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ON THE ACTIVITIES OF UNIDIRECTIONAL NONLINEAR WATER WAVES ON A VERTICAL WALL

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Abstract

This study is a generalization of the stochastic family previously expanded to second order in nonlinearity Consequently the Fourier and Stokes series are infused, Stokes coefficients playing a major role. Thus, the expansion is a stochastic family with the coefficients randomly distributed within specified limits. From the expansion, the wave crest elevation height and fluctuating wave pressure are calculated in front of a vertical wall and the data are in agreement with observations. Further, from the random nonlinearity parameters derived from the stochastic family, exceedence probability sketch is constructed as a function of nonlinear wave parameters and the data so obtained are in agreement with observed wave heights.

1.0 INTRODUCTION

The understanding of the behavior of nonlinear wave activities in the front of an offshore vertical wall is fundamental to the design of marine engineering structures[2,3]. This applies to both shallow and deep water offshore structures[2]. Consequently, detail knowledge of wave crest elevation and trough depth in the locality is a key factor in this consideration[3,4]; for this leads to calculation of wave pressure force on the marine structures[1].

However, the inherent complexity[5] associated with the interaction of wave evolution and the adjoining offshore structures makes the calculations related of wave parameters a difficult task analytically and even numerically. One of the essential factors contributing to this complexity is that water wave phenomenon is essentially nonlinear with its parameters randomly distributed[4].

A new and effective method for the statistical study of second order wave activities in an undistributed wave field and that in front of a vertical wall was presented by Arena and Fedele[1,5]). This follows the related investigations associated with Longuet-Higgius[4]. Okeke[5] extended the second order stochastic family formulated by Arena[1] to higher order nonlinearity. The extension revealed the singular behavior of some of the nonlinear parameters associated with the family.

In this study, the theory of nonlinear stochastic family associated with narrow banded and unidirectional wave processes is generalized. It is based on basic Fourier series expansion but the coefficients expressed in form of those of Stokes nonlinear wave processes. With this approach, subsequent calculations are based.

2. Statistical description of wave activities in front of a vertical barrier

The model is that for which x-axis is along the direction of wave motion, x=0 being the location of the vertical wall. z-axis along the wave front and $-\infty < z < \infty$. y-axis is perpendicular to the xz plane. $y = 0$ is the undisturbed sea surface and $y = \eta(x, \epsilon, t)$ describes the wave surface profile, $\epsilon = kh$, k is the weave number of the dominant wave component with frequency ω. The limiting case of long wave length is described by $\epsilon \to 0$. $h =$ depth of water level from the seabed when undistributed. The above description is the case of a narrow banded spectrum associated with a unidirectional and evolutional wave process. Following Fenton[4] we shall assume likewise that all time variations can be represented in the form of $X(t) = \omega t + \theta$, θ being a stochastic variable distributed uniformly in $(0, 2\pi)$, t is the time. Thus, the rather stochastic family $\psi(x, y, t)$ is in the form;

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$$\psi(x,y,t) = \sigma\{\epsilon[f_1(x,y)cosX + f_2(x,y)sinX] + \epsilon^2[g_1(x,y)cos2X + g_2(x,y)sin2X] + \epsilon^3[h_1(x,y)cos3X + h_2(x,y)sin3X]\} + 0(\epsilon^4) \ldots \quad (1)$$

But cos2x = $\cos^2$X- $\sin^2$X, sin2X = 2sinXcosX

cos3X = 4$\cos^2$X – 3cosX, sin3X = 4$\sin^3$X – 3sinX

Thus 2.1 takes the form

$$\psi(x,y,t) = \sigma\{ \epsilon[(f_1(x,y) - 3h_1(x,y))cosX + (f_2 + 3h_2(x,y))sinX] + \epsilon^2[g_1(x,y)cos^2X - g_2(x,y)sin^2X + 2g_2(x,y)cosXsinX] + \epsilon^3[(h_1(x,y)cos^3X + h_2(x,y)sin^3X)] + 0(\epsilon^4)$$

Define the following parameter $\mathbb{z}_1 = \epsilon cosX$, $\mathbb{z}_2 = \epsilon sinX$, $\bar{\mathbb{z}}_1 = \bar{\mathbb{z}}_2 = 0$. Bar indicates mean over a wave length.

