# ON THE PERFORMANCE OF FLOYD'S ALGORITHM AND ALL-OR-NOTHING ASSIGNMENT IN DETERMINING SHORTEST PATHS FOR COMMERCIAL MOTORCYCLES TOWN SERVICE 

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#### Abstract

The paper explored Urban Traffic Planning and Management in Bida town of Niger State using conventional transportation models of All-or-Nothing Traffic Assignment Technique and floyd's Algorithm to find minimum links and associated link flow volumes of commercial motorcycles that ply the routes in the town. The paper presented traffic counts (inflows and outflows) of commercial motorcycles in eight selected Centroids in the town during three peak periods (7am - 8am, 1pm - 2pm and $4 p m-5 p m)$. Minimum paths were identified to ensure good movement/flow of traffic and to reduce cost for the commercial motorcyclists.


Keyword: Transportation Model, Trip Generation, Trip Distribution, Modal Choice, Assignment Technique, All-orNothing, Floyd's, Algorithm.

## 1. Introduction

Complicated road environments, dense highway traffic networks, and random congestion in urban systems aggravate the difficulties of logistic transportation. Traffic jams result in decreased speed of commercial motorcycles, increase in urban logistic transportation costs, and decrease in the customer service level. Hence, establishing rational urban transportation routes with high efficiency, improving the timeliness of the logistic system, and reducing generalized logistic costs are urgently required [1].
The all-or-nothing assignment method involves the concept of traffic distribution, planning, and management. Traffic assignment refers to the process in which existing Origin-Destination (OD) trips are assigned to various paths of the network according to a specific assignment algorithm to obtain the assignment flow of each OD at each road segment and the total flow of each road segment. Supposing that the impedance of a road segment is a constant (the traveling time is not influenced by the traffic flow of this road segment), all trips of a producing point are assigned; an attracting point is likewise assigned to the shortest path between such points at one time, and no point is assigned to other road segments. Such assignment method is called the all-or-nothing assignment method [2, 3, 4].
The Floyd Algorithm, also called Floyd-Warshall algorithm is a shortest path algorithm. It is an example of dynamic programming which was published in its currently recognized form by Robert Floyd in 1962. The Floyd-Warshall algorithm compares all possible paths through the graph between each pair of vertices. Floyd's algorithm is more general than Dijkstra's because it determines the shortest route between any two nodes in the network. The algorithm represents an n-node network as a square matrix with $n$ rows and $n$ columns .Entry $(i, \mathrm{j})$ of the matrix gives the distance $d i j$ from node $i$ to node j , which is finite if $i$ is linked directly to j , and infinite otherwise [5, 6, 7].

### 1.1 Objectives of the Study

The objective of this paper is to identify the minimum link path and volume of trips between given origins to certain destinations in Bida town using all-or-nothing assignment technique and Floyd's algorithm.
2. Materials and Methods

According to [8], the idea of Floyd's algorithm is straightforward. Given three nodes $I, \mathrm{j}$, and $k$ in Figure 1 with the connecting distances shown on the three arcs, it is shorter to reach $j$ from $i$ passing through $k$ if
$d_{i k}+d_{k j}<d_{i j}$

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Fig. 1: Floyd's Triple Operation
In this case, it is optimal to replace the direct route from $i \rightarrow j$ with the indirect route $i \rightarrow k \rightarrow j$. This triple operation exchange is applied systematically to the network using the following steps:

Step 0: Define the starting distance matrix $D_{0}$ and node sequence matrix $\mathrm{S}_{0}$ as given below. The diagonal elements are marked with $(-)$ to indicate that they are blocked. Set $k=1$.

$$
\begin{aligned}
&
\end{aligned}
$$

General step $\boldsymbol{k}$ : Define row $k$ and column $k$ as pivot row and pivot column. Apply the triple operation to each element $\mathrm{d}_{\mathrm{ij}}$ in $D_{k-l}$, for all $i$ and $j$. If the condition
$d_{i k}+d_{k j}<d_{i j},(\mathrm{i} \neq \mathrm{k}, \mathrm{j} \neq \mathrm{k}$, and $\mathrm{I} \neq j)$
is satisfied, make the following changes:
(a) Create $\mathrm{D}_{\mathrm{k}}$ by replacingd $\mathrm{ij}_{\mathrm{ij}}$ in $\mathrm{D}_{\mathrm{k}-1}$ with $d_{i k}+d_{k j}$
(b) Create $\mathrm{S}_{\mathrm{k}}$ by replacing $\mathrm{S}_{\mathrm{ij}} \mathrm{inS}_{\mathrm{k}-1}$ with k . Set $k=k+1$. If $k=n+1$, stop; else repeat step $k$.

