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A THREE STATE MARKOV MODEL OF THE PERFORMANCE OF A BANKING INDUSTRY IN NIGERIA STOCK MARKET: A CASE STUDY OF ACCESS BANK PLC

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Abstract

The daily stock price of Access bank plc has been modeled by a three state Markov model considered for continuous time. The effect of seasonal variation was also studied. The model was used to analyze the data of industry. The result shows that the daily stock price of the bank do not depend on the periods. It was observed also that the stock price shows a tendency of increase in price in the future. The model could be used to predict the stock price of other banks. The information could help both researchers and investors in identifying the future trends in Access bank stock market and making informed decision regarding investment in the stock market.

***Keywords*:** Markov Chain, Continuous time, Transition Probability, Stock Market, Closing price, Access Bank

1. Introduction

Stock Market is a public market for trading the company's stocks and derivative at an approved stock price through their brokers and jobbers. These are called securities listed on a stock exchange. The market is highly organized and self-regulating. The securities traded are shares, debentures, and development stocks. It is a part of capital market. The participants are officially registered and they must comply with the rules and regulations. The major reason for the establishment of the market is to provide avenue for people to invest their surplus fund to purchase securities. The Nigeria stock exchange market was established in 1960.

Stocks are form of investment in Nigeria just like any other countries of the world. It has suffered losses as a result of global economic meltdown [1]. This has resulted to loss of public confidence in Nigeria stocks especially bank stocks. Most of the investors do not even know whether there is hope for Nigerian bank stocks as there has been a consistent downward trend in their closing prices. Although, scholars have come to agree that the movement of stocks prices is random and as such making the process to be unpredictable, nonetheless, Markov chain has proved to be relevant in the analysis of Bank stock prices in Nigeria [2]. Also in [3], a Markov Chain model was applied to study the performance of two top Banks in the stock market in Nigeria. The long-run behavior of Eight Commercial Banks in the Nigerian Bank stocks prices is reported in [4]. The papers reported above considered the future prices of the selected Banks in discrete states and time only (Markov Chain). It is therefore very important to consider the effect of seasonal Changes in the future prices of Banks stocks and also to consider the model in the continuous time.

2 Theoretical Background

The family of continuous random variable $\{X(t), t \geq 0\}$ indexed by the time parameter t is called a stochastic process. The values assumed by the process are called "States" and the set of possible values is called the state space and is denoted by S. The set of possible value of the indexing parameter is called the parameter state which can be either continuous or discrete [5].

The Stochastic process occurring in most real life situation are such that for a discrete set of parameters $t_1, t_2, t_3 .... t_n, t \in T$, the random variable $X(t_1), X(t_2), X(t_3) .... X(t_n)$ exhibit some sort of dependence which if the

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conditional distribution of $X(t)$ for given value of $X(t_1), X(t_2), X(t_3)............X(t_n)$ depends only on $X(t_n)$, the most recent known value of the process. That is if $P[X(t)\le X|X(t_n)=X_n, X(t_{n-1})=X_{n-1},.........X(t_0)=X_0]$

$P[X(t)\le X|X(t_n)=X_n]$

$F(X_n, X, t_n, t)$

The stochastic process exhibiting the above property is called "Markov process". In a Markov process, therefore, if the state is known for any specific value of the time parameter t, that information is sufficient to predict the behavior of the process beyond that point. A stochastic process for a discrete random variable (X_n) is called a Markov Chain if for all times $n \ge 0$ and all states $i_0, \dots\dots\dots, i, j \in S$,

$$P(X_{n+1}=j|X_n=i, X_{n-1}=i_{n-1}.........X_0=i_0)=P(X_{n+1}=j|X_n=i)=P_{ij} \quad (1)$$

3 Modeling the Periodic Effect

Suppose that the probable course and outcome of stock price of Access bank changes with periods. Let the Periods be described by each quarter in a year as follows:

Period 1: January to March. (First quarter)
Period 2: April to June. (Second quarter)
Period 3: July to September. (Third quarter)
Period 4: October to December. (Fourth quarter)

It is assumed that the outcome of stock price changes in every quarter of the year. Accordingly, each period has its own transition counts and transition probability matrices (5).

We denote the transition counts as follows:

M_1 transition count for period 1
M_2 transition count for period 2
M_3 transition count for period 3
M_4 transition count for period 4

The transition probabilities P_1, P_2, P_3, P_4 are estimated for each period respectively.

Now let

$$M_k = F_{ij}(k), i, j = 1,2,3 \text{ and } k = 1,2,3,4 \quad (2)$$

And $P_k = P_{ij}(k), i, j = 1,2,3$ and $k = 1,2,3,4$ (3)

$F_{ij}(k)$ denotes the transition count from state i to state j for the period k. $P_{ij}(k)$ is the transition probability from state i to state j for the period k.

