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INVESTIGATING THE INVERSE RELATIONSHIP BETWEEN ACTUARIAL LIABILITY AND DISCOUNT RATES UNDER GRATUITY RETIREMENT PLANNING

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Abstract

The main purpose of this paper is to investigate whether there is inverse relationship between discount rates and gratuity actuarial liability. Obtaining the appropriate normal cost for the future population of gratuity of defined benefits scheme is a vital component of gratuity valuation process for issues ranging from governmental budgets to gratuity systems where pension experts depend on a deterministic approach by using different scenarios. In developing countries such as Nigeria, a few employers in the organised private sector and even in the public sector still offer gratuity as ex-gratia in the staff welfare package scheme in order to further boost staff morale. To this effect, a set of salary data of a small sized organization who offers gratuity scheme to her staff was collected for investigating the inverse relationship. Theories of measurement was developed and thereafter tested for sensitivity analysis. In this paper, the inverse relationship between pension gratuity liabilities and discount rate was investigated through the process of projected credit unit method. The valuation of gratuity exercise was based on employee demographic characteristics such as dates of birth, entry dates to employment, salary information and employee category. Commutation functions were employed in the process. The result shows that interest rate decreases with increase in liabilities and an interest rate increases with decrease in liabilities.

Keywords: commutation, pensionable, gratuity, interest, inverse, normal cost, liability.

1.0 THEORETICAL BACKGROUND

Discounting rate is a foremost parameter employed in the valuation of gratuity especially when fixed interest rate model is applied. The liability of gratuity plan is much akin to debt instruments such as bonds and the benefits paid out to exit members could be taken to be the cash flows of the debt instrument and that is why the relative liability-interest rate sensitivity can be accessed from the above argument. Interest rate sensitivity is higher than that of actuarial liability because liability has a higher maturity period. Furthermore, liabilities have extensively longer duration to maturity than assets and consequently, it is expected that the interest rate sensitivity for liability will be of higher value than that of assets. Discounting under fixed and variable interest rates differs and are usually analyzed to ensure that the funds can match future obligations. Gratuity liability and insurance valuations comprise part of statutory requirement which insurance and pension trustees must undergo at law to determine their state of affairs. Actuaries are the only professionals qualified to carry out, certify and stamp the results of actuarial valuation. A hydra-headed problem which actuaries often face amounts to calculating actuarial liability akin to debt instruments, normal cost and the liability changes which occur thereof for which exact values functionally depend on life tables and other assumptions with respect to funding criteria. The actuarial liabilities of active members in force either as at the valuation date is calculated taking into account all types of decrement functions [1]. The implication on these calculations is that pensionable pay will be projected from a particular date up to the appropriate date of retirement or exit. Because the variables of interest may not be roughly estimated at the moment therefore, the latest available service tables for the population of scheme members would be used. Consequently, one can calculate the worth of monetary values required now to meet those future payments. The projections will depend on the rates used to discount. An infinitesimally small discount rate bearing a low investment strategy automatically leads to higher present value of liabilities. This problem of estimating the correct liability is essentially challenging for some Nigerian states and some companies in the organized private sector which although have recently experienced quick paradigm shift from defined benefit to defined contribution but still run gratuity schemes as additional staff welfare packages.

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Gratuity scheme is a defined benefit plan and provides a one off sum payment at the time of separation. Based on scheme terms and conditions, the benefits are calculated on the basis of both last earned regular monthly income and the period of service (t_e- t_R)at the time of separation and a lump sum is paid as severance benefit. In gratuity computing, lump sum benefit levels are determined ahead of retirement or qualifying period and then guaranteed at an assumed constant rate of interest irrespective of how the underlying pension funds are invested. Based on the number of years an individual has spent in the scheme in force, gratuity benefits will be actuarially calculated by discounting. Actuarial valuation of gratuity benefit is payable to a plan member on termination of his employment after he or she must have rendered continuous service for not less than five years under the following conditions: (i) superannuation, (ii) resignation, (iii) death or disablement or injuries suffered from work related operations [2]. The accounting for defined benefit planning general is a challenging task since actuarial assumptions are required to measure both obligation and the expense hence actuarial gains and losses arise. Moreover, the obligations are measured on a discounted basis because they may be settled many years after the employees render the related service. The standard actuarial practice is that the reporting sponsor is expected to measure gratuity obligations under the defined benefit gratuity plans, it is therefore sufficient for such sponsor to engage the services of a professional pension actuary. A technique employed here for the gratuity valuation is the projected unit credit method which can be defined as a method which considers each period of service as giving rise to an additional unit of benefit entitlement and measures each unit separately to build up the final obligation.

