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SENSITIVITY ANALYSIS FOR PRODUCTION PLANNING MODEL AND ITS APPLICATION TO DECISION MAKING (A CASE STUDY OF SUNSEED NIGERIA PLC ZARIA)

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Abstract

Sunseed Nigeria plc Zaria as a production and distribution company is characterized with various degree of challenges. We observed that for the company to achieve efficiency in its operations, careful attention must be paid on its decision making process and sales of output. This is only achievable when certain strategies are employed to overcome production and distribution problems. Sensitivity Analysis presents a post optimality investigation of how a change in the model data changes the optimal solution. Therefore interpretations of the results from the analysis allow decision makers to determine how "sensitive" the optimal solution is to changes in data values.

Keywords**:** Sensitivity Analysis, Production planning, Integer Programming, Optimality

1.0 Introduction

Production and distribution is the core process of every manufacturing organization and so the efficiency and quality of decisions taken by the organization determines the performance of the organizations quality management system. Production planning can be viewed as planning of the acquisition of resources and raw materials as well as planning of the production activities required to transform materials into finished products. All of the above, meeting customer demand in the most efficient or economical way possible, i.e. minimizing total costs [1].

Production and distribution firms are confronted with decision in production and distribution planning problems. Amongst such are the problems of making decisions regarding the size of production and distribution levels, for each of the time periods in a planning horizon and decisions on the quantities of raw materials to purchase, store, order or process and distributed. These problems have led to a loss of goodwill and even a total collapse of many firms over the years. In a bid to addressing these problems, many scholars have explore various techniques with much results but the numerous problems cannot be treated completely in a single article, hence our work intends to address the problems associated with decision in distribution confronting manufacturers as they distribute products.

In this research we formulate production and distribution processes and present relevance solution approaches for the models formulated. Sensitivity analysis was tested on the distribution model and the result obtained showed how the optimal solution may be affected by changes in the objective function coefficients or the right- hand side values is also given. The remainder of this paper is organized as follows: in section 2, the Integer Programming production planning problem formulation that maximizes profit under manufacturing and selling constraints is presented and the sensitivity analysis formulation procedure that is used for the model. In section 3, we present the model formulations of the production and distribution process. In section 4, an application problem of distribution cost minimization and optimal production is given where the optimal solution is determined using the LIPS solver (Linear Program Solver). Finally the concluding remarks are given in section 6

2.0 IP and SA Problem Formulation for Production Planning

Production planning process consists of three stages, namely, manufacturing and marketing data preparation, generation of production items and selling alternatives, and production plan formulation. [2,3,4]. IP model is used for optimization of the produced and sold items under manufacturing constraints that will maximize the profit.

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2.1 Formulation of Integer linear programming for Production Planning

The problem of IP is the search for the optimal (minimum or maximum) of a linear function of variables constrained by linear relations (equations or inequalities) [5, 6, 7, 8]. The IP optimizes a linear objective function subject to a set of linear equalities or inequalities. The general production planning maximization models is given as;

Model (P1)

$$\left.\begin{array}{l} Optimize\, Z : c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ Subject\ to: \\ \qquad b_{11}x_1 + b_{12}x_2 + \ldots + b_{1n}x_n \le R_1 \\ \qquad b_{21}x_1 + b_{22}x_2 + \ldots + b_{2n}x_n \le R_2 \\ \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \qquad b_{m1}x_1 + b_{m2}x_2 + \ldots + b_{mn}x_n \le R_m \\ \qquad x_j \ge 0 \\ \qquad j = 1, 2, 3 \ldots n; \quad i = 1, 2, 3, \ldots, m \end{array}\right\} \qquad (1)$$

Where,

Z = objective function that maximized selling profits.

x_j = choice variable (production item) for which the problem solved.

c_j = coefficient measuring the contribution of the j^{th} choice variable to the objective function.

R_i = constraint or restrictions placed upon the problem.

b_{ij} = coefficient measuring the effect of the i^{th} constraint on the j^{th} choice variable.

2.2 Sensitivity Analysis of the Model

The following procedure is used for the sensitivity analysis of the model (P1):

For the objective functions coefficients;

$\sum_j \frac{\delta c_j}{\Delta c_j} \le 1$, the optimal solution will not change

δc_j is the actual increase (decrease) in the coefficient,

and,

Δc_j is the maximum allowable increase (decrease) from the sensitivity analysis.