Accordingly: $P_{ij}(k) = \frac{F_{ij}(k)}{F_i(k)}, k = 1,2,3,4.$ and $i, j = 1,2,3$ (4)

Where $F_i = \sum_{j=1}^{3} F_{ij}(K)$ (5)

4 Test for Stationary of the Probability Matrix P_k

To test for the independence of P_k on k, the null hypothesis is stated thus:

$H_0 : P_{ij}(k) = P_{ij}$ $for\ all\ i, j = 1,2,3\ and\ for\ all\ k$

$H_1 : P_{ij}$ depends on k

The likelihood ratio test for above hypothesis is

$$M = \sum_{K=1}^{4} M_K = F_{ij} \quad (6)$$

Where $F_{ij} = \sum_{K=1}^{4} F_{ij}(K)$ (7)

The maximum likelihood estimate of the stationary transition probability matrix is

$$P_{ij} = \frac{F_{ij}}{F_i} \quad (8)$$

Where $F_{ij} = \sum_{K=1}^{3} F_{ij}(K)$ (9)

The β, the likelihood ratio criterion is given by

$$\beta = \prod_{ij=1}^{3}\prod_{k=1}^{4}\left(\frac{P_{ij}}{P_{ij}^{(k)}}\right)^{F_{ij}(k)} \tag{10}$$

$$-2\ln\beta = \chi^2_{m(m-1)(t-1)} \tag{11}$$

Where 'm' is the number of states and t is the time parameter. We evaluate β and calculate $-2ln\beta$. We then get the critical value of χ^2 at $\alpha-$ significant level and compare it with $-2ln\beta$. It is then decided whether to accept or reject the null hypothesis [5].

5 Formulation of the Model

Suppose that the value of stock of Access bank in the market in a day is considered as a random variable X, let the collection of these values for some days (n) be a stochastic process $(X_n), n = 1,2,3, \dots \dots,$. A stochastic process of first order dependable called a Markov chain is assumed for this study.

Markov process requires identification of states and accordingly, let the process be described by the following states:

State 1: (below 7.00) – drop state.
State 2: (7.01 to 9.00) – stable state.
State 3: (9.01 and above) – rise state.

Therefore we have 3 states for the process which is described as drop (d), stable(s) and rise (r), which is used to describe the three basic possible price movement of a stock. With this, we can derive the probabilities of the stock price rising, dropping, and remaining stable, and on the basis of these probabilities attempt to predict the future price direction of a stock of Access bank in the market.

The transitions between the states are explained by the following transition diagram.

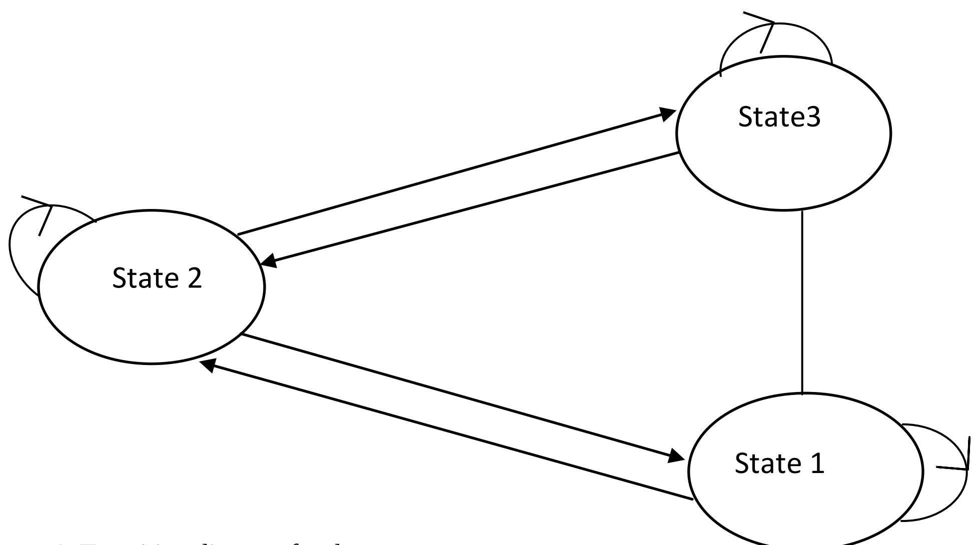

Figure 1. Transition diagram for the process.