In the view of [1, 3-6], it is necessary and sufficient to formulate a table of decrement first displaying all appropriate rates for mortality in performing actuarial valuation of gratuity. From the earlier discussions, demographic and economic assumptions will depend on the past trends and expectations in future relating to operations of the plan sponsor being investigated. Gratuity benefit is usually payable at the time of retirement since at that material time, employees generally have no other source of regular income and as a life (x) approaches life table ω chosen, medical expenses, family commitments and inflation and day to day expenses increase [7]. In order to cope with all these uncertainty costs, gratuity lump sum can partially offset necessary expenses. Consequently, this paper takes the perspective of pensionable wage using the more integral analysis framework of a whole population of active members. The starting point is a recursive functional comparison of wage $W_y(t)$ at time t and the promotional wage W_y so that the normal cost and actuarial liability on the aggregate domain can be determined. The design of the analytical framework used here references techniques from both integral calculus and mathematics of life contingencies. However, the work is purely hinged on the projected unit credit method by theoretically establishing an inverse functional relationship between discount rates j and L(.) and consequently validated by wage data.

2. RIEMANN INTEGRAL MODEL FOR ESTIMATING PAY ROLL

Using the contributions of [8-10] as applied in life insurance mathematics for pension valuation, the total payroll for gratuity is given as TMPG.

That is,

$$\text{the total monthly payroll for gratuity (TMPG)} = \frac{(1+r_e)^t \int_e^R E(L_y) w_y d_y}{12}, e < y < R \qquad (1)$$

where e and r are the entry age and retirement ages respectively.

Following [11-13], let W_y be the promotional wage of an employee at age y and let $W_y(t)$ be the pensionable wage function at a further time t. Then

$$W_y(t) = e^{t\ln(1+r_e)} * W_y = (1+r_e)^t W_y, \qquad (2)$$

where r_e is the rate of wage escalation.

The rate of change in wage with respect to time is

$$W_y'(t) = \ln(1+r_e) * W_y * e^{t\ln(1+r_e)}. \qquad (3)$$

If the gratuity is set up at initial time, then t = 0,and the initial change in wage is independent of time as

$W_y'(0) = W_y \ln(1+r_e)$ but $W_y(0) = (1+r_e)^0 W_y = W_y$, so that

$$W_y'(0) = W_y(0)\ln(1+r_e)$$

$$\frac{W_y'(0)}{W_y(0)} = \ln(1+r_e) \Rightarrow d\ln W_y(0) = \ln(1+r_e) \qquad (4)$$

Then

$$W_y(t) = e^{\ln(1+r_e)} * W_y(0) \qquad (5)$$

The number of active member within the intermediate ages y and (y + δy) is

$$E[L(y)]\delta y = l_y \delta y, \tag{6}$$

where L(y) is binomially distributed and the random variable l_y is the number of active members who survive employment to age y. The mean number of active members who survive employment to age $y = l_0 *_y p_0$ where y is the random life time.

$$\text{Total employee age related wage} = E(L(y))\delta y. \tag{7}$$

$$\text{Escalated total employee age related wage} = \mathrm{E(L(y))(1+r_e)^t W_y} \tag{8}$$

The total gratuity employee wage for computation for all ages

$$= \int_e^R \mathrm{E(L(y))(1+r_e)^t W_y}\, dy \tag{9}$$

$$E_S^A = (1+\mathrm{r_e})^t \int_e^R \mathrm{l_y W_y}\, dy, \tag{10}$$

where e and r are the entry and retirement ages respectively and $E_S{}^A$ is the total escalated wage for all ages. Usually $E_S{}^A$ represents annual total wage hence for computational purposes it must be divided by 12, hence,

$$\frac{E_S^A}{12} = \frac{(1+\mathrm{r_e})^t}{12} \int_e^R \mathrm{E(L(y))W_y}\, dy = \frac{(1+\mathrm{r_e})^t}{12} \int_e^R \mathrm{l_y W_y}\, dy \tag{11}$$

where e, r are the entry and retirement ages respectively.

$$\frac{E_S^A}{12} = \frac{(1+\mathrm{r_e})^t}{12} l_0 \int_e^R \mathrm{f_Y(y) W_y}\, dy \tag{12}$$

where $\mathrm{f_Y(y)}$ is the probability of surviving employment.

Depending on the funding policy, the accrued gratuity liability V_{L+t} at time t corresponds to the target amount of liability at a specific point in time. The targeted liability represents the actuarial present value of future benefits based on employees' service period rendered to the measurement date using the selected actuarial cost method.

