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BUYS-BALLOT MODELING OF THE EXTERNAL RESERVES OF NIGERIA

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Abstract

This study focused on modeling of the external reserves of Nigeria. The buy-ballot procedure was used to make choice of model, estimation of trend and assessment of seasonal effect. The results showed that the additive model is appropriate for modeling external reserves of Nigeria between the periods considered in the study. The three measures of accuracy (MAPE, MAD and MSD) were computed for the linear, quadratic, exponential and s-curve trend curves. The quadratic trend curve had the least measures of accuracy when compared with other trend curves; hence the quadratic trend curve was fitted to the external reserves of Nigeria. The trend-cycle component revealed a downward trend showing that between these periods, the external reserves of Nigeria has been on the decrease. The data also revealed the influence of seasonal effect. The study recommended that the Nigerian government should focus more on increasing the external reserves so as to build international community confidence in the nation's policies and creditworthiness.

Keywords: Buys-ballot, External reserve, Additive model, Quadratic trend, Seasonal effect.

1.0 INTRODUCTION

External reserves, also known as International Reserves, "consists of official public sector foreign assets that are readily available to and controlled by the monetary authorities for direct financing of payment imbalances and regulating the magnitude of such imbalances through intervention in the exchange market to affect the currency exchange rate and/or for other purposes" [1]. External reserves are needed to guard against possible financial crisis [2]. Adequate reserves do contribute to confidence in a nation by guaranteeing the availability of foreign exchange to domestic borrowers to meet international debt servicing and enhance its credit rating [3]. The main reasons for a country holding external reserves include foreign exchange market stability, exchange rate stability, exchange rate targeting, creditworthiness, transactions buffer, and emergency such as natural disasters [4]. [5] in their work discovered that Nigeria external reserve have suffered instability in recent years due to economic global financial crisis and it is known that the growth or decline of a country's external reserves is an indispensable aspect of her economy. Hence, this work seeks to statistically model the external reserves of Nigeria using the Buys-Ballot approach.

2.0 LITERATURE REVIEW

The uses of the Buys-Ballot table were discussed in [6] to include choice of appropriate transformation (using the Bartllet technique). Assessment of trend and seasonal components and choice of model for time series decomposition. They described in great detail the Buys-Ballot table for better understanding of the methods used to achieve these objectives. Furthermore, they stated that when any of the assumptions underlying the time series analysis is violated, one of the options available to an analyst is to transform the study series. The choice of appropriate transformation for a study series using Buys-Ballot was also described in their study, the presence and nature of trend and seasonal averages. Assessment of trend and seasonal component of the actual series from the Buy-Ballot table was discussed. A major problem in the use of the descriptive time series analysis is the choice of appropriate model for time series decomposition.

[7] compared the Least Squares Estimation (LSE) and Buys-Ballot Estimation (BBE) methods for the choice of appropriate model for decomposition and detection of presence of seasonal effect in a series model. The comparison of the two methods is done using the Accuracy Measures; Mean Error (ME), Mean Square Error (MSE), the Mean Absolute Error (MAE), and the Mean Absolute Percentage Error (MAPE). The results from simulated series show that for the additive model; the summary statistics (ME, MSE and MAE) for the two estimation methods and for all the selected trending curves are equal in

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all the simulations both in magnitude and direction. For the multiplicative model, results show that when a series is dominated by trend, the estimates of the parameters by both methods become less precise and differ more widely from each other. However, if conditions for successful transformation (using the logarithmic transform in linearizing the multiplicative model to additive model) are met, both of them give similar results.

A time series analysis of Nigeria domestic crude oil production was conducted by [8] using the descriptive approach of time series analysis. Buys-Ballot table procedure was used in assessing variance stability (transformation), choice of model and seasoned effects. Also, inverse square root transformation was carried out to stabilize the variance, quadratic trend was fitted and the error component was found to be randomize and normally distributed with mean zero and some constant variance, that is e~N (0, 0.2227) t.