Step $k$ :Here, row $k$ and column $k$ define the current pivot row and column. Rowire presents any of the rows $1,2, \ldots$, and $k$ $1, n$ and column $j$ represents any of the columns $1,2, \ldots$, and $k-1, n$. The triple operation can be applied as follows. If the sum of the elements on the pivot row and the pivot column is smaller than the associated intersection element, then it is optimal to replace the intersection distance by the sum of the pivot distances.

After $n$ steps, we can determine the shortest route between nodes $i$ and $j$ from the Matrices $D_{\mathrm{n}}$ and $\mathrm{S}_{\mathrm{n}}$ using the following rules:

1. From $D_{n}$,dijgives the shortest distance between nodes $i$ and $j$.
2. From $S_{n}$, determine the intermediate node $k=\mathrm{S}_{\mathrm{ij}}$ that yields the route $i \rightarrow k \rightarrow j$.

If $\mathrm{s}_{\mathrm{ik}}=k$ ands $_{\mathrm{kj}}=j$, stop; all the intermediate nodes of the route have been found. Otherwise, repeat the procedure between nodes $i$ and $k$, and betweennodes $k$ and $j[8]$.

## 3. Results and Discussion

3.1 Data Presentation

The survey on motorcycle (popular called Byke or Okada or Kabukabu) was carried out in Bida, Bida Local Government Area of Niger State. Below is the road network digraph of 8 zones which were considered along with arc weights as the average time (in minutes) it takes a motorcycle at $40 \mathrm{~km} / \mathrm{h}$ to move from an origin to a destination.


Figure 2: A Digraph of TAZs
Table 1: TAZs Defined


Table 2: Link Array Table

| From/To | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | - | 2 | 4 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $\mathbf{2}$ | 2 | - | $\infty$ | 2 | 4 | $\infty$ | $\infty$ | $\infty$ |
| $\mathbf{3}$ | 4 | $\infty$ | - | 5 | $\infty$ | $\infty$ | 8 | $\infty$ |
| $\mathbf{4}$ | $\infty$ | 2 | 5 | - | 4 | 5 | $\infty$ | $\infty$ |
| $\mathbf{5}$ | $\infty$ | 4 | $\infty$ | 4 | - | 7 | $\infty$ | $\infty$ |
| $\mathbf{6}$ | $\infty$ | $\infty$ | $\infty$ | 4 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | $\infty$ | $\infty$ | 8 | $\infty$ | $\infty$ | 2 | - | 4 |
| $\mathbf{8}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 5 | 4 | - |

Note: Table 2 indicates the zones that have direct link with their associate travel times, for instance zone 1 has direct link to zone 2 and 3 with travel times of 2 and 4 minutes respectively. Those without direct link are marked with $\infty$.

Table 3: Traffic Productions (Outflow) and Attraction (Inflow) during peak periods of the TAZs

|  | 7 - 8AM |  | 1-2PM |  | 5-6PM |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zone | Production <br> Outflow | Attraction <br> Inflow | Production <br> Outflow | Attraction <br> Inflow | Production <br> Outflow | Attraction Inflow |
| $\mathbf{1}$ | 550 | 700 | 450 | 650 | 500 | 600 |
| $\mathbf{2}$ | 670 | 860 | 560 | 800 | 750 | 500 |
| $\mathbf{3}$ | 350 | 280 | 300 | 430 | 400 | 550 |
| $\mathbf{4}$ | 700 | 950 | 640 | 880 | 850 | 720 |
| $\mathbf{5}$ | 550 | 750 | 650 | 600 | 720 | 450 |
| $\mathbf{6}$ | 400 | 640 | 300 | 520 | 320 | 550 |
| $\mathbf{7}$ | 420 | 400 | 330 | 480 | 360 | 440 |
| $\mathbf{8}$ | 580 | 800 | 500 | 740 | 750 | 560 |