From the diagram in figure 1, we can have the transition count matrix M and the transition probability P as:

$$M = \begin{pmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & f_{23} \\ 0 & f_{32} & f_{33} \end{pmatrix}$$

$$P = \begin{pmatrix} p_{11} & p_{12} & 0 \\ p_{21} & p_{22} & p_{23} \\ 0 & p_{32} & p_{33} \end{pmatrix} = P_{ij,\ i,j=1,2,3.}$$

6 The Continuous Time Markov Model

Markov process is a special case of Markov chain in which the time parameter is continuous. We shall now consider three (3) state model of the value/price of stock in continuous time which will allow us to obtain information about stock exchange at any given point in time.

Following [6], we let b_{ij} represent the transition of price of stock from state i to state j, $i \neq j$ in a short time interval $(t, t+\Delta t)$ the price of stock currently in state$i$, will make a transition to state $j$with probability $b_{ij}\ \Delta t,\ i \neq j$. If X_t is the state of the process at time t, then we have

$$P(X_{t+1} = j | X_t = i) = b_{ij}\Delta t \quad (12)$$

The probability of two or more state transitions is of order $(\Delta t)^2$or more and it is negligible if Δt is sufficiently small.
Supposed that the transition rate do not change with time (b_{ij}, are constants) and

$$b_{ij} = -\sum_{i\neq j} b_{ij}, \quad i, j = 1,2,3 \quad (13)$$

We describe the process by a transition rate matrix B with component b_{ij}
Suppose $P_i(t)$ is the probability that stock price is in state i and time t after the start of the process and let $P_j(t + \Delta t)$ be the probability that stock price will be in state j a short time Δt later, then

$$P_j(t+\Delta t) = P_j(t)\left[1 - \sum_{i\neq j} b_{ij}\Delta t\right] + \sum_{i\neq j} P_i(t) b_{ij}\Delta t, \quad j = 1,2,3 \quad (14)$$

Equation (14) is obtained by multiplying the probabilities and adding over all i that are not equal to j because the stock price could have entered j from any other state i. Putting (13) in (14) and rearranging the terms gives

$$P_{jj}(t+\Delta t) - P_j(t) = \sum_{t=1}^{3} P_i(t) b_{ij}\Delta t \quad (15)$$

Thus we have,

$$\frac{dP_j(t)}{dt} = \sum_{i=1}^{3} P_i(t) b_{ij} \quad j = 1,2,3 \quad (16)$$

In matrix form, we have $\frac{dP(t)}{dt} = P(t)B$ (17)

In fact equation (17) is an exact (not approximate) differential equation for $P_{ij}(t)$ in

$$\frac{dP_{ij}(t)}{dt} = \sum P_{ik} b_{kj} \quad (18)$$

(Chapman Kolmogorov differential equation). It is a linear first order differential equation with constant coefficient b_{ij} s
The elements of B may be further related by extending the properties of $P(t)$. In particular since for each i

$$\sum_j P_{ij}(t) = 1 \quad (19)$$

Then

$$\frac{d}{dt}\left[\sum_j p_{ij}(t)\right]_{t=0} = \left[\frac{d(1)}{dt}\right]_{t=0} \quad (20)$$

$$\sum_j \frac{d}{dt} P_{ij}(t)\Big|_{t=0} = 0 \quad (21)$$

$$\sum_j b_{ij} = 0 \quad (22)$$

That is each row of B must be zero since every off diagonal is non-negative, hence equation (13). The development of the equation that determine the $p_{ij}(t)$ functions, for this process can be simplified if the following assumption are made

1. the process satisfies the Markov property.
2. the process is stationary.
3. the probability of a transition from one state to a different state in a short time interval is proportional to Δt.
4. the probability of two or more changes of state in a short interval Δt is zero

$P(t)$ is a row vector of the state probabilities at time t. To obtain the solution of (17), the initial condition $P_i(0)$, $i = 1,2,3$ must be specified. Taking the Laplace transform of (17) we have:

$$P(s) = P(0)(SI - B)^{-1} \quad (23)$$

Thus $P(t)$ is obtained as the inverse transform of $P(s)$ [7]. One other method that could be used to solve the equation is the classical adjoint reported in [8]

7 Application

A summary of daily stock price of Access bank from January to December 2017 is presented in table 1. The data used for the research work was obtained through the cashcraft website.

Table 1: A summary of daily of stock price or value of Access bank.