THE GRATUITY LIABILITY FUNCTION MODEL

Exley *et al*. [11] in (1997) observes the numerical value of a defined benefit obligation to plan members is the equivalent of the cost of the same obligation promised to the shareholder of the plan sponsor, given that values and costs are measured in an economically consistent manner. Thus, we can only reduce the cost of pension benefits to companies by reducing their value to employees. From the actuarial perspective, the value of the gratuity liability can then be measured under the following discount framework:

$$V_{L+t} = F_L^1(1+j_L^1)^{-1} + F_L^2(1+j_L^2)^{-2} + \ldots = F_L^k(1+j_L^k)^{-k} \tag{13}$$

here F_L^k is the cash flow generated by the liability and V_{L+t} represents the value of liability while j_L^k is the discount rate applied, each at time t. When the cash-flows extend over a defined number of years n, then $V_{L+t} = F_L^k(1+j_L^k)^{-k}$. The liabilities which represent the benefits to all members of gratuity plans are guaranteed before knowing the uncertain outcomes of future long investment environment. The plan sponsor obliges to honour the agreement even though the assets may fall short of the promised benefits obligation. Liabilities should be valued at bond linked discount rate because the discount rate as at the time of valuation will depend on the market yield of government debt instruments whose finite term corresponds to the term of the gratuity liability. The discount rate j is then applicable to the period over which the obligation is to be settled. However, the perspective on which matching asset premise the argument in favour of equity is that gratuity liabilities are worse hit by wage inflationary trends and consequently equity returns will be hit by wage inflation also and thus the best matching asset for wage related liabilities are equities. Furthermore in the long run, equities such as stocks may eventually perform better than debt instruments such as bonds so that to further reduce the cost intensity of funding gratuity liabilities, gratuity fund should be invested mostly in equities and

$$V_{L+t} = F_L^k(1+j_L^k)^{-k}. \tag{14}$$

The liability generated cashflows are discounted because a unit value of cash flow generated through liability if invested now is worth more than a risky one and possesses the capacity to earn further interest continuously.

Extended Farid Jawwad's Numerical Algorithm for Actuarial Valuation of Gratuity

(a) Gratuity plan funding as in any other defined benefit plan requires actuarial assumptions otherwise called projections about future uncertain events which divides basically into demographic and economic data. The former discusses the plan's membership while the latter concerns about growth in wage income, growth in investment return on assets of the fund and interest rates. The initial work therefore in the valuation process is to collect demographic and economic statistics of plan members. Examples of such data are: wage income, dates of birth, dates of employment, fund asset details such as portfolio selection and investment, prior valuation reports such as the actuary who carried out the last valuation and the last valuation report.

(b) We then consequently establish all actuarial assumptions concerning economic and demographic variables such as discount rate, future wage income, wage growth rate, benefits growth rate, employee withdrawal rate, pre and post retirement mortality rate, members' disability rate, members' early retirement rate, proportion of plan members with dependent relatives who are qualified for gratuity benefits, expected return on plan assets, rate of claim under medical plans.

(c) Furthermore, based on a projected final wage, the total projected benefit amount in form of a lump sum payable on the retirement date B_R is then computed. For a pension plan the projected benefit amount corresponds to the actuarial present value (APV) of all future pension benefits payable from the date of retirement of plan members upwards till he dies assuming no bequest or death of his dependents if there is bequest in accordance with the plan benefits defined. The projected pension obligation is then discounted to the date of retirement for post-employment mortality and interest.

However, for a post-retirement plan which pays for the expense of medical care, this will result to the actuarial present value of all future medical care expense discounted for post-retirement mortality, claim rates on medical plans and interest. A deterministic actuarial present value for decrements together with interest rates may involve the use of the appropriate commutation functions.

(d) Under the projected unit credit (PUC) method as regulated by [14], a prorate amount is applied on the benefit value projected for each year of service accruing up to the date of valuation mathematically expressed as,

$$\mathrm{B_x} = \frac{B_R * (x-e)}{(R-e)}. \quad (15)$$

It is usually assumed that the each unit of benefit accrual is uniform for each service year.

(e) The retirement or post-retirement benefit to be accrued for the following year is then computed in the form

$$\mathrm{b_x} = \frac{B_R}{(R-e)}. \quad (16)$$

We assume that the benefit accrual unit is the same for each year of service.

(f) The values of B_x and b_x must be discounted for decrements such as pre-retirement mortality, early retirement, disability, turnover and interest to the valuation date to estimate the actuarial liability for the year including the normal cost or current service cost for the following year. The IAS 19 disclosures and we test the changes to key assumptions and identify key risks by computing a table of stress testing [14].

