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PREVENTIVE REPLACEMENT MODEL OF A REPAIRABLE DETERIORATING SYSTEM SUBJECT TO THREE TYPES OF FAILURE

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Abstract

This paper considers a preventive replacement model of arepairable deteriorating system subject to three types of failure. If the failure is of type 1(minor failure), the system is imperfectly repaired. In the case of type 2 failure (moderate failure), the system is minimally repaired and the type three failure (major failure) is non repairable in which the system is replaced. The system is replaced at a planned time T or at a non repairable failure 2 failure whichever occurs first. Explicit expressions for the expected total cost and the optimal preventive replacement time minimizing it are presented analytically and discussed numerically.

Keywords: Failure, Minimal Repair, Optimization, Reliability, Replacement Cycle.

1. Introduction

In many manufacturing organizations, system failure interrupts operation. Such interruptions have negative effects on revenue, production of defective items and causes delay in customer services. To reduce the incidence of system failure, management of these organizations are often interested with implementing appropriate maintenance and replacement policies. A policy of periodic replacement with minimal repair at failure is the one in which the system is replaced at time T while performing minimal repair at any intervening failures. This is the basic minimal repair policy presented by Barlow and Hunter [1]. In this model, they derived the optimal replacement time T, assuming the cost of minimal repair is constant and using as an optimality criterion the minimization of the total expected cost per unit time over an infinite time horizon. This model has been modified by many authors in different ways. Tilquin and Cleroux[2] modified the basic minimal repair replacement policy by introducing a term which takes adjustment costs into accounts and a unit is replaced at planned times kT, k=1, 2,... Cleroux et al [3] studied a generalized age replacement policy, where failures are corrected by a minimal repair or replacement. Boland and Proschan [4] generalized the policy by considering the cost of a minimal repair as an increasing function of the number of previous repairs to the system since the last replacement. Beichelt [5] considered the repair cost limit policy. When a unit fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a predetermined limit, otherwise the unit is replaced. Nakagawa [6] presented age replacement policy by replacing a unit at time T or at a number of N failures, whichever occurs first and undergoes minimal repairs at failure. Yakasai [7] studied two replacement policies for a repairable system whose working time after sequence of repairs follow a geometric process. In policy I, the system is replaced by a new one if the expected working time after $(n-1)$ repairs is within a tolerance limit, ε. In policy II, repairs are allowed until the return benefit after the nth repair is a non positive valued function.Sheu et al [8] studied a generalized age replacement policy with age dependent minimal repair and random lead time and derived the average cost per unit time based on the stochastic behavior of the assumed system which reflects the cost of storing a spare as well as the cost of system downtime. Mamabolo and Beichelt[9] presented a maintenance policy where each failure is removed by a minimal repair and on the first failure after a given system age Y, an unscheduled replacement is carried out. Zhao and Nakagawa [10] Studied age and periodic replacement last models with working cycles, where a unit is replaced before failure at a total operating time T or at a random working cycle whichever occurs last.

Aven and Castro [11] presented another dimension to the basic minimal repair model by considering a system subject to two types of failure. The type 1 failure is minimally repaired. In the case of type 2 failure, the system is minimally repaired with probability p and replaced with probability $q(1-p)$. The system is replaced at a planned time T or at a non repairable type 2 failure whichever occurs first. This model determined the optimal replacement time T which minimizes the expected discounted cost subject to the safety constraint.

This paper extends the model in [11] by introducing three types of failure. In the case of type 1 failure (minor failure), an imperfect repair is carried out on the system. The type 2 failure (moderate failure) is rectified by a minimal repair and the type 3 failure (major failure) which is non repairable, the system is replaced. The system is replaced at planned period T or at non repairable type 3 failure whichever occurs first. Costs are associated with repairs and replacements and the objective is to determine an optimal planned replacement period T minimizing the total expected cost.

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Using Ross [12], we define the total expected cost per unit time as follows

$$C(T) = \frac{E(\text{cost per cycle})}{E(\text{length of cycle})} \quad (1)$$

2. Model Formulation and Assumptions

We consider a system subject to three types of failure where repairs and replacement take place according to the following scheme:

1. The failure time X has a general distribution $F(t)$ and probability density function $\mathrm{f}(t)$.

2. The type 1 failure (minor failure) arrives with intensity function $r_1(t)$ and cumulative intensity function

$$h_1(t) = \int_0^t r_1(s)ds,$$

where t is the age of the system. A type 1 failure is always corrected by an imperfect repair. We denote the expected cost of an imperfect repair of type 1 failure as c_i

3. The type 2 failure (moderate failure) arrives with intensity function $r_2(t)$ and cumulative intensity function

$$h_2(t) = \int_0^t r_2(s)ds$$

In the case of type 2 failure, the system is minimally repaired. The expected cost associated with a minimal repair of type 2 failure is c_m

4. The type 3 failure (major failure) is non repairable and arrives with intensity function $r_3(t)$ and cumulative intensity function

$$h_3(t) = \int_0^t r_3(s)ds$$

In the case of type 3 failure, the system is replaced replacement at a cost c_r .