Note: This was head count of motorcycles as they were produced from and attracted to zones

### 3.2. Problem Formulation and Data Analysis

Table 4: Trip Generation: Average Trip Production and Attraction in the TAZs

| Zone | Production (Outflow) | Attraction (Inflow) |
| :---: | :---: | :---: |
| $\mathbf{1}$ | 500 | 650 |
| $\mathbf{2}$ | 660 | 720 |
| $\mathbf{3}$ | 350 | 420 |
| $\mathbf{4}$ | 730 | 850 |
| $\mathbf{5}$ | 640 | 600 |
| $\mathbf{6}$ | 340 | 570 |
| $\mathbf{7}$ | 370 | 440 |
| $\mathbf{8}$ | 610 | 700 |
| TOTAL | 4200 | 4950 |

Observe from table 4that the total outflow is not equal to total inflow, i.e
$\sum_{i=1}^{N} P_{i}=4200 \neq \sum_{i=1}^{N} A_{j}=4950$
Thus, we adjust trip attractions to match total attraction equal to total production as shown in the table 5. Usually, the attractions are modified to make their sum equal to that of the production. This is done by applying the following formula to each attraction for each zone:
Adjusted Trip Attraction $\left(A T A_{j}\right)=\frac{\sum_{i=1}^{N} P_{i}}{\sum_{j=1}^{N} A_{j}} \mathrm{x} A_{j}$; where $P$ is Production and $A$ is attraction.
Table 5: Adjusted trip inflow in the TAZs

| Zone | Trip Production (Outflow) | Adjusted Trip Attraction (Inflow) |
| :---: | :---: | :---: |
| $\mathbf{1}$ | 500 | 552 |
| $\mathbf{2}$ | 660 | 611 |
| $\mathbf{3}$ | 350 | 356 |
| $\mathbf{4}$ | 730 | 721 |
| $\mathbf{5}$ | 640 | 509 |
| $\mathbf{6}$ | 340 | 484 |
| $\mathbf{7}$ | 370 | 373 |
| $\mathbf{8}$ | 610 | 594 |
| TOTAL | 4200 | 4200 |

### 3.3 Trip Distribution

After the trip attraction and trip production corresponding to each zone are obtained, the number of trips between any two zones is obtained as shown in the table 6. This is done using the formula
$D_{i j}=\frac{P_{i} \times A_{j}}{T}$; where $D_{i j}$ is the distance from $i$ to $j, \mathrm{P}_{i}$ is Production at $i, \mathrm{~A}_{j}$ is Attraction at $j$ and $T$ is the Total of productions and Attractions which is the same. For example
$D_{11}=0, D_{12}=\frac{500 \times 611}{4200} \approx 73, D_{13}=\frac{500 \times 356}{4200} \approx 42, D_{14}=\frac{500 \times 721}{4200} \approx 86, D_{15}=\frac{500 \times 509}{4200} \approx 61$

Table 6: Trip Distribution Table

| Zone | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | Obtained $\left(\mathbf{P}_{\mathbf{i}}\right)$ | Actual $\left(\mathbf{P}_{\mathbf{i}}\right)$ | Row Factor $\left(\mathbf{F}_{\mathbf{i}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 73 | 42 | 86 | 61 | 58 | 44 | 71 | 435 | 500 | 1.15 |
| $\mathbf{2}$ | 87 | 0 | 56 | 113 | 80 | 76 | 59 | 93 | 564 | 660 | 1.17 |
| $\mathbf{3}$ | 46 | 51 | 0 | 60 | 42 | 40 | 31 | 49 | 320 | 350 | 1.09 |
| $\mathbf{4}$ | 96 | 106 | 62 | 0 | 88 | 84 | 65 | 103 | 605 | 730 | 1.21 |
| $\mathbf{5}$ | 84 | 93 | 54 | 110 | 0 | 74 | 57 | 91 | 562 | 640 | 1.14 |
| $\mathbf{6}$ | 45 | 49 | 29 | 58 | 41 | 0 | 30 | 48 | 301 | 340 | 1.13 |
| $\mathbf{7}$ | 49 | 54 | 31 | 64 | 45 | 43 | 0 | 52 | 337 | 370 | 1.10 |
| $\mathbf{8}$ | 80 | 89 | 52 | 105 | 74 | 70 | 54 | 0 | 524 | 610 | 1.16 |
| Obtained $\left(A_{j}\right)$ | 486 | 515 | 327 | 596 | 432 | 444 | 340 | 508 | 3647 | 4200 |  |
| Actual $\left(A_{j}\right)$ | 552 | 611 | 356 | 721 | 509 | 484 | 373 | 594 | 4200 |  |  |
| Column <br> Factor $\left(F_{j}\right)$ | 1.14 | 1.19 | 1.09 | 1.21 | 1.18 | 1.09 | 1.10 | 1.17 |  |  |  |