(g) Compute the actuarial gain and loss from where we can track the number of plan members over the valuation year taking cognizance of the number entering and exiting the plan taking cognizance of the intent for leaving and check the expected interest cost and return against those that are really obtained so as to ensuring that the economic and demographic assumptions are effective so that actuarial gain and loss accounted for.

(h) We suggest actuarial audit should be the last exercise though not included in the original Jawwad's algorithm of the valuation process. Actuarial audit is performed to reflect its sensitivities to changes Δj, in discount rates. The liability covers a wider period of time than asset hence liabilities are very sensitive to interest rate changes [15, 16]. The intent of actuarial audit is to ensure that the actuarial liability, normal cost and contribution estimated are reasonable, thus reviewing the entire valuation process against key assumptions such as demographic and economic statistics, the benefit calculated and the funding policy.

3. METHODOLOGY AND DATA

A medium medical sized firm sets up a gratuity scheme which pays a one-off sum when retired at normal retirement age R = 65. The lump sum value benefit is expressed as follows: Gratuity benefit is the last monthly wage per year of service in employment. It is assumed that the monthly wage will increase at a rate of r_e% yearly. The following conditions apply (i)There would not be death benefits for death in service and (ii) there are no other prior-retirement decrement other than death. The staff data are extracted as follows under the following sub-headings (i) Date of birth (ii)date of employment (iii) current monthly wage income. As at the time of collecting the data from the firm, the discount rate was 13% and the company's fair value of plan asset from her investment manager stood at₦4,000,000.00.

It is imperative to note that, liability obligations as at 31-Dec-2017 and the funding cost level for the following year for each member must be estimated(the normal cost). It is assumed that the contribution made by the plan sponsor is greater than or equal to the normal cost calculated so that the question of unfunded actuarial liability does not arise.

In PUC methodology, as guided by the international accounting standards dealing with employee benefits [14], the wage income is calculated based on information already known by projection to the retirement date applying the growth rate r_e.

Where the unit benefit becomes uniform for all service years, will imply that the distribution of the projected retirement benefit must be uniform over the years of service of the employee. For us to estimate the present value of the retirement benefit obligations that is, the actuarial liability, the benefit has to be calculated with respect to the current and previous years of service by pro-rata. It is after this that actuarial liabilities can be shown to have functional relationship to the normal cost and furthermore depend on the actuarial cost method used.

Projected Unit Method

Projected unit credit attempts to compute the correct present value of the benefits as it accrues. This benefit accrual method gives smaller cost values at the beginning of an employee's career but costs later grow when members approach retirement age. It is the most commonly used method in private sector valuations. The model used here will be partitioned into its segmented equations where the segmented equations indicate some patterns in pension fund variables, given the appropriate economic and demographic variables and the funding policy decisions.

1) $\text{Entry age (e)} = \text{Date of employment - Date of birth}$
2) $\text{Age nearest birth date on 31-Dec-2017 (x)} = \text{Valuation date - Date of birth}$
3) $B_r = \text{Projected Final wage} = [(\text{current wage}) * (1 + r_e)^{\text{retage-x}}]$
4) $\text{Projected gratuity amount} = \text{projected Final wage} * \text{number of service year.}$
5) $\text{Total number of years in service} = \text{Normal Retirement Age (r) - Age at entry (e)}$
6) $\text{Projected Gratuity Amount} = [\text{Current wage} * (1 + \text{re})^{\text{ret age-x}}] * (\text{r - e})$
7) $\text{Past service years} = \text{x - e}$
8) $B_x = \text{Proportion of accrued projected gratuity at age x}$
   $= \text{projected Final wage} * \text{past service years} = B_r * (\text{x - e}).$

The accruing benefit in the year which follows assumes that the retirement benefit will be distributed uniformly throughout the service years.

Mathematically expressed the unit benefit, will be $\frac{\text{Gratuity value Projected}}{\text{Total service years}}$

The value of benefit computed in respect of the fraction of projected benefit and unit benefit applies effective the date of employees' retirement from active service. In computing actuarial liability and normal cost values, the gratuity benefit amounts must be discounted for interest and mortality but we note that there will be no other prior-retirement decrements. The discount function must be calculated to enable us discount the benefit amounts up to the date of valuation, $v^{R-x} {}_{r-x}p_x^{\alpha}$ by using commutation functions, $\frac{D_r(\alpha)}{D_x(\alpha)}$

where $D_x = (1+j)^{-x} l_x$, and $v = \frac{1}{(1+j)}$

Hence,

$$(1+j)^{x-R} {}_{R-x}^{\ n}p_x = (1+j)^{x-R} * \frac{l_{x+r-x}}{l_x} = (1+j)^{x-R} * \frac{l_R}{l_x} = \frac{l_R v^R}{l_x v^x} = \frac{D_r(\alpha)}{D_x(\alpha)} \qquad (17)$$

The superscript, (α) indicates that the actuarial function has considered only one decrement which by reason of this paper is only termination by death.