Also, $\omega = \frac{2\pi}{T}, \sigma^2 = \frac{1}{T}\int_d^{d+T} \eta^2(t)dt$ (2)

d = field constant

$$\psi(x,y,\mathbb{z}_1,\mathbb{z}_2) = [f_1(x,y) - 3h_1(x,y)]\mathbb{z}_1 + [f_2 + 3h_2(x,y)]\mathbb{z}_2] + g_1(x,y)({\mathbb{z}_1}^2 + {\mathbb{z}_2}^2) + 2g_2(x,y)\mathbb{z}_1\mathbb{z}_2 + 4[(h_1(x,y){\mathbb{z}_1}^3 + h_2(x,y){\mathbb{z}_2}^3] + 0(\epsilon^4) \quad (3)$$

3. The wave field $\eta(x, kh, t)$

We consider the wave field obstructed by a vertical wall located at x= 0, -h<y<b, where b is the height of the wall above the mean sea-level. The following parameters will take the form in this considerations as follows:

$f_1(x,y) = \bar{f}_1(kh)coskx$, $f_2(x,y) = \bar{f}_2(kh)coskx$, $g_1(x,y) = \bar{g}_1(kh)cos2kx$, $g_2(x,y) = \bar{g}_2(kh)cos2kx$, $h_1(x,y) = \bar{h}_1(kh)cos3kx$, $h_2(x,y) = \bar{h}_2(kh)cos3kx$ for $\eta = \eta(x,t,kh)$

Equation (3) can be re-arranged in terms of three terms in Fourier expansion and gives

$$\eta(kh,x,t) = N_1coskx + N_2cos2kx + N_3cos3kx + \cdots \quad (4)$$

Where $N_1 = N_1(kh) = \bar{f}_1\mathbb{z}_1 + \bar{f}_2\mathbb{z}_2$, $N_2(kh) = [\bar{g}_1({\mathbb{z}_1}^2 + {\mathbb{z}_2}^2) + \bar{g}_2\mathbb{z}_1\mathbb{z}_2]$, $N_3 = 3[(\bar{h}_2\mathbb{z}_2 - \bar{h}_1\mathbb{z}_1)] + 4(\bar{h}_2\mathbb{z}_2 + \bar{h}_1\mathbb{z}_1)$

By Stokes expansion, we obtained the following representations

$$\bar{f}_1(kh) = \frac{4}{coshkh}, \quad \bar{f}_2(kh) = \frac{-1}{sinhkh}, \quad \bar{g}_1(kh) = \frac{cosh2kh}{cosh^2(kh)},$$

$$\bar{g}_2(kh) = \frac{tanhkh - 2kh}{sinh^2kh}, \quad \bar{h}_1(kh) = \frac{2\sinh(kh) + \sinh(2kh)}{cosh^3kh},$$

$$\bar{h}_2(kh) = \frac{3sinh3kh + cosh^2kh}{sinh^2kh + cosh^2(2kh)}$$

The wave height fluctuations on the vertical wall is calculated from

$$\eta(kh,t) = \sigma(N_1 + N_2 + N_3) \quad x = 0 \ \ldots \quad (5)$$

4. Optimum behavior of the wave on the vertical wall offshore

Take the wave T = 10mins, sea depth (h) = 8 meters, then from eqn. (5)

$d\eta(0,kh,t) = 0$; this equation gives kh = 0.75ωt (a non-dimensional parameter). Correspondingly

$\eta(0,kh,t)$= $\sigma[3{\mathbb{z}_1}^2 + 5\mathbb{z}_1\mathbb{z}_2]$, $0 < t < 17$ seconds ………… (6)

Method of calculating the extreme values of a function suggest that 6 provides the maximum possible height of the incident wave crest on the vertical wall