Observe from table 6 that the targeted trip productions and attractions are not equal to the obtained trip productions and attractions. Thus, an Adjustment of Trip Distribution is done using column factors from the table above i.e
$F_{j}^{0}=\frac{A_{j}^{0}}{A_{j}^{1}} ;$ For example, $\quad F_{1}^{0}=\frac{A_{1}^{0}}{A_{1}^{1}}=\frac{552}{486} \approx 1.14 ; F_{2}^{0}=\frac{A_{2}^{0}}{A_{2}^{1}}=\frac{611}{515} \approx 1.19 ; F_{3}^{0}=\frac{A_{3}^{0}}{A_{3}^{1}}=\frac{356}{336} \approx 1.09 ; F_{4}^{0}=\frac{A_{4}^{0}}{A_{4}^{1}}=\frac{721}{596} \approx 1.21 ; \ldots F_{8}^{0}=\frac{A_{8}^{0}}{A_{8}^{1}}=\frac{594}{508} \approx$
1.17

Calculations of New trip distributions can now be done by obtaining the column and row balancing factors. This is done iteratively by multiplying column factors column-wise and row factors row-wise starting with column factor. At the fourth iteration, it was observed that all the actual trip productions and attractions are equal to the obtained trip productions and attractions as can be seen in table 7 .
$D T_{i j}^{1}=F_{j}^{0} \times D_{i j}^{0}$, for example
$D_{12}^{1}=F_{2}^{0} \times D_{12}^{0}=1.19 \times 73 \approx 87 ; D_{13}^{1}=F_{3}^{0} \times D_{13}^{0}=1.09 \times 42 \approx 46 ; D_{14}^{1}=F_{4}^{0} \times D_{14}^{0}=1.21 \times 86 \approx 104 \ldots D_{87}^{1}=F_{7}^{0} \times D_{87}^{0}=1.09 \times 54 \approx 59$.
Table 7: Adjusted Trip Distribution Table (Fourth Iteration)

| Zone | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | Obtained | Actual | Row Factor |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 87 | 46 | 104 | 71 | 62 | 48 | 82 | 500 | 500 | 1.00 |
| $\mathbf{2}$ | 100 | 0 | 62 | 141 | 96 | 84 | 65 | 112 | 660 | 660 | 1.00 |
| $\mathbf{3}$ | 49 | 57 | 0 | 69 | 47 | 41 | 32 | 55 | 350 | 350 | 1.00 |
| $\mathbf{4}$ | 115 | 134 | 71 | 0 | 110 | 97 | 75 | 128 | 730 | 730 | 1.00 |
| $\mathbf{5}$ | 95 | 109 | 58 | 133 | 0 | 79 | 61 | 105 | 640 | 640 | 1.00 |
| $\mathbf{6}$ | 49 | 57 | 30 | 69 | 47 | 0 | 33 | 55 | 340 | 340 | 1.00 |
| $\mathbf{7}$ | 52 | 61 | 32 | 74 | 50 | 44 | 0 | 57 | 370 | 370 | 1.00 |
| $\mathbf{8}$ | 92 | 107 | 57 | 130 | 88 | 77 | 59 | 0 | 610 | 610 | 1.00 |
| Obtained | 552 | 611 | 356 | 721 | 509 | 484 | 373 | 594 |  |  |  |
| Actual | 552 | 611 | 356 | 721 | 509 | 484 | 373 | 594 |  |  |  |
| Column Factor | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |  |  |  |