[9] modeled the levels and trend of external reserves in Nigeria using Time Series. The relevance of this lies in the fact that it could help to monitor the reserves and throw early warning signal about any economic crisis. Data was analyzed using ARIMA model. Results of the analyses show that (i) the data requires logarithmic transformation to stabilize the variance and make the distribution normal (ii) the appropriate model that best describes the pattern in the transformed data is the Autoregressive- Integrated Moving Average process of order (2,1,0). This model is recommended for use until further analysis proves otherwise.

3.0 METHODOLOGY

The data for this work are monthly external reserves of Nigeria from 2008-2017, retrievable from the Data and Statistics publication of the Central Bank of Nigeria website: www.Cenbank.org.

The Buys-Ballot procedure will be used for data analysis in this study. The Buys-Ballot method arranges the series (External reserves) in a tabular form called the Buys-Ballot table as shown in Table 3.1.

Table 3.1: Buys-Ballot Table

	1	2	…	J	…	S	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	…	X_j	…	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_{1.}$
2	X_{s+1}	X_{s+2}	…	X_{s+j}	…	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_{2.}$
3	X_{2s+1}	X_{2s+2}	…	X_{2s+j}	…	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_{3.}$
	…	…	…	…		…	…	…	…
I	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	…	$X_{(i-1)s+j}$	…	$X_{(i-1)s+s}$	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
	…	…	…		…	…	…	…	…
M	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	…	$X_{(m-1)s+j}$	…	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	…	$T_{.j}$	…	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$		$\bar{X}_{.j}$		$\bar{X}_{.s}$			
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	…	$\hat{\sigma}_{.j}$	…	$\hat{\sigma}_{.s}$		$\bar{X}_{..}$	
			…		…				$\hat{\sigma}_{..}$

Now, we define the row and column totals, averages and standard deviations as follows:

$$T_{i.} = \sum_{j=1}^{s} X_{(i-1)s+j} \quad , \; i = 1, 2, ..., m \qquad (3.1)$$

$$T_{.j} = \sum_{i=1}^{m} X_{(i-1)s+j} \quad , \; j = 1, 2, ..., s \qquad (3.2)$$

$$T_{..} = \sum_{i=1}^{m} T_{i.} = \sum_{j=1}^{s} T_{.j} \qquad (3.3)$$

$$\overline{X}_{i.} = \frac{T_{i.}}{s} \quad , \; i = 1, 2, ..., m \qquad (3.4)$$

$$\overline{X}_{.j} = \frac{T_{.j}}{m} \quad , \; j = 1, 2, ..., s \qquad (3.5)$$

$$\overline{X}_{..} = \frac{T_{..}}{ms} = \frac{T_{..}}{n} \quad , \; n = ms \qquad (3.6)$$

$$\hat{\sigma}_{i.} = \sqrt{\frac{1}{s-1}\sum_{i=1}^{m}\left(X_{(i-1)s+j} - \overline{X}_{i.}\right)^2} \quad , \; i = 1, 2, ..., m \qquad (3.7)$$

$$\hat{\sigma}_{.j} = \sqrt{\frac{1}{m-1}\sum_{j=1}^{s}\left(X_{(i-1)s+j} - \overline{X}_{.j}\right)^2} \quad , \; j = 1, 2, ..., s \qquad (3.8)$$

$$\hat{\sigma}_{..} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{(i-1)s+j} - \overline{X}_{..}\right)^2} \qquad (3.9)$$

However, in order to decompose the time series, it is important to choose the appropriate model for the time series.

Choice of Appropriate Times Series Model

According to [6], the relationship between the seasonal mean $\left(\overline{X}_{.j} \;, j = 1, 2, ..., s\right)$ and seasonal standard deviation gives an indication of the appropriate time series model. Thus, the plot of the seasonal means $\left(\overline{X}_{.j}\right)$ and the seasonal standard deviation $\left(\hat{\sigma}_{.j}\right)$ gives an indication of the desired model. With this plot, an additive model is appropriate when the seasonal standard deviations show no appreciable increase or decrease relative to any increase or decrease in the seasonal means. On the other hand, a multiplicative model is appropriate when the seasonal standard deviations show appreciable increase or decrease relative to any increase or decrease in the seasonal means. Inspection of the graph of the seasonal means and seasonal standard deviation (see fig.4.2) suggested the additive model as the appropriate model for the external reserves of Nigeria; hence we explain the additive model.