CLASS INTERVAL	STATES	FREQUENCY
Below 7.00	1	12
7.01 to 9.00	2	110
9.01 and above	3	123
TOTAL		245

From the data recorded in table 1, we obtained the transition counts and transition probability matrices for each period as follows:

$M_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 36 & 1 \\ 0 & 2 & 22 \end{pmatrix}$, $P_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.9730 & 0.0270 \\ 0 & 0.0833 & 0.9167 \end{pmatrix}$

$M_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 21 & 2 \\ 0 & 1 & 35 \end{pmatrix}$, $P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.9130 & 0.0870 \\ 0 & 0.0278 & 0.9722 \end{pmatrix}$

$M_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 60 \end{pmatrix}$, $P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.0164 & 0.9836 \end{pmatrix}$

$M_4 = \begin{pmatrix} 10 & 1 & 0 \\ 2 & 44 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $P_4 = \begin{pmatrix} 0.9167 & 0.0833 & 0 \\ 0.0426 & 0.9362 & 0.0213 \\ 0 & 0.5000 & 0.5000 \end{pmatrix}$

Using equations (6) and (8), then

$$M = \begin{pmatrix} 11 & 1 & 0 \\ 2 & 104 & 4 \\ 0 & 5 & 118 \end{pmatrix} \quad (24)$$

and $$P = \begin{pmatrix} 0.9167 & 0.0833 & 0 \\ 0.0182 & 0.9454 & 0.0364 \\ 0 & 0.0407 & 0.9593 \end{pmatrix} \quad (25)$$

From equation (10). We have

$$\beta = \prod_{ij=1}^{3} \prod_{k=1}^{4} \left(\frac{P_{ij}}{P_{ij}^{(k)}} \right)^{F_{ij}(k)} \quad (26)$$

Where k = 4

$\beta = 0.003645421181$

Here $m = 3,\ t = 4$

$\chi^2_{m(m-1)(t-1)} = \chi^2_{18}$

Therefore $-2ln\beta = -2\ln(0.\mathbf{003645421181})$

$= -2(-5.61428337)$

$= 11.22856674.$

The critical value of $\chi^2_{18}\ at\ \propto = 0.05\ is\ 28.87.$ Since the calculated value of $-2ln\beta$ is less than the critical value of χ^2_{18} at α significant level the null hypothesis of constant transition probability matrix is accepted. Then we have homogenous Markov chain

The model considered in this section will enable us to present the result on a continuous time scale.

First we shall consider the transition count matrix of the stationary Markov chain discussed in the last section. From equation (6), we have

$$M = \begin{pmatrix} 11 & 1 & 0 \\ 2 & 104 & 4 \\ 0 & 5 & 118 \end{pmatrix} \quad (27)$$

Normalizing this matrix using equation (13) we have

$$B = \begin{pmatrix} -1 & 1 & 0 \\ 2 & -6 & 4 \\ 0 & 5 & -5 \end{pmatrix} \quad (28)$$

The matrix B can therefore be expressed as the expected value of the exponential distribution $\left(\frac{1}{\beta}\right)$. Thus

$$C = \begin{pmatrix} -1 & 1 & 0 \\ 0.50 & -0.75 & 0.25 \\ 0 & 0.20 & -0.20 \end{pmatrix} \quad (29)$$

Substituting matrix C in equation (23) we have

$$P(S) = (SI - C)^{-1} = P(0)\left[\begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix} - \begin{pmatrix} -1 & 1 & 0 \\ 0.50 & -0.75 & 0.25 \\ 0 & 0.20 & -0.20 \end{pmatrix}\right]^{-1} \quad (30)$$

$$P(S) = \begin{pmatrix} s+1 & -1 & 0 \\ -0.50 & s+0.75 & -0.25 \\ 0 & -0.20 & s+0.20 \end{pmatrix}^{-1} \quad (31)$$

P(0) is assumed to (0.3, 0.4, 0.3)
Solving equation (31) by maple software, we obtain the following functions.

$P_{12}(t) = 0.4444 - 1.4545e^{-t} + 1.0101e^{-0.45t}$

$P_{21}(t) = 0.2222 - 0.7273e^{-t} + 0.5051e^{-0.45t}$

$P_{23}(t) = 0.5556 + 0.0000e^{-t} - 0.5556e^{-0.45t}$

$P_{32}(t) = 0.4444 + 0.0000e^{-t} - 0.4444e^{-0.45t}$

The values of these functions for $t = 0,1,2,3, \dots \dots . . ,30$ are presented in table 2 and illustrated in figure 2 respectively.