For the right hand side constants

$\sum_j \frac{\delta a_j}{\Delta a_j} \le 1$, the optimal basis will not change,

δa_j is the actual increase (decrease) in the coefficient

and,

Δa_j is the maximum allowable increase (decrease) from the sensitivity analysis.

3.0 Model Formulation

Let x_{ij} denote the quantity to be distributed from location of a certain warehouse i to another demand point j; the optimization problem will be formulated as model P2

Model (P2)

$$\left.\begin{aligned} &\underset{x_{ij}}{\text{minimize}}\ f=\sum_{i,j} c_{ij}x_{ij} \\ &Subject\ to: \quad \sum_j x_{ij}=w_i, \quad i=1,\ldots,m \\ &\qquad\qquad\quad \sum_i x_{ij}=d_j, \ i=1,..,n \\ &\qquad\qquad\quad x_{ij}\geq 0 \end{aligned}\right\} \quad (2)$$

where,

c_{ij} = denote the distribution cost from warehouse *(i)* to distribution point *(j)*

d_j = demand at distribution point j

w_i = Capacity of each warehouse or (supply point).

4.0 Application problem of Distribution Cost Minimization and Optimal Production Sunseed Nigeria plc Zaria has five warehouses w_1, w_2, w_3, w_4 and w_5 for storing palm oil, there are four major distribution centers d_1, d_2, d_3 and d_4 where the palm oil are taken to for onward sale. The estimated capacity of these warehouses are cw_1, cw_2, cw_3, cw_4 and cw_5 , while the demand at each distribution center are dd_1, dd_2, dd_3 and dd_4 The distance chart in kilometers between the warehouse and distribution centers is given in Table 2.In addition trucking company in charge of transporting the palm oil charges ₦108/Km per drum of palm oil. The transportation cost per drum of palm oil on different routes is given in Table 3. We want to find the optimal quantities x_{ij} to be distributed along these routes to minimize the total distribution cost.

Table 1: Capacity at Each Warehouse and Demand at Each Distribution Centers

	cw_i		dd_i
w_1	**450**	d_1	**500**
w_2	**320**	d_2	**340**
w_3	**250**	d_3	**380**
w_4	**400**	d_4	**900**
w_5	**700**		

Table 2: Distance in Kilometers between Warehouse and Distribution Centers

	d_1	d_2	d_3	d_4
w_1	**10**	**12**	**16**	**35**
w_2	**12**	**16**	**15**	**10**
w_3	**14**	**16**	**20**	**12**
w_4	**20**	**22**	**10**	**15**
w_5	**25**	**20**	**18**	**12**

We proceed by calculating the cost schedule as represented in Table 2

Table 3: Cost in (Naira) incurred Transporting between Warehouse and Distribution Centers

	d_1	d_2	d_3	d_4
w_1	**1080**	**1296**	**1728**	**3780**
w_2	**1296**	**1728**	**1620**	**1080**
w_3	**1512**	**1728**	**2160**	**1296**
w_4	**2160**	**2376**	**1080**	**1620**
w_5	**2700**	**2160**	**1944**	**1296**

4.1 Solution to Distribution Cost Minimization and Optimal Production problem

From Table 1, 2 and 3, we formulate the mathematical programming as follows;

$$\min imize\ f:\ 1080x_{11}+1296x_{12}+1728x_{13}+3780x_{14}+1296x_{21}+1728x_{22}+1620x_{23}+1080x_{24}+1512x_{31}+1728x_{32}+2160x_{33}+1296x_{34}+2160x_{41}+2376x_{42}+1080x_{43}+1620x_{44}+2700x_{51}+2160x_{52}+1944x_{53}+1296x_{54}$$

$$\text{Subject to:}\quad \left.\begin{array}{ll} x_{11}+x_{12}+x_{13}+x_{14} & =450 \\ x_{21}+x_{22}+x_{23}+x_{24} & =320 \\ x_{31}+x_{32}+x_{33}+x_{34} & =250 \\ x_{41}+x_{42}+x_{43}+x_{44} & =400 \\ x_{51}+x_{52}+x_{53}+x_{54} & =700 \\ x_{11}+x_{21}+x_{31}+x_{41}+x_{51} & =500 \\ x_{12}+x_{22}+x_{32}+x_{42}+x_{52} & =340 \\ x_{13}+x_{23}+x_{33}+x_{43}+x_{53} & =380 \\ x_{14}+x_{24}+x_{34}+x_{44}+x_{54} & =900 \end{array}\right\} \quad (3)$$