Additive Model of Time Series

The additive model of a time series is given by

$$X_{t=}T_t + S_t + C_t + I_t \qquad (3.10)$$

where;

T_t Is the trend component
S_t Is the seasonal component
C_t Is the cyclical component
I_t Is the irregular component

Since the Buys-Ballot procedure is used for short series, it means that the trend and cycle components are joined together. On this note, the additive model in (3.10) reduces to;

$$X_t = M_t + S_t + I_t \qquad (3.11)$$

Where; M_t is the trend-cycle component

Choice of Appropriate Trend Curve

In order to reliably choose the appropriate trend curve, three measures of accuracy (MAPE, MAD and MSD) will be computed for the linear, quadratic, exponential and s-curve curves to check which best suits the time series data. Thus, the trend curve having the least of the three accuracy measures is appropriate for the research data. The accuracy measures are computed as follows;

MAPE= Mean absolute percentage error is computed by;

$$MAPE = \frac{\sum_{t=1}^{n}\left|\frac{\left(X_t - \hat{X}_t\right)}{X_t}\right|}{n} \times 100 \qquad provided \;\; X_t \neq 0 \qquad (3.12)$$

where;

X_t = Variable of interest
$\hat{X}_t$ = fitted values of X_t
n = number of observations

MAD- Mean absolute deviation is computed by;

$$MSD = \frac{\sum_{t=1}^{n} \left| X_t - \hat{X}_t \right|}{n} \quad (3.13)$$

where; X_t , $\hat{X}_t$ and n are as defined under MAPE

MSD-Mean squared deviation is computed by

$$MSD = \frac{\sum_{t=1}^{n} \left| X_t - \hat{X}_t \right|^2}{n} \quad (3.14)$$

Buys-Ballot Estimates of Parameters of the Quadratic Trend-Curve under the Additive Model

The quadratic trend curve is given by:

$$M_t = a + bt + ct^2 \qquad t= 1,2,..,n \quad (3.15)$$

The estimate of the parameters of the trend of the entire series is given by the relation;

$$\overline{X}_{i.} = X_{t_i} = a + bt_i + ct_i^2 = a^1 + b^1 i + ci^2 \quad (3.16)$$

Where;

$$a^1 = a - b\left(\frac{s-1}{2}\right) + c\left(\frac{s-1}{2}\right)^2 \quad (3.17)$$

$$b^1 = \left(bs - cs(s-1)\right) \quad (3.18)$$

$$c^1 = c\left(s\right)^2 \quad (3.19)$$

Here, a^1 is the intercept of the quadratic trend curve, b^1 is the slope (i.e. linear effect of t on $\overline{X}_{i.}$) and c^1 is the slope coefficient (i.e. curvilinear effect of t on $\overline{X}_{i.}$).

Using the estimates in Equations (3.16), (3.17), (3.18) and (3.19) respectively we have the quadratic trend-cycle component as;

$$m_t = a + bt + ct^2 \quad (3.20)$$

Where;

$$a = a^1 + b\left(\frac{s-1}{2}\right) - c\left(\frac{s-1}{2}\right)^2 \quad (3.21)$$

$$c = \frac{c^1}{s^2} \quad (3.22)$$

$$b = \frac{b^1 + cs\left(s-1\right)}{s} \quad (3.23)$$

Here, a is the intercept of the quadratic trend curve, b is the slope (i.e. linear effect of t on X_t) and c is the slope coefficient (i.e. curvilinear effect of t on X_t).