The actuarial accrued liability is the value of assets targeted at a specific point in time in accordance with the funding objectives. The gratuity liability represents the estimated benefits payable to exiting members at retirement, however the exact time and amount of the benefit which are not known in advance will depend on the future experiences such as (i)material time a plan member will retire (ii) his survival probability in retirement and (iii) economic conditions such as inflationary trends, fiscal policies, interest rate uncertainties before and after retirement. As a condition to establish the level to which actual experience has fallen in line with these economic assumptions, the valuation results of these liabilities must be reviewed constantly. The fund's assets grow from the contributions paid by plan sponsor and include returns on investment. The instruments in which these assets are to be invested is germane to the fund. The optimal selection and performance of the fund's investment portfolio are the principal responsibilities of the fund investment managers [15-16]. The actuarial value of the normal cost due to the fund's liabilities is constantly checked to ascertain the amount of assets required to meet such accrued liability. The size of the assets required is a moving target and hence the fund may technically enter into surplus or deficit status.

4. RESULTS

As can be seen in Table 1, the actuarial liability increases with age and past service years hence the cost of accrual rises with age and past service. This implies that the actuarial liability assigned to older members is much greater than the values attributable to younger members. The implication is that members who are in their early career, has lower normal costs than members approaching retirement age , hence for each member, the projected unit credit creates lower costs early in a member's career but the costs progressively increase as such employee approaches retirement age because there are larger accruals and shorter discount period.

Table 1: Actuarial Liabilities

COUNT	AGE = x	ENT AGE = e	R-e	x-e	ACT LIAB = (WGE,DIS = 8.00, 13.00)	ACT LIAB = (WGE,DIS = 8.00, 14.00)	ACT LIAB = (WGE,DIS = 8.00, 12.00)	ACT LIAB = (WGE,DIS = 9.00, 13.00)	ACT LIAB = (WGE,DIS = 7.00, 13.00)
A1	58	44	21	14	1718769.6	1428443.6	2071507.5	2085812.5	1413768
A2	50	37	28	13	520951.51	407059.48	668173.24	674332.49	401492.95
A3	47	34	31	13	233022.77	177328.98	306952.66	310086.86	174646.16
A4	48	36	29	12	347579.89	269208.53	449786.95	454081.89	265396.56
A5	45	35	30	10	72393.54	55578.6	94517.54	95451.35	54764.67
A6	49	39	26	10	226921.59	180463.44	285921.47	288368.04	178170.75
A7	55	42	23	13	107765.12	87997.6	132211.1	133211.37	87007.91
A8	38	27	38	11	30973.97	22161.2	43420.3	43964.39	21750.92
A9	50	39	26	11	72661.03	57784.98	91552.98	92336.38	57050.85
A10	43	30	35	13	58870.17	43248.5	80354.47	81281.42	42510.49
A11	57	44	21	13	82906.95	68902.72	99921.69	100611.71	68194.82
A12	43	30	35	13	65934.58	48438.31	89996.99	91035.18	47611.74
A13	42	30	35	12	43076.11	31645.52	58796.46	59474.73	31105.5
A14	38	26	39	12	33881.86	24029.09	47920.76	48537.15	23572.64
A15	40	27	38	13	50561.07	36175.35	70878.13	71766.28	35505.62
A16	56	43	22	13	111321.64	91706.18	135365.77	136345.22	90719.38
A17	46	40	25	6	25881.31	20764.72	32321.9	32587.79	20511
A18	46	36	29	10	153569.47	118943.05	198727.1	200624.71	117258.82
A19	46	40	25	6	258796.03	207633.5	323197.6	325856.34	205096.46
A20	31	25	40	6	15284.13	10744.46	21810.1	22097.87	10535.17
A21	33	26	39	7	21158.68	15005.78	29925.74	30310.67	14720.73
A22	45	41	24	4	18899.47	15297.34	23393.74	23578.46	15117.85
A23	58	45	20	13	145285.63	121813.24	173552.58	174693.81	120621.05
A24	42	36	29	6	74472.58	57680.7	96371.49	97291.73	56863.95
A25	36	30	35	6	63488.41	46641.25	86658.1	87657.77	45845.34
A26	36	31	34	5	14365.94	10647.22	19435.17	19652.92	10470.68
A27	34	28	37	6	34165.3	24660.86	47470.17	48049.26	24216.2
A28	35	30	35	5	33610.54	24691.71	45876.5	46405.72	24270.36
A29	36	31	34	5	32702.94	24237.56	44242.65	44738.36	23835.68
A30	30	26	39	4	9096.47	6451.24	12865.58	13031.07	6328.69
A31	30	26	39	4	20707.42	14685.75	29287.5	29664.21	14406.77
A32	34	30	35	4	12823.53	9420.71	17503.4	17705.32	9259.95
A33	30	27	38	3	8356.92	5979.2	11715	11861.8	5868.5
A34	28	25	40	3	9734.31	6843.04	13890.63	14073.91	6709.75
A35	33	31	34	2	14888.88	11034.79	20142.64	20368.33	10851.83
A36	42	40	25	2	22726.06	18233.24	28381.45	28614.93	18010.45
A37	32	30	35	2	12436.27	9136.21	16974.81	17170.63	8980.31
A38	41	39	26	2	40975.11	32586.19	51628.69	52070.46	32172.2
A39	31	29	36	2	11872.27	8645.36	16349.67	16543.7	8493.66
A40	46	44	21	2	52134.24	43327.99	62833.59	63267.5	42882.84
A41	32	30	35	2	14212.88	10441.38	19399.79	19623.58	10263.2
A42	29	27	38	2	10821.81	7742.77	15170.37	15360.46	7599.43
TOTAL	1721	1406	1324		4910058.1	3913461.4	6186404	6239598.3	3864459.8