5. The system is replaced at a planned time T $(T > 0)$ after installation or at a non repairable type 3 failure, whichever occurs first. The cost of a planned replacement is c_p and it is assumed that $c_r > c_p$.

6. After a replacement, the system is as good as new and the replacement time is negligible.

The problem is to determine a replacement time which balances the costs of unplanned repair/replacements and the cost of planned replacement.

Consider a replacement cycle defined by the interval between replacements of the system caused by non repairable type 3 failures or by a planned replacement at time T. Let Z be the waiting time until the first non repairable type 3 failures. Using Beichelt [13], the reliability function of Z is given by

$$\tilde{F}(t) = e^{-h_3(t)}, \ \mathrm{t} \geq 0. \quad (2)$$

The probability that the system is replaced at age T is

$$P(Z > T) = \tilde{F}(\mathrm{T}), \quad (3)$$

and the probability that it is replaced after the first type 3 failure is

$$P(Z \leq T) = \int_0^T dF(t). \quad (4)$$

The expected length of a replacement cycle can be expressed as

$$T\tilde{F}(T) + \int_0^T tdF(t) = \int_0^T \tilde{F}(t)dt. \quad (5)$$

The expected number of type 1 failures (imperfect repairs) in $[0,t]$ is $h_1(t)$ and the total expected number of type 1 failures before replacement is

$$h_1(\mathrm{T})\,\tilde{\mathrm{F}}(\mathrm{T}) + \int_o^T h_1(t)dF(t) = \int_0^T \tilde{F}(t)r_1(t)dt \cdot \quad (6)$$

Similarly, the expected number of type 2 failures (minimal repairs) in $[0,t]$ is $h_2(t)$ and the total expected number of type 2 failures before replacement is

$$h_2(\mathrm{T})\,\tilde{\mathrm{F}}(\mathrm{T}) + \int_o^T h_2(t)dF(t) = \int_0^T \tilde{F}(t)r_2(t)dt. \quad (7)$$

Next, the expected cost of imperfect repairs associated with repairable type 1 failure before replacement is

$$E(\mathrm{CIR}) = c_i \int_0^T \tilde{F}(t)r_1(t)dt, \quad (8)$$

the expected cost of minimal repairs associated with type 2 failure before replacement is

$$E(CMR)=c_m\int_0^T \tilde{F}(t)\,\mathrm{r}_2(t)dt, \tag{9}$$

the cost of unplanned replacement associated with non repairable type 3 failure is

$$E(CCR)=\mathrm{c}_r\int_0^T \tilde{F}(t)r_3(t)dt \tag{10}$$

and the cost of planned replacement at time T is

$$E(CPR)=\mathrm{c}_p\,\tilde{F}(T). \tag{11}$$

The total expected cost in a replacement cycle is given by,

$$C(T)=\frac{\int_0^T (c_i r_1(t)+c_m r_2(t)+c_r r_3(t))\tilde{F}(t)dt+c_p\tilde{F}(T)}{\int_0^T \tilde{F}(t)dt}. \tag{12}$$

Now using that,

$$\int_0^T dF(t)=1-\tilde{F}(T),$$

we obtain

$$\tilde{F}(T)=1-\int_0^T \tilde{F}(t)dt, \tag{13}$$

and we can express the total expected cost given by (12) as

$$C(T)=\frac{c_p+\int_0^T [c_i r_1(\mathrm{t})+c_m r_2(\mathrm{t})+(\mathrm{c}_r-c_p)r_3(\mathrm{t})]\tilde{\mathrm{F}}(t)dt}{\int_0^T \tilde{F}(t)dt}. \tag{14}$$

3. OPTIMIZATION

The problem is to find a value of T that minimizes $C(T)$ given by (14). Let T^* be an optimal value of T. To do that, we state the following:

Theorem 1. Let the total expected cost be given by (14) and $F(t)$ is strictly increasing failure rate with respect to $t>0.$ If $c_i r_1'(T^*)+c_m r_2'(T^*)+(c_r-c_p)r_3'(T^*)>0,$ then there exist a finite optimal replacement time $T^*(0<T^*<\infty)$ that minimizes $\mathrm{C}(T)$ and the corresponding total expected cost is $C(T)=c_i r_1\left(T^*\right)+c_m r_2\left(T^*\right)+\left(c_r-c_p\right)r_3\left(T^*\right).$

Proof.