Assessment of Seasonal Indices

According to [6], the overall mean ($\overline{X}_{..}$) and the seasonal mean $\left(\overline{X}_{.j}\right)$ of the Buys- ballot table are used to assess the seasonal effects of the time series. Since the model for External Reserves is additive, we shall use the difference of the seasonal averages to the overall averages $\left(\overline{X}_{.j} - \overline{X}_{..}\right)$ to assess the presence or otherwise of seasonality. These are seen as deviations and the wider the deviations the greater the seasonal effects.

Assessment of Irregular Component for Additive time Series Model

Recall that the additive model in equation (3.11) is given by

$$X_t = M_t + S_t + I_t$$

Once the trend-cycle component, M_t and the seasonal component, S_t have been estimated, then the irregular component can be estimated by

$$I_t = X_t - M_t - S_t \quad (3.24)$$

4.0 Results and Discussions

According to [10], the first approach to time series analysis is to make a time plot of the original series. Thus, Figure 4.1 shows the plot of the External Reserves of Nigeria (2008-2017).

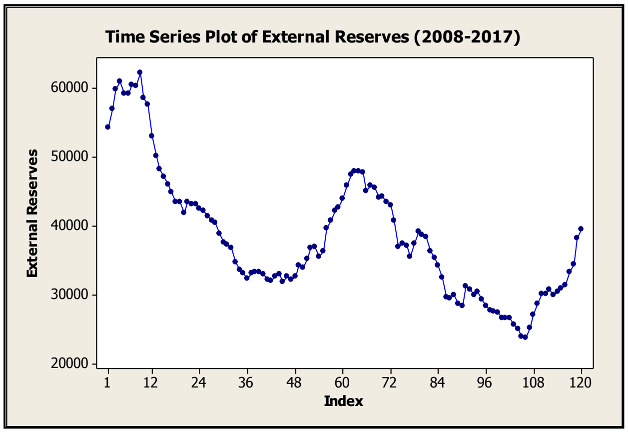

Figure 4.1: Time Plot of External Reserves of Nigeria (2008-2017)

Careful examination of Figure 4.1 shows that External Reserves of Nigeria has fluctuated up and down over the years under investigation. In recent years, the plot has shown downfall in external reserves of Nigeria in recent years, which started moving up again.

Estimation of Parameters of Buys-Ballot Table Using Monthly External Reserves of Nigeria

Applying equation (3.1) through (3.9) to the external reserves of Nigeria, we obtain results in Table 4.1**:**

Table 4.1 Buys-Ballot Table for External Reserves of Nigeria (2008-2017)

	Seasons								
Period	**1**	**2**	**3**	**…**	**11**	**12**	Ti.	Mean i.	SD i.
1	54,215.80	56,908.40	59,756.50	…	57,480.50	53,000.40	701,674.60	58,472.90	2681.940329
2	50,108.70	48,113.10	47,081.90	…	43,024.70	42,382.50	536,428.20	44,702.30	2572.952547
3	42,075.70	41,410.10	40,667.00	…	33,059.30	32,339.30	448,268.50	37,355.70	3390.018679
4	33,131.80	33,246.10	33,221.80	…	32,125.20	32,639.80	390,963.40	32,580.30	518.8417137
5	34,136.60	33,857.40	35,197.40	…	42,568.30	43,830.40	457,105.90	38,092.20	3491.501892
6	45,824.40	47,295.80	47,884.10	…	43,414.20	42,847.30	547,355.40	45,613.00	1784.598801
7	40,667.60	36,923.60	37,399.20	…	35,248.70	34,241.50	446,644.00	37,220.30	1802.12868
8	32,385.70	29,567.00	29,357.20	…	29,263.00	28,284.80	357,665.80	29,805.50	1213.832622
9	27,607.90	27,568.40	27,336.40	…	25,081.20	26,990.60	312,409.00	26,034.10	1378.059544
10	28592.98	29990.36	29996.38	…	38207.96	39353.49	386,713.50	32,226.10	3428.261088
T.j	388,747.10	384,880.20	387,898.00	…	379,473.00	375,910.00	4,585,228.20		
Mean.j	38,874.71	38,488.02	38,789.80	…	37,947.30	37,591.00		38,210.24	
SD.j	9150.004658	9663.624529	10208.45395	…	9217.486168	8112.818999			22,262.14

Now, Figure 4.2 shows the plot of the seasonal means and the seasonal standard deviations of the buys ballot table for the external reserves of Nigeria..