Source: Authors' Computation, 2017.

Unfunded actuarial liability = 4910058.1 − 4000000.00 = 910,058.1

Table 2 shows the normal cost computed which represents the present value of future benefits earned by employees during the current fiscal year. It is a fraction of the actuarial present value of benefits and expenses which is allocated to a valuation year by the actuarial cost method. The normal cost will be the cost of accruing next year's benefit or the cost of providing benefits to new employees. Employers must contribute the normal cost plus a closed amortization of any unfunded actuarial liability (UAL) that exists under the stipulated ordered pair (WAGES, DISCOUNT). The total sum in each column represents the normal cost which plan sponsor must contribute at each level of wage and discount rates respectively.

Table 2: Normal Cost of retirement benefits

COUNT	AGE = x	ENT AGE = e	r-e	x-e	NORMAL COST = (WGE,DIS = 8.00, 13.00)	NORMAL COST = (WGE,DIS = 8.00, 14.00)	NORMAL COST = (WGE,DIS = 8.00, 12.00)	NORMAL COST = (WGE,DIS = 9.00, 13.00)	NORMAL COST = (WGE,DIS = 7.00, 13.00)
A1	58	44	21	14	122769.26	102031.69	147964.82	148986.61	100983.43
A2	50	37	28	13	40073.19	31312.27	51397.94	51871.73	30884.07
A3	47	34	31	13	17924.83	13640.69	23611.74	23852.84	13434.32
A4	48	36	29	12	28964.99	22434.04	37482.25	37840.16	22116.38
A5	45	35	30	10	7239.35	5557.86	9451.75	9545.14	5476.47
A6	49	39	26	10	22692.16	18046.34	28592.15	28836.8	17817.07
A7	55	42	23	13	8289.62	6769.05	10170.08	10247.03	6692.92
A8	38	27	38	11	2815.82	2014.65	3947.3	3996.76	1977.36
A9	50	39	26	11	6605.55	5253.18	8323	8394.22	5186.44
A10	43	30	35	13	4528.47	3326.81	6181.11	6252.42	3270.04
A11	57	44	21	13	6377.46	5300.21	7686.28	7739.36	5245.76
A12	43	30	35	13	5071.89	3726.02	6922.85	7002.71	3662.44
A13	42	30	35	12	3589.68	2637.13	4899.71	4956.23	2592.13
A14	38	26	39	12	2823.49	2002.42	3993.4	4044.76	1964.39
A15	40	27	38	13	3889.31	2782.72	5452.16	5520.48	2731.2
A16	56	43	22	13	8563.2	7054.32	10412.75	10488.09	6978.41
A17	46	40	25	6	4313.55	3460.79	5386.98	5431.3	3418.5
A18	46	36	29	10	15356.95	11894.3	19872.71	20062.47	11725.88
A19	46	40	25	6	43132.67	34605.58	53866.27	54309.39	34182.74
A20	31	25	40	6	2547.36	1790.74	3635.02	3682.98	1755.86
A21	33	26	39	7	3022.67	2143.68	4275.11	4330.1	2102.96
A22	45	41	24	4	4724.87	3824.33	5848.44	5894.61	3779.46
A23	58	45	20	13	11175.82	9370.25	13350.2	13437.99	9278.54
A24	42	36	29	6	12412.1	9613.45	16061.92	16215.29	9477.33
A25	36	30	35	6	10581.4	7773.54	14443.02	14609.63	7640.89
A26	36	31	34	5	2873.19	2129.44	3887.03	3930.58	2094.14
A27	34	28	37	6	5694.22	4110.14	7911.7	8008.21	4036.03
A28	35	30	35	5	6722.11	4938.34	9175.3	9281.14	4854.07
A29	36	31	34	5	6540.59	4847.51	8848.53	8947.67	4767.14
A30	30	26	39	4	2274.12	1612.81	3216.4	3257.77	1582.17
A31	30	26	39	4	5176.85	3671.44	7321.88	7416.05	3601.69
A32	34	30	35	4	3205.88	2355.18	4375.85	4426.33	2314.99
A33	30	27	38	3	2785.64	1993.07	3905	3953.93	1956.17
A34	28	25	40	3	3244.77	2281.01	4630.21	4691.3	2236.58
A35	33	31	34	2	7444.44	5517.4	10071.32	10184.16	5425.91
A36	42	40	25	2	11363.03	9116.62	14190.73	14307.46	9005.23
A37	32	30	35	2	6218.14	4568.1	8487.41	8585.32	4490.15
A38	41	39	26	2	20487.56	16293.09	25814.34	26035.23	16086.1
A39	31	29	36	2	5936.14	4322.68	8174.84	8271.85	4246.83
A40	46	44	21	2	26067.12	21663.99	31416.8	31633.75	21441.42
A41	32	30	35	2	7106.44	5220.69	9699.89	9811.79	5131.6
A42	29	27	38	2	5410.91	3871.39	7585.18	7680.23	3799.71
TOTAL	**1721**	**1406**	**1324**		**528036.79**	**416878.99**	**671941.34**	**677971.87**	**411444.92**