Solving the above equation (3) using LIPS (Linear Program Solver), we arrive at,

$$x_{ij}^{*}=\begin{Bmatrix} x_{11},x_{12},x_{13},x_{14},x_{21},x_{22},x_{23},x_{24},x_{31},x_{32} \\ x_{33},x_{34},x_{41},x_{42},x_{43},x_{44},x_{51},x_{52},x_{53},x_{54} \end{Bmatrix}=\begin{Bmatrix} 110 & 340 & 0 & 0 & 140 & 0 & 0 & 180 & 250 & 0 \\ 0 & 0 & 0 & 0 & 380 & 20 & 0 & 0 & 0 & 700 \end{Bmatrix} \quad (4)$$

Therefore, we have obtain the total drum of palm oil to be distributed between the warehouse and various distribution centers as shown above with a total minimum distribution cost of **₦2,663,280.00.**

4.2 Sensitivity Analysis for the Distribution Process

Demand for palm oil will be govern by the laws of demand, which predicts that the demand at distribution centers may be uncertain. In any case we should have three (3) possible demand scenarios (i.e. Low, Average or Most likely and High value demand) may be available. Now if,

(i) Low value demand at d_1, d_2, d_3 and d_4 equal 75% and occur with a probability $P_L = 0.3$.

(ii) Average or most-likely value demand equal 90% and occur with $P_A = 0.4$.

(iii) High value demand equal 100% and occur with probability $P_H = 0.3$,

i.e.

- Low values equals; 375, 255, 285, 675 carton for d_1, d_2, d_3 and d_4 respectively.
- Average Values equals; 450, 306, 342, 810 carton for d_1, d_2, d_3 and d_4 respectively.
- High values equals; 500, 340, 380, 900 carton for d_1, d_2, d_3 and d_4 respectively.

The possible distribution scenarios and the corresponding probabilities form distribution that can be used to describe future demands at distribution centers. We present the distribution scenarios in Table 4

Table 4: Distribution Possible Scenarios

Distribution Centers			
Scenarios	I	II	III
d_1	375	450	500
d_2	255	306	340
d_3	285	342	380
d_4	675	810	900
Probability of occurrence	**0.3**	**0.4**	**0.3**

When demand is uncertain, it is important to know when it will be revealed to the decision maker. Will it be known before his drum of palm oil is acquired, between acquisition and distribution, or after distribution decisions are made? That is, to determine the point during the decision sequence at which demands are known and to have complete information about the

demand before making any decisions .At the other extreme, demands might not be known until after drum of palm oil are acquired. The demand determines the actual distribution quantities and consequently the minimum distribution cost.
If demand is known at the start, our decisions are not exposed to uncertainty, and we need no cross-scenario evaluation. Because all uncertainty is resolved before we make any decisions, we adapt any decision to the specific scenario realized, and the problem collapses into a collection of deterministic problems;

An integer Programming model for this problem will be formulated from equation (3) as;

$$\left.\begin{array}{ll} \min imize\ f: & 1080x_{11}+1296x_{12}+1728x_{13}+3780x_{14}+1296x_{21}+1728x_{22}+1620x_{23}+1080x_{24}+ \\ & 1512x_{31}+1728x_{32}+2160x_{33}+1296x_{34}+2160x_{41}+2376x_{42}+1080x_{43}+1620x_{44}+ \\ & 2700x_{51}+2160x_{52}+1944x_{53}+1296x_{54} \\ & x_{11}+x_{12}+x_{13}+x_{14} = 450 \\ & x_{21}+x_{22}+x_{23}+x_{24} = 320 \\ & x_{31}+x_{32}+x_{33}+x_{34} = 250 \\ & x_{41}+x_{42}+x_{43}+x_{44} = 400 \\ \text{Subject to:} & x_{51}+x_{52}+x_{53}+x_{54} = 700 \\ & x_{11}+x_{21}+x_{31}+x_{41}+x_{51} \geq demand\ at\ d_1 \\ & x_{12}+x_{22}+x_{32}+x_{42}+x_{52} \geq demand\ at\ d_2 \\ & x_{13}+x_{23}+x_{33}+x_{43}+x_{53} \geq demand\ at\ d_3 \\ & x_{14}+x_{24}+x_{34}+x_{44}+x_{54} \geq demand\ at\ d_4 \end{array}\right\} \quad (5)$$

