frac{aC_3\beta_{23}}{bk_4} \tag{2.55}$$

Let $I_3^* = f$ (2.56)

Substitute for I_3^* and I_2^* in (2.27) and simplifying gives

$$S_1^* = \frac{\theta_1\omega_1\Lambda_1}{\mu_1} - \frac{C_1}{\mu_1}(\beta_1 f + \beta_{21}\frac{a}{b}) \tag{2.57}$$

Substitute for E_1^* and I_1^* in (2.30) and simplifying gives

$$R_1^* = \frac{(1-\omega_1)\Lambda_1 + \gamma_1 e + \gamma_{R_1} d}{\mu_1} \tag{2.58}$$

Substitute for I_1^* and I_2^* in (2.31) and simplifying gives

$$S_3^* = \frac{\Lambda_3 - C_3(\beta_{13} d + \beta_{23}\frac{a}{b})}{k_4} \tag{2.59}$$

Substitute for I_1^* and I_3^* in (2.33) and simplifying gives

$$S_2^* = \frac{\theta_2\omega_2\Lambda_2}{\mu_2} - \frac{C_2}{\mu_2}(\beta_8 f + \beta_{12} d) \tag{2.60}$$

Substitute for E_2^* and I_2^* in (2.36) and simplifying gives

$$R_2^* = \frac{(1-\omega_2)\Lambda_2}{\mu_2} + \frac{a\gamma_2}{b\sigma_2\mu_2}(k_{11} + \sigma_2\gamma_{R_2}) \tag{2.61}$$

From (2.43), if $\sigma_1 = \sigma_2 = 0$ then $I_2^* = 0$ implies

$$\left.\begin{array}{l} I_{11}^* = I_{12}^* = I_{21}^* = I_{22}^* = I_3^* = R_1^* = R_2^* = 0 \\ S_1^* = \dfrac{\theta_1\omega_1\Lambda_1}{\mu_1}, S_3^* = \dfrac{\Lambda_3}{k_4} = \dfrac{\Lambda_3}{\mu_3+\delta}, S_2^* = \dfrac{\theta_2\omega_2\Lambda_2}{\mu_2} \end{array}\right] \tag{2.62}$$

4. Conclusion

We obtained two points at the equilibrium of the model, which are the disease free equilibrium point (DFE) and endemic equilibrium (EE) point. The points are; (i) when $I_{11}^* = I_{12}^* = I_{21}^* = I_{22}^* = I_3^* = R_1^* = R_2^* = 0$ gives (2.63) (ii) when $I_{11}^* \neq I_{12}^* \neq I_{21}^* \neq I_{22}^* \neq I_3^* \neq R_1^* \neq R_2^* \neq 0$ gives (2.64)

$$\varepsilon_0 = (S_1^*, E_1^*, I_{11}^*, I_{12}^*, R_1^*, S_3^*, I_3^*, S_2^*, E_2^*, I_{21}^*, I_{22}^*, R_2^*) = \left(\frac{\theta_1\omega_1\Lambda_1}{\mu_1}, 0,0,0,0, \frac{\Lambda_3}{\mu_3+\delta}, 0, \frac{\theta_2\omega_2\Lambda_2}{\mu_2}, 0,0,0,0\right) \tag{2.63}$$

The endemic equilibrium (EE) point is obtained when $\sigma_1 \neq 0 \neq \sigma_2$ and $I_2^* \neq 0$ as

$$\varepsilon_E = (S_1^{**}, E_1^{**}, I_{11}^{**}, I_{12}^{**}, R_1^{**}, S_3^{**}, I_3^{**}, S_2^{**}, E_2^{**}, I_{21}^{**}, I_{22}^{**}, R_2^{**}) = (S_1^*, E_1^*, I_{11}^*, I_{12}^*, R_1^*, S_3^*, I_3^*, S_2^*, E_2^*, I_{21}^*, I_{22}^*, R_2^*) \tag{2.64}$$

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