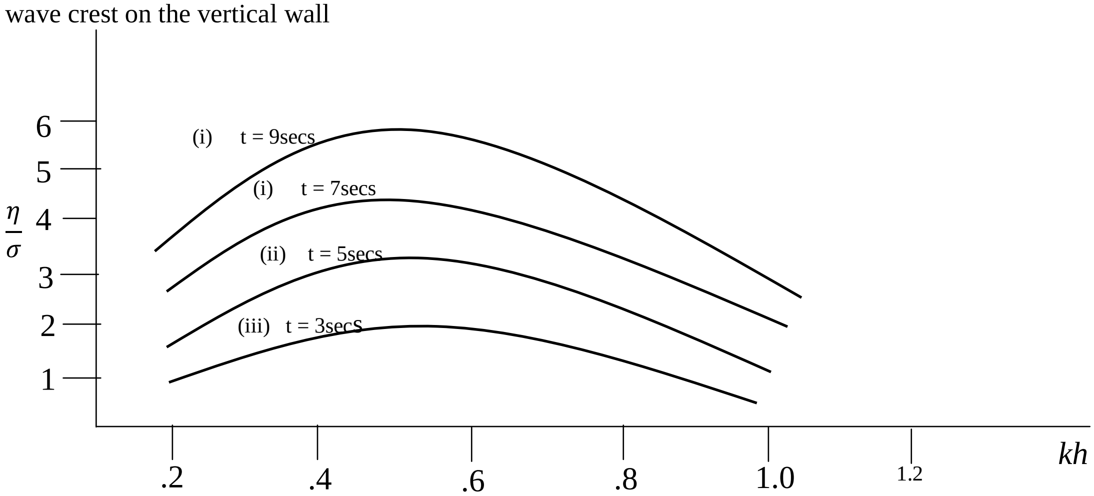


Fig. I: The distribution of the maximum wave crest amplitude in the presence of a vertical wall

5. Relative wave crest elevation and the height of the wall

In marine operations, the safe of design of any fixed structure depends on the accurate estimate of wave height distribution on the structure.

The mathematical tool to solve this situation is by the use of exceedence probability related to wave crest elevation.

From eqn. (2.4) $\alpha_1 = \frac{N_2}{N_1}, \quad \alpha_1 = \frac{N_3}{N_1}. \quad \beta = [1 - 2(\alpha_1{}^2 + \alpha_2{}^2)]^{\frac{1}{2}}$

Take the spatial mean of $\psi(x, y, t)$ as ξ and $\xi = \frac{\bar{\psi}}{\sigma} = \alpha_1 \beta U + \alpha_2 \beta^2 U^2$

U is a random Gaussian variable normally distributed

Thus, $U^2 + \frac{\alpha_1}{\beta} U - \frac{\xi}{\alpha_2 \beta^2} = 0$

$$U = \frac{-\frac{\alpha_1}{\beta} \pm \sqrt{\frac{\alpha_1{}^2}{\beta^2} + 4\frac{\xi}{\alpha_2 \beta^2}}}{2}$$

Let $\delta_1 = \frac{-\alpha_1}{\beta} + \frac{\alpha_1}{\beta}\sqrt{\alpha_2{}^2 + 4\frac{\xi}{\alpha_2}}, \quad \delta_1 = \frac{-\alpha_1}{\beta} - \frac{\alpha_1}{\beta}\sqrt{\alpha_2{}^2 + 4\frac{\xi}{\alpha_2}}$

If U_1 and U_2 are variables related to height of the wall and wave crest elevation on the wall respectively, the probability of exceedence is calculated from the expression

$$P(U_2 > U_1) = \frac{1}{2\pi\sigma}\left[e^{\delta_1} + e^{\delta_2}\right] = \frac{e^{-\frac{\alpha_1}{\beta}}}{\pi\sigma} \cosh\left[\frac{1}{\beta}\left(\alpha_2 + \frac{4\xi}{\alpha_2}\right)\right] \quad \text{……………………} \quad (7)$$