Source: Authors' Computation, 2017.

Sensitivity analysis

This section discusses the deterministic results of the model used. By changing an economic variable and holding others constant, functional relationship among differing economic variables is established. The stress testing starts with an analysis of creating route which all standard assumptions enumerated above must follow so as to allow a change in result when economic variables are adjusted while holding others constant. The cross-examination of the gratuity plan stress testing can be efficiently done using stochastic analysis but the consequences of changes on some economic variables while holding else constant can simply be demonstrated clearly in a deterministic setting. Following Joshi and Pitt (2009), efficient calculation of the sensitivity of key valuation results to model inputs is a very vital information top practicing pension actuaries as it provides guidance as to the relative importance of various actuarial judgments arrived at, in the calculation of the gratuity liability valuation basis. For the sensitivity analysis, we adopt the following ordered pair notations (WAGE, DISCOUNT)for ease of computation.

$(\overline{W}, \overline{D}) = (8, 13)$; $(\overleftarrow{W}, \overline{D}) = (7, 13)$; $(\overrightarrow{W}, \overline{D}) = (9, 13)$;$(\overline{W}, \overleftarrow{D}) = (8, 12)$;$(\overline{W}, \overrightarrow{D}) = (8, 14)$

The result of the gratuity calculation is tested for their sensitivity to key actuarial assumptions. We carry out a sensitivity analysis on the discount rate and salary increase assumptions below.

First, we calculate the actuarial liability by varying the assumption one at a time while holding else constant and see the impact on the funded status of the gratuity plan. We have considered only two key economic assumptions both of which relate to the liability calculations. The sensitivity analysis used here determines how the value of gratuity asset will be impacted on, by event changes occurring to some factors in the market such as discount rate, time and salary volatility. The entries in the following Tables 3,4,5 & 6 below were extracted from tables 1 and 2.

Table 3: Summary of Gratuity and Normal cost @ (D = 13%)

DISCOUNT RATE @ D = 13%	GRATUITY LIABILITY @D = 13%	NORMAL COST @ D = 13%
S = 7	3864459.81	411444.92
S = 8	4910058.06	528036.79
S = 9	6239598.26	677971.87

Source: Authors' Computation, 2017.

Table 4: Summary of Gratuity and Normal cost @ (s = 8%)

DISCOUNT RATE @ S = r_e = 8%	GRATUITY LIABILITY @ S = r_e = 8%	NORMAL COST @ S = r_e = 8%
D = 12	6186403.98	671941.34
D = 13	4910058.06	528036.79
D = 14	3913461.36	416878.99

Source: Authors' Computation, 2017.