### 3.4 Trip Assignment: Minimum Path Finder (Floyd's Algorithm)

Table 8: Showing Matrix $\mathrm{D}_{0}$ andMatrix $\mathrm{S}_{0}$ (Distance and Sequenceof the network)

| $\mathrm{D}_{0}=$ |  |  |  |  |  |  |  |  | $\mathrm{S}_{0}=$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | - | 2 | 4 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 1 | - | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | - | $\infty$ | 2 | 4 | $\infty$ | $\infty$ | $\infty$ | 2 | 2 | - | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 4 | $\infty$ | - | 5 | $\infty$ | $\infty$ | 8 | $\infty$ | 3 | 3 | 3 | - | 3 | 3 | 3 | 3 | 3 |
| 4 | $\infty$ | 2 | 5 | - | 4 | 5 | $\infty$ | $\infty$ | 4 | 4 | 4 | 4 | - | 4 | 4 | 4 | 4 |
| 5 | $\infty$ | 4 | $\infty$ | 4 | - | 7 | $\infty$ | $\infty$ | 5 | 5 | 5 | 5 | 5 | - | 5 | 5 | 5 |
| 6 | $\infty$ | $\infty$ | $\infty$ | 5 | 7 | - | 2 | 5 | 6 | 6 | 6 | 6 | 6 | 6 | - | 6 | 6 |
| 7 | $\infty$ | $\infty$ | 8 | $\infty$ | $\infty$ | 2 | - | 4 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | - | 7 |
| 8 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 5 | 4 | - | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | - |

All elements along the main diagonal of matrix $T_{0}$ equal zero since by definition $t_{i, j}^{0}=0$ for $\mathrm{i}=\mathrm{j}$.
We note element $d_{12}^{0}$ and $d_{13}^{0}$ of matrix $D_{0}$ have elements equal to 2 and 4 since the length of the branches connecting nodes $1-2$ and $1-3$ are 2 and 4 respectively. Element $d_{14}^{0}$ equals infinity since the network has no branch which is oriented from node 1 to node 4 . Element $d_{15}^{0}$ of matrix $D_{0}$ equals infinity as well since there is no direct branch linking nodes 1 and 5 and so on.
Note from the Sequence table that node $i$ is the immediate predecessor of node $j$ on the shortest path leading from node i to node j (for $\mathrm{i} \neq \mathrm{j}$ ). For this reason we have, elements of $s_{12}^{0}=s_{13}^{0}=s_{14}^{0} \ldots=1$ and $s_{21}^{0}=s_{23}^{0}=s_{24}^{0} \ldots=2$ and so on in matrix $\mathrm{S}_{0}$.

Table 9: Iteration $\mathrm{k}=1$

| $\mathrm{D}_{1}=$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | $\infty$ | $\infty$ | $\infty$ |
| $\mathbf{3}$ | 4 | 6 | - | 5 | $\infty$ | $\infty$ | 8 | $\infty$ |
| $\mathbf{4}$ | $\infty$ | 2 | 5 | - | 4 | 5 | $\infty$ | $\infty$ |
| $\mathbf{5}$ | $\infty$ | 4 | $\infty$ | 4 | - | 7 | $\infty$ | $\infty$ |
| $\mathbf{6}$ | $\infty$ | $\infty$ | $\infty$ | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | $\infty$ | $\infty$ | 8 | $\infty$ | $\infty$ | 2 | - | 4 |
| $\mathbf{8}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 5 | 4 | - |


| $\mathrm{S}_{1}=$ |  | $\mathbf{1}$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mathbf{2}$ | 2 | - | 1 | 2 | 2 | 2 | 2 | 2 |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 3 | 3 | 3 | 3 |
| $\mathbf{4}$ | 4 | 4 | 4 | - | 4 | 4 | 4 | 4 |
| $\mathbf{5}$ | 5 | 5 | 5 | 5 | - | 5 | 5 | 5 |
| $\mathbf{6}$ | 6 | 6 | 6 | 6 | 6 | - | 6 | 6 |
| $\mathbf{7}$ | 7 | 7 | 7 | 7 | 7 | 7 | - | 7 |
| $\mathbf{8}$ | 8 | 8 | 8 | 8 | 8 | 8 | 8 | - |