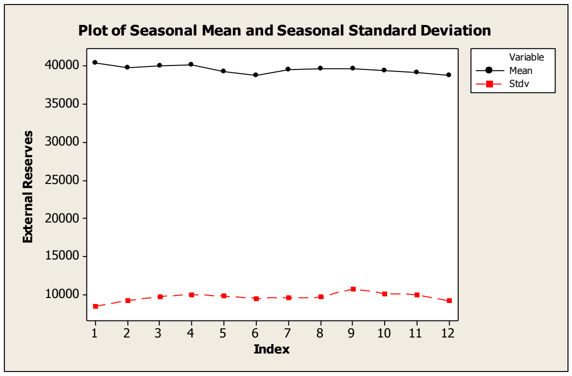

Figure 4.2 Plot of Seasonal Mean and Seasonal Standard Deviation

Careful examination of Figure 4.2 reveals that the additive model is appropriate for modeling External Reserves of Nigeria. Hence we consider the best trend curve using the three measures of accuracy (MAPE, MAD and MSD). Minitab 16 is used to compute these measures of accuracy.

Table 4.2: Accuracy Measures for the Best Trend Curve

Trend Curve	MAPE	MAD	MSD
Linear	14	5385	40505038
Quadratic	13	5089	35832269
Exponential	14	5226	38903986
S-Curve	14	5554	48119738

The quadratic trend has the least error of estimate when compared with the linear, exponential and s-curve trends. This shows that quadratic trend is the line of best fit for the data on External Reserves. We estimate the parameters of the quadratic trend line.

Now, Applying Equations (3.17), (3.18) and (3.19) to data in Table 4.3, we obtain the following results:

$$\bar{X}_{i.} = 54817 - 436.3t + 2.014t^2 \qquad (4.1)$$

Applying Equations (3.22), (3.23) and (3.21) in that order to Equation (4.1), we obtain the results

$$c = \frac{c^l}{s^2} = \frac{2.014}{12^2} = 0.0140 \qquad (4.2)$$

$$b = \frac{b^1 + cs(s-1)}{s} = \frac{-436.3 + (0.0140)(12)(12-1)}{12} = -36.204 \qquad (4.3)$$

$$a = a^1 + b\left(\frac{s-1}{2}\right) - c\left(\frac{s-1}{2}\right) \qquad (4.4)$$

$$= 54817 + (-36.204)\left[\frac{12-1}{2}\right] - (0.0140)\left[\frac{12-1}{2}\right]^2 = 54617.452$$

$$m_t = 54617.452 - 36.204t + 0.0140t^2 \qquad (4.5)$$

Equation (4.5) shows that the Nigeria External Reserves is on the decline because the linear effect of t on X_t is negative (-36.204)

Substituting the values of $t = 1,2,3,\ldots,120$, we obtain the trend values thus fig.4.3 shows the plot of the de-trended series.

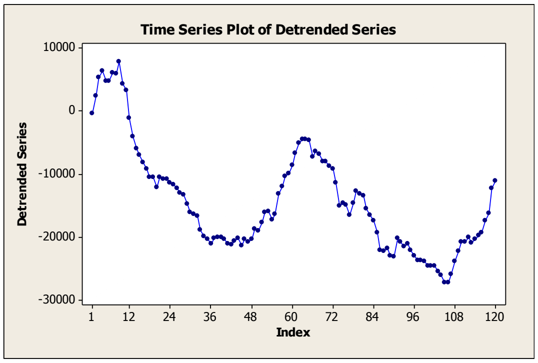

Figure 4.3 Plot of De-trended Series

REMARKS: Figure 4.3 shows that when trend was removed from the actual series, the effects of season and irregular components were left, and are characterized by downward movements. More external reserves are recorded during January followed by April and March respectively. Similarly, the external reserves seem to be on the decline during December followed by June, and November respectively, which may be attributed festive periods where many national products that yield the external reserves like the crude oil are scarce. We then estimate the seasonal indices by the difference of the seasonal averages to the overall averages $\left(\overline{X}_{.j} - \overline{X}_{..}\right)$ thus fig. 4.4 shows the plot of the seasonal indices.