Table 5: Summary of Gratuity and Normal cost @ (s = 8%):

DISCOUNT RATE @ S = 8%	GRATUITY LIABILITY @ S = 8%	NORMAL COST @ S = 8%
D = 12 = (j −1)	6186403.98 = L(j −1)	671941.34
D = 13 = j	4910058.06 = L(j)	528036.79
D = 14 = (j + 1)	3913461.36 = L(j + 1)	416878.99

Source: Authors' Computation, 2017.

From our summary above, it is apparent from our analysis that the interest rate decrease from 13% to 12% is consistent with the increase from 4910058.06 units of money to 6186403.98units of money in liabilities. The implication for the gratuity plan is that it needs more assets currently to ensure enough investment returns to pay a projected value of benefit when a member retires. but the interest rate increase from 13% to 14% is also consistent with the decrease in liabilities from 4910058.06 units of money to 3913461.36units of money thus establishing an inverse relationship between discount rates j and L(.). From the tables above we can establish that $L \; \alpha \; \frac{1}{D}$ implying $LD = K$

Table 6: Sensitivity Analysis

ASSUMPTIONS	EXISTING	i	i	S	s
Discount rate (j)	13%	14%	12%	13%	13%
Wage income increase factor (S)	8%	8%	8%	9%	7%
Actuarial liability as at 31-12-2017	4910058.06	3913461.36	6186403.98	6239598.26	3864459.81
Fair value of plan assets as at 31-12-2017	4,000,000.00	4,000,000.00	4,000,000.00	4,000,000.00	4,000,000.00
Surplus or Deficit	(910058.06)	86538.64	(2186403.98)	(2239598.26)	135540.19

Source: Authors' Computation, 2017.

We have assumed a change $\Delta j = 1\%$ over the existing assumptions. The current position shows an actuarial deficit of₦910,058.06. When the discount rate is increased by 1%, all other factors held constant, it results in an actuarial surplus of ₦86,538.64. The increase in surplus reflects the good investment performance of the fund. On the other hand if the discount rate were to decrease by 1%, it would result in a much larger deficit of ₦2,186,403.98.When the wage growth rate is increased by 1%, the deficit increases by almost 2-fold whereas a 1% decline in the growth rate results in a decline in the actuarial liability and consequently a significant surplus. The sensitivity of the actuarial liability and hence surplus appears to be greater for the salary increase assumption than the discount rate.

5. CONCLUSION AND RECOMMENDATION

A full professional valuation technique under this model is a task not within the scope of this paper. This paper restricts itself to a few interesting points of inverse relationship between actuarial liabilities and discount rates methodologies and sensitivities construction ideas.

This not-withstanding, the accuracy of our results functionally depended upon the accuracy and of the underlying demographic data used. The summary of the valuation results entails actuarial calculations which require assumptions about future events. The assumptions used in this valuation are adequate for the purposes for which they have been used. It is also assumed that all calculations fell in line with the applicable funding policy requirements. The high level of estimation required in setting all the key assumptions and relative differing sensitivities of presented results to changes in those assumptions have serious implications on gratuity plan administration since it may require that gratuity scheme trustees be charged with plan management to understand those key assumptions driving the result instead of accepting the valuation results hook line and sinker. The result therefore will be much relevant to scheme trustees or insurance firms who have been assigned the responsibility of scheme administration. One basic fact is that the scheme's trustees cannot contribute meaningfully to the degree of appropriateness of the estimated contribution arrived at nor discharge their trusteeship obligations efficiently unless they can interpret the effect of future changes on the key assumption holding else constant on the valuation result.

Based on the above discussions, the study suggests that the unfunded liability of ₦910,058.06which occurs in the gratuity valuation results and which gives rise to a funded ratio of 81.46% should be amortized for a fixed period of five years management. Even though, the funded ratio (FR) is above the 60% benchmark, all contributions should still be tested against overriding minimum contribution calculation. The value of the unfunded liability may have arisen from a couple of factors such as underfunding. We suggest that the revenue income to deal with the unfunded liability amount can be sourced from either the plan sponsor or members or a combination of the two. A worthy point to note here is that, regular funding of the actuarially required contribution over time is more important than the amount of contributions in a single year. The accumulated assets at any point in time are supposed to match the computed accrued or the actuarial liability. Therefore the actuarial liability determined based on projected unit funding method is a guide for the gratuity plan sponsor to the tune of ₦4,910,058.16which must be set aside to fully meet scheme benefits accrued in respect of service up to that point of time.

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