The derivative of $C(T)$ is

$$C'(T)=\frac{\tilde{F}(T)[c_i r_1(T)+c_m r_2(T)+(c_r-c_p)r_3(T)-C(T)]}{\int_0^T \tilde{F}(t)dt}. \tag{15}$$

The optimal replacement time T^* can be found by setting $C'(T^*)$ equal to zero and solving we have

$$c_i r_1(\mathrm{T}^*)+c_m r_2(\mathrm{T}^*)+(c_r-c_p)r_3(\mathrm{T}^*)-C(T^*)=0. \tag{16}$$

The second derivative of $C(T)$ is

$$C''(T)=\frac{H(T)}{(\int_0^T \tilde{F}(t)dt)^2}, \tag{17}$$

where,

$$H(T)=[\tilde{F}'(T)+\tilde{F}'(T)(c_i r_1(T)+c_m r_2(T)+(\mathrm{c}_r-c_p)r_3(T)-C(T)$$

$$+\tilde{\mathrm{F}}(\mathrm{T})(\mathrm{c}_i\, r_1'(T)+c_m r_2'(T)+(c_r-c_p)r_3'(T)-C'(T)]\int_0^T \tilde{F}(t)dt$$

$$-\tilde{F}^2(T)[c_i r_1(T)+c_m r_2(T)+(\mathrm{c}_r-c_p)r_3(T)-C(T)].$$

At $T^*,$ the second derivative becomes

$$C''(T^*)=\frac{\tilde{F}(T^*)[c_i r_1(T^*)+c_m r_2(T^*)+(c_r-c_p)r_3(T^*)]}{\int_0^{T^*} \tilde{F}(t)dt}.$$

If $c_i r_1'(T^*)+c_m r_1'(T^*)+(c_r-c_p)\mathrm{r}_3'(T^*)>0,$ (18)

then $C''(T)>0,$ and the T^* obtained from (16) is a minimizer of $C(T)$.

4. Numerical example.

Let the failure time distribution of the system has a gamma distribution of order 2 i.e. $f(t)=\beta^2 te^{-\beta t}$, $t>0$, $c_i=30$, $c_m=40$, $c_r=1500$, $c_p=300$, $\beta_1=1$, $\beta_2=0.5$ and $\beta_3=0.2.$ The intensity functions of the processes are given by

$$r_i(t)=\frac{\beta_i^2 t}{1+\beta_i t},\quad \beta_i>0,\ t>0,\ i=1,2,3. \quad (19)$$

The failure intensities of the processes are given by,

$$r_1(t)=\frac{t}{1+t},\quad r_2(t)=\frac{0.25t}{1+0.5t}\ \text{ and }\ r_3(t)=\frac{0.04t}{1+0.2t}. \quad (20)$$

Furthermore,

$$r_i'(t)=\frac{\beta_i^2}{1+\beta},\ i=1,2,3. \quad (21)$$

Notice that $r_i'(\mathrm{T}^*)>0,$ for $0<\mathrm{T}^*<\infty,$ and $c_i r'_1(T^*)+c_m r'_1(T^*)+(c_r-c_p)\mathrm{r}'_3(T^*)>0.$

Using Theorem 1, a finite value T^* minimizes $C(\mathrm{T}).$

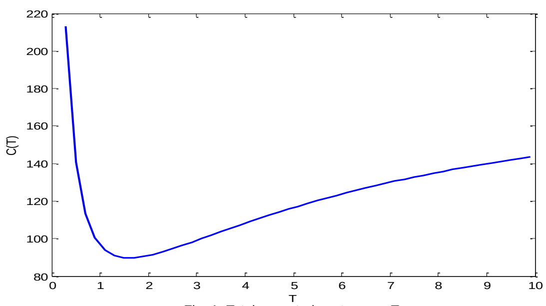

Fig. 1. Total expected cost versus T.

We can see from fig. 1, that $C(\mathrm{T})$ is non-increasing for $\mathrm{T}<T^*$ and non-decreasing for $\mathrm{T}>T^*$ and the value T^* is unique. The value T^* which minimizes $C(T)$ is equal to 1.7, with $C(T)=89.71.$

5. Conclusion

In this paper we developed a preventive replacement model of arepairable deteriorating system subject to three types of failure. If the failure is of type 1(minor failure), the system is imperfectly repaired. In the case of type 2 failure (moderate failure), the system is minimally repaired. The type three failure (major failure) is non repairable in which the system is replaced. The system is replaced at a planned time T or at a non repairable failure 2 failure whichever occurs first. Explicit expressions for the expected total cost and the optimal preventive replacement time minimizing it presented analytically and discussed numerically. Finally, the result shows that the minimum expected total cost and the optimal preventive replacement time minimizing it exist.

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