Table 10: Iteration $\mathrm{k}=2$

| $\mathrm{D}_{2}=$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | 4 | 6 | $\infty$ | $\infty$ | $\infty$ |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | $\infty$ | $\infty$ | $\infty$ |
| $\mathbf{3}$ | 4 | 6 | - | 5 | 10 | $\infty$ | 8 | $\infty$ |
| $\mathbf{4}$ | 4 | 2 | 5 | - | 4 | 5 | $\infty$ | $\infty$ |
| $\mathbf{5}$ | 6 | 4 | 10 | 4 | - | 7 | $\infty$ | $\infty$ |
| $\mathbf{6}$ | $\infty$ | $\infty$ | $\infty$ | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | $\infty$ | $\infty$ | 8 | $\infty$ | $\infty$ | 2 | - | 4 |
| $\mathbf{8}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 5 | 4 | - |


| $\mathrm{S}_{2}=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |  |
| $\mathbf{1}$ | - | 1 | 1 | 2 | 2 | 1 | 1 | 1 |  |
| $\mathbf{2}$ | 2 | - | 1 | 2 | 2 | 2 | 2 | 2 |  |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 2 | 3 | 3 | 3 |  |
| $\mathbf{4}$ | 2 | 4 | 4 | - | 4 | 4 | 4 | 4 |  |
| $\mathbf{5}$ | 2 | 5 | 2 | 5 | - | 5 | 5 | 5 |  |
| $\mathbf{6}$ | 6 | 6 | 6 | 6 | 6 | - | 6 | 6 |  |
| $\mathbf{7}$ | 7 | 7 | 7 | 7 | 7 | 7 | - | 7 |  |
| $\mathbf{8}$ | 8 | 8 | 8 | 8 | 8 | 8 | 8 | - |  |

Table 11: Iteration $\mathrm{k}=3$

| $\mathrm{D}_{3}=$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | 4 | 6 | $\infty$ | 12 | $\infty$ |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | $\infty$ | 14 | $\infty$ |
| $\mathbf{3}$ | 4 | 6 | - | 5 | 10 | $\infty$ | 8 | $\infty$ |
| $\mathbf{4}$ | 4 | 2 | 5 | - | 4 | 5 | 13 | $\infty$ |
| $\mathbf{5}$ | 6 | 4 | 10 | 4 | - | 7 | 18 | $\infty$ |
| $\mathbf{6}$ | $\infty$ | $\infty$ | $\infty$ | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | 12 | 14 | 8 | 13 | 18 | 2 | - | 4 |
| $\mathbf{8}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 5 | 4 | - |


| $\mathrm{S}_{3}=$ |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 1 | $\mathbf{1}$ | 2 | 2 | 1 | 3 | 1 |
| $\mathbf{2}$ | 2 | - | 1 | 2 | 2 | 2 | 3 | 2 |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 2 | 3 | 3 | 3 |
| $\mathbf{4}$ | 2 | 4 | 4 | - | 4 | 4 | 3 | 4 |
| $\mathbf{5}$ | 2 | 5 | 2 | 5 | - | 5 | 3 | 5 |
| $\mathbf{6}$ | 6 | 6 | 6 | 6 | 6 | - | 6 | 6 |
| $\mathbf{7}$ | 3 | 3 | 7 | 3 | 3 | 7 | - | 7 |
| $\mathbf{8}$ | 8 | 8 | 8 | 8 | 8 | 8 | 8 | - |

Table 12: Iteration $\mathrm{k}=4$


Table 13: Iteration $\mathrm{k}=5$

| $\mathrm{D}_{5}=$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | 4 | 6 | 9 | 12 | $\infty$ |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | 7 | 10 | $\infty$ |
| $\mathbf{3}$ | 4 | 6 | - | 5 | 9 | 10 | 8 | $\infty$ |
| $\mathbf{4}$ | 4 | 2 | 5 | - | 4 | 5 | 13 | $\infty$ |
| $\mathbf{5}$ | 6 | 4 | 9 | 4 | - | 7 | 17 | $\infty$ |
| $\mathbf{6}$ | 9 | 7 | 10 | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | 12 | 10 | 8 | 13 | 17 | 2 | - | 4 |
| $\mathbf{8}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 5 | 4 | - |


| $\mathrm{S}_{5}=$ |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 1 | 1 | 2 | 2 | 4 | 3 | 1 |
| $\mathbf{2}$ | 2 | - | 1 | 2 | 2 | 4 | 3 | 2 |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 4 | 4 | 3 | 3 |
| $\mathbf{4}$ | 2 | 4 | 4 | - | 4 | 4 | 3 | 4 |
| $\mathbf{5}$ | 2 | 5 | 4 | 5 | - | 5 | 4 | 5 |
| $\mathbf{6}$ | 4 | 4 | 4 | 6 | 6 | - | 6 | 6 |
| $\mathbf{7}$ | 3 | 3 | 7 | 3 | 4 | 7 | - | 7 |
| $\mathbf{8}$ | 8 | 8 | 8 | 8 | 8 | 8 | 8 | - |

Table 14: Iteration $\mathrm{k}=6$

| $\mathrm{D}_{6}=$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | 4 | 6 | 9 | 11 | 14 |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | 7 | 9 | 12 |
| $\mathbf{3}$ | 4 | 6 | - | 5 | 9 | 10 | 8 | 15 |
| $\mathbf{4}$ | 4 | 2 | 5 | - | 4 | 5 | 7 | 10 |
| $\mathbf{5}$ | 6 | 4 | 9 | 4 | - | 7 | 9 | 12 |
| $\mathbf{6}$ | 9 | 7 | 10 | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | 11 | 9 | 8 | 7 | 9 | 2 | - | 4 |
| $\mathbf{8}$ | $\mathbf{1 4}$ | $\mathbf{1 2}$ | $\mathbf{1 5}$ | $\mathbf{1 0}$ | $\mathbf{1 2}$ | $\mathbf{5}$ | 4 | - |


| $\mathrm{S}_{6}=$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |  |  |  |  |  |  |  |  |
| $\mathbf{1}$ | - | 2 | 3 | 2 | 2 | 4 | 6 | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{2}$ | 1 | - | 1 | 2 | 2 | 4 | 6 | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 4 | 4 | 3 | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{4}$ | 2 | 4 | 4 | - | 4 | 4 | 6 | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{5}$ | 2 | 5 | 4 | 5 | - | 5 | 6 | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{6}$ | 4 | 4 | 4 | 6 | 6 | - | 6 | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{7}$ | 6 | 6 | 7 | 6 | 6 | 7 | - | 8 |  |  |  |  |  |  |  |  |
| $\mathbf{8}$ | $\mathbf{6}$ | 6 | 6 | 6 | 6 | 6 | 8 | - |  |  |  |  |  |  |  |  |

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Table 15: Iteration $\mathrm{k}=7$

| $\mathrm{D}_{7}=$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | 4 | 6 | 9 | 11 | 14 |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | 7 | 9 | 12 |
| $\mathbf{3}$ | 4 | 6 | - | 5 | 9 | 10 | 8 | 12 |
| $\mathbf{4}$ | 4 | 2 | 5 | - | 4 | 5 | 7 | 10 |
| $\mathbf{5}$ | 6 | 4 | 9 | 4 | - | 7 | 9 | 12 |
| $\mathbf{6}$ | 9 | 7 | 10 | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | 11 | 9 | 8 | 7 | 9 | 2 | - | 4 |
| $\mathbf{8}$ | 14 | 12 | 12 | 10 | 12 | 5 | 4 | - |


| $\mathrm{S}_{7}=$ |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 3 | 2 | 2 | 4 | 6 | 6 |
| $\mathbf{2}$ | 1 | - | 1 | 2 | 2 | 4 | 6 | 6 |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 4 | 4 | 3 | 7 |
| $\mathbf{4}$ | 2 | 4 | 4 | - | 4 | 4 | 6 | 6 |
| $\mathbf{5}$ | 2 | 5 | 4 | 5 | - | 5 | 6 | 6 |
| $\mathbf{6}$ | 4 | 4 | 4 | 6 | 6 | - | 6 | 6 |
| $\mathbf{7}$ | 6 | 6 | 7 | 6 | 6 | 7 | - | 7 |
| $\mathbf{8}$ | 6 | 6 | 7 | 6 | 6 | 8 | 8 | - |

Table 16: Iteration $\mathrm{k}=8$

| $\mathrm{D}_{\mathbf{8}}=$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 2 | 4 | 4 | 6 | 9 | 11 | 14 |
| $\mathbf{2}$ | 2 | - | 6 | 2 | 4 | 7 | 9 | 12 |
| $\mathbf{3}$ | 4 | 6 | - | 5 | 9 | 10 | 8 | 12 |
| $\mathbf{4}$ | 4 | 2 | 5 | - | 4 | 5 | 7 | 10 |
| $\mathbf{5}$ | 6 | 4 | 9 | 4 | - | 7 | 9 | 12 |
| $\mathbf{6}$ | 9 | 7 | 10 | 5 | 7 | - | 2 | 5 |
| $\mathbf{7}$ | 11 | 9 | 8 | 7 | 9 | 2 | - | 4 |
| $\mathbf{8}$ | 14 | 12 | 12 | 10 | 12 | 5 | 4 | - |


| $\mathrm{S}_{8}=$ |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| $\mathbf{1}$ | - | 1 | 1 | 2 | 2 | 4 | 6 | 6 |
| $\mathbf{2}$ | 2 | - | 1 | 4 | 5 | 4 | 6 | 6 |
| $\mathbf{3}$ | 3 | 1 | - | 3 | 4 | 4 | 3 | 7 |
| $\mathbf{4}$ | 2 | 4 | 4 | - | 4 | 4 | 6 | 6 |
| $\mathbf{5}$ | 2 | 5 | 4 | 5 | - | 5 | 6 | 6 |
| $\mathbf{6}$ | 4 | 4 | 4 | 6 | 6 | - | 6 | 6 |
| $\mathbf{7}$ | 6 | 6 | 3 | 6 | 6 | 7 | - | 7 |
| $\mathbf{8}$ | 6 | 6 | 7 | 6 | 6 | 8 | 8 | - |

Matrices $\mathrm{D}_{8}$ and $\mathrm{S}_{8}$ furnish us with complete information on the lengths of the shortest paths and the nodes on those paths between all pairs of nodes in the transportation network respectively as shown in the table 17.
Table 17: Recommended Shortest Path and Trip Volumes

| NODE |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| FROM | TO | MINIMUM LINK PATH | TRAVE TIME <br> (Min) | TRIPS | LINK VOLUME |
| 1 | 2 | 1-2 | 2 | 86 | 454 |
|  | 3 | 1-3 | 4 | 46 | 46 |
|  | 4 | 1-2-4 | 4 | 105 | 297 |
|  | 5 | 1-2-5 | 6 | 71 | 71 |
|  | 6 | 1-2-4-6 | 9 | 62 | 192 |
|  | 7 | 1-2-4-6-7 | 11 | 48 | 48 |
|  | 8 | 1-2-4-6-8 | 14 | 82 | 82 |
| 2 | 1 | 2-1 | 8 | 100 | 162 |
|  | 3 | 2-1-3 | 6 | 62 | 62 |
|  | 4 | 2-4 | 2 | 141 | 402 |
|  | 5 | 2-5 | 4 | 96 | 96 |
|  | 6 | 2-4-6 | 7 | 84 | 261 |
|  | 7 | 2-4-6-7 | 9 | 65 | 65 |
|  | 8 | 2-4-6-8 | 12 | 112 | 112 |
| 3 | 1 | 3-1 | 4 | 49 | 106 |
|  | 2 | 3-1-2 | 6 | 57 | 57 |
|  | 4 | 3-4 | 5 | 69 | 157 |
|  | 5 | 3-4-5 | 9 | 47 | 47 |
|  | 6 | 3-4-6 | 10 | 41 | 41 |
|  | 7 | 3-7 | 8 | 32 | 87 |
|  | 8 | 3-7-8 | 12 | 55 | 55 |



## 4. Conclusion

This study can be generalized to cover other means of transportation apart from motorcycles; and the methodology can also be adopted in other cities or towns with modification in respective data. The study provides both present and future motorcyclists and other road users with minimum link paths and link flow volumes in Bida town. Table 17 furnishes us with the information as a recommendation which can practically be implemented. The optimized method retains the characteristic of simple calculation in the all-or-nothing algorithm. Owing to the consideration of road network conditions in the assignment process, the assignment results are more practical and can adapt to the characteristics of the town system, such as complicated road network and heavy traffic volume.

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