8 RESULTS

Table 2. The values of $P_{12}(t), P_{21}(t), P_{23}(t), P_{32}(t)$ for $t = 0,1,2,3, \dots \dots \dots ,30$

Time	$P_{12}(t)$	$P_{21}(t)$	$P_{23}(t)$	$P_{32}(t)$
0	0.000000	0.000000	0.000000	0.000000
1	0.553388	0.276707	0.201334	0.161038
		0.329129	0.329710	0.263720
2	0.658231			
3	0.633843	0.316932	0.411566	0.329194
4	0.584728	0.292373	0.463760	0.370941
5	0.541063	0.270537	0.497040	0.397561
6	0.508679	0.254343	0.518261	0.414534
7	0.486359	0.243181	0.531791	0.425357
8	0.471512	0.235757	0.540419	0.432257
9	0.461819	0.230910	0.545920	0.436657
10	0.455555	0.227781	0.549428	0.439463
11	0.444376	0.225766	0.551664	0.441252
12	0.448953	0.224477	0.553091	0.442393
13	0.447306	0.223653	0.554000	0.443120
14	0.446254	0.223127	0.554580	0.443584
15	0.445582	0.222791	0.554950	0.443880
16	0.445154	0.222577	0.555185	0.444068
17	0.444881	0.222139	0.555336	0.444188
18	0.444707	0.222353	0.555431	0.444265
19	0.444595	0.222298	0.555492	0.444314
20	0.444525	0.222262	0.555531	0.444345
21	0.444480	0.222240	0.555556	0.444365
22	0.444451	0.222225	0.555572	0.444378
23	0.444432	0.222216	0.555582	0.444386
24	0.444421	0.222210	0.555589	0.444391
25	0.444413	0.222207	0.555593	0.444394
26	0.444408	0.222204	0.555595	0.444396
27	0.444405	0.222203	0.555597	0.444398
28	0.444403	0.222202	0.555598	0.444399
29	0.444402	0.222201	0.555599	0.444399
30	0.444401	0.222201	0.555599	0.444399

These results is as illustrated in figure 2

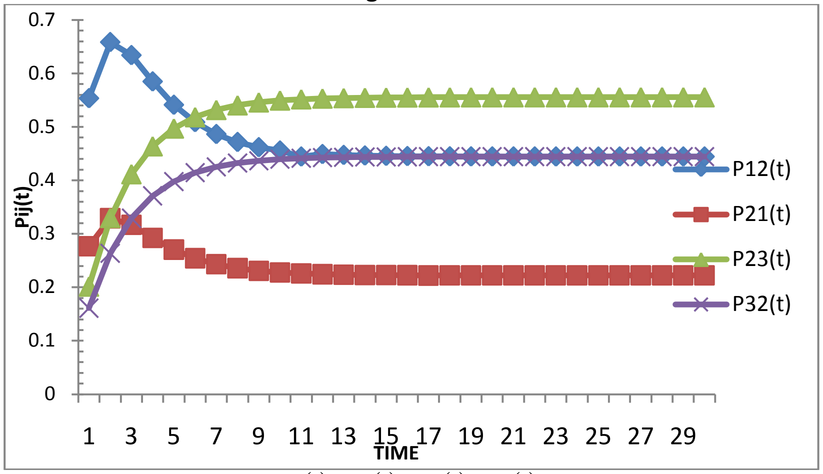

Figure 2. The graph of $P_{12}(t), P_{21}(t), P_{23}(t), P_{32}(t)$

9 Discussion of Results.

This research work considered Markov model to study movement of price of stock of Access bank plc. in continuous time. The data consists of daily closing price for a period of one year. The test of independence of P_K on K, shows the acceptance of the null hypothesis of constant probability. We have the result for the continuous time model presented in the table 2 and illustrated with graph shown in figure 2. The result shows that the model converges to the equilibrium probabilities at t = 30. Some of the P_{ij}, i = 1,2,3 even converge earlier than thirty days. $P_{12}(t)$ attains maximum value of 0.66 (corrected to 2 decimal places) at t = 2. $P_{21}(t)$ has maximum value of 0.33 at t = 2. $P_{23}(t)$ rises gradually to 0.55 at t = 12. $P_{32}(t)$ increases steadily to 0.44 at $t = 12$

Thus, the maximum transition probability are recorded for $P_{12}(t)$, and $p_{23}(t)$

At equilibrium state $P_{12}(t) = P_{32}(t) = 0.444400$, $P_{23}(t) = 0.555600$ and $P_{21}(t) = 0.222200$. This shows that the stock price of Access bank has tendency of increase in the near future.

10 Conclusion

This study has demonstrated some of the ways in which the theory of Markov chain process may be applied to the analysis of security price movements. It has shown that the daily prices of Access Bank stock do not depend on the periods of the year. It also shows that there shall be rise in the prices of the Bank stock in the near/far future. The continuous time model could be used at any point in time to predict the movement of stock price on the floor of Nigeria stock market unlike the Markov chain which gives discrete times only.

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