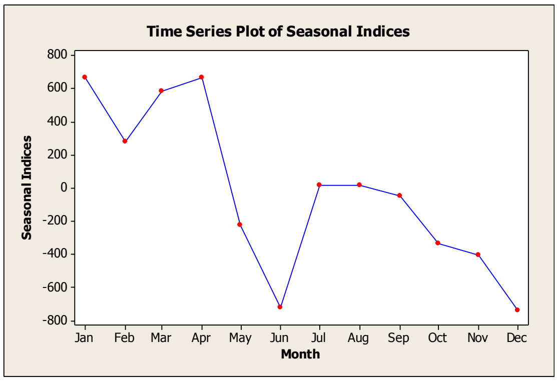

Figure 4.4 Time series plot of the Seasonal indices

REMARKS: The plot above clearly shows that there is presence of seasonal effect on the series.

Estimation of Irregular Component for External Reserves of Nigeria

To estimate the irregular components of the series, we apply equation (3.24), hence fig.4.5 shows the graph of the irregular components.

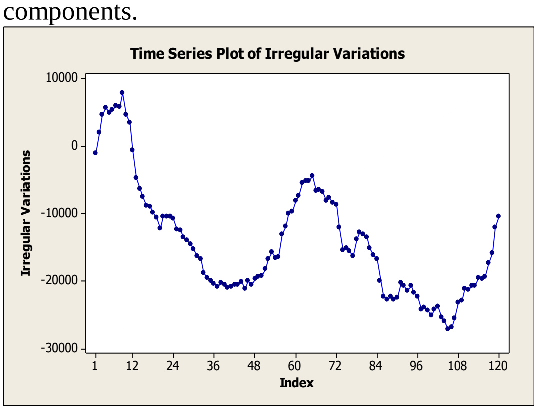

Figure 4.5 Time series plot of the Irregular components

4.2.3 Validation of Normality of Error Component

In time series analysis, we make assumptions that the error component is random when we use Ordinary Least Squares (OLS) in estimating the trend parameters. This may not always be the case. Hence, to verify this; we conduct a test on the randomness of the error component. In this work, we used the Anderson-Darling test to determine that the error component is normal. The plot is shown in Figure 4.6 below:

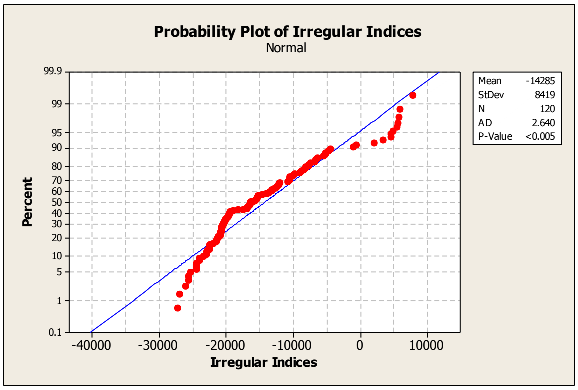

Figure 4.6 Anderson-Darling test.

REMARKS: AD-statistic is estimated to be 2.604 and the approximate p-value is <0.005. This shows that the error component is not normal with mean -14285 and standard deviation 8419.

5.0 CONCLUSION

The result reveals that the additive model is appropriate for modeling external reserves of Nigeria between the periods under study. The trend-cycle component revealed that the external reserves of Nigeria have not been stable between the periods under study, it goes up and down and this pattern was characterized by a parabola. As expected of the data, there is influence of seasonal effect where some seasons (months) are seen as the peak while some are seen as the low point in the production. It is recommended that the government should focus more on increasing the external reserves of Nigeria so as to build international community confidence in the nation's policies, creditworthiness, market stability, and exchange rate stability.

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