Based on this equation (5) and Table 4 it is possible to calculate RHS for each scenario: i.e
Demand at distribution centre 1 (d_1) = $(375\times0.3)+(450\times0.4)+(500\times0.3)=150$
Demand at distribution centre 2 (d_2) = $(255\times0.3)+(306\times0.4)+(340\times0.3)=301$
Demand at distribution centre 3 (d_3) = $(285\times0.3)+(342\times0.4)+(380\times0.3)=114$
Demand at distribution centre 4 (d_4) = $(675\times0.3)+(810\times0.4)+(900\times0.3)=270$
It follows that equation (5) becomes (6)

$$\left.\begin{array}{ll} \min imize\ f: & 1080x_{11}+1296x_{12}+1728x_{13}+3780x_{14}+1296x_{21}+1728x_{22}+1620x_{23}+1080x_{24}+ \\ & 1512x_{31}+1728x_{32}+2160x_{33}+1296x_{34}+2160x_{41}+2376x_{42}+1080x_{43}+1620x_{44}+ \\ & 2700x_{51}+2160x_{52}+1944x_{53}+1296x_{54} \\ & x_{11}+x_{12}+x_{13}+x_{14} = 450 \\ & x_{21}+x_{22}+x_{23}+x_{24} = 320 \\ & x_{31}+x_{32}+x_{33}+x_{34} = 250 \\ \text{Subject to:} & x_{41}+x_{42}+x_{43}+x_{44} = 400 \\ & x_{51}+x_{52}+x_{53}+x_{54} = 700 \\ & x_{11}+x_{21}+x_{31}+x_{41}+x_{51} \geq 150 \\ & x_{12}+x_{22}+x_{32}+x_{42}+x_{52} \geq 310 \\ & x_{13}+x_{23}+x_{33}+x_{43}+x_{53} \geq 114 \\ & x_{14}+x_{24}+x_{34}+x_{44}+x_{54} \geq 270 \end{array}\right\} \quad (6)$$

Solving equation (6) using LIPS (Linear program solver), we get the optimal solution as;

$$x_{ij}^{*}=\begin{Bmatrix} x_{11},x_{12},x_{13},x_{14},x_{21},x_{22},x_{23},x_{24},x_{31},x_{32} \\ x_{33},x_{34},x_{41},x_{42},x_{43},x_{44},x_{51},x_{52},x_{53},x_{54} \end{Bmatrix}=\begin{Bmatrix} 149 & 301 & 0 & 0 & 0 & 0 & 0 & 320 & 1 & 0 \\ 0 & 249 & 0 & 0 & 400 & 0 & 0 & 0 & 0 & 700 \end{Bmatrix} \quad (7)$$

The total distribution cost in this case reduces to **₦2, 560,032.00** as compared to(4) which was**₦2, 663,280.00.**

5.0 Discussion of Results

For better understanding of the problem solved we present a two-step summary of our findings based on the problem given, phase one is known as the constraints certification stage, while phase two is summary of the findings.

Phase I: Constraint check

We proceed to check for the constraint certification.

C1 = $x_{11} + x_{12}$ *= 110 + 340 = 450*

C2 = $x_{21} + x_{24}$ *= 140 + 180 = 320*

C3 = x_{31} *= 250*

C4 = $x_{43} + x_{44}$ *= 380 + 20 = 400*

C5 = x_{54} *= 700*

C6 = $x_{11} + x_{21} + x_{31}$ *= 110 + 140 + 250 = 500*

C7 = x_{12} *= 340*

C8 = x_{43} *= 380*

C9 = $x_{24} + x_{14} + x_{54}$ *= 180 + 20 +700 = 900*

All the constraints are hence satisfied.

Phase II

In order to minimize cost of distribution, the following must be satisfied

$x_{13} = x_{14} = x_{22} = x_{23} = x_{32} = x_{33} = x_{34} = x_{41} = x_{42} = x_{52} = x_{32} = x_{53} = 0$, this implies that no item should be distributed along those routes.

If the company wishes to distribute items this routes the following must be considered;

(i) Alternate routes with cheaper distribution cost should be created.

(ii) Since distance is fixed, the company needs to establish other distribution points so as to maximize its supply and thereby increasing profits.

6.0 Conclusion

In this paper, Sensitivity Analysis is used to determine how the optimal solution is affected by changes in the value of decision variables, within specified ranges in the objective function coefficients and the right-hand side values. So in the contest of optimization in a production and distribution setting, this work is significant in the sense that it will assist the top management of any company in making corrective decisions well in time. This will determine the future production and distribution patterns and outlook resulting in the establishment of new production and distribution units, while planning for cost minimizing and maximizing profits of the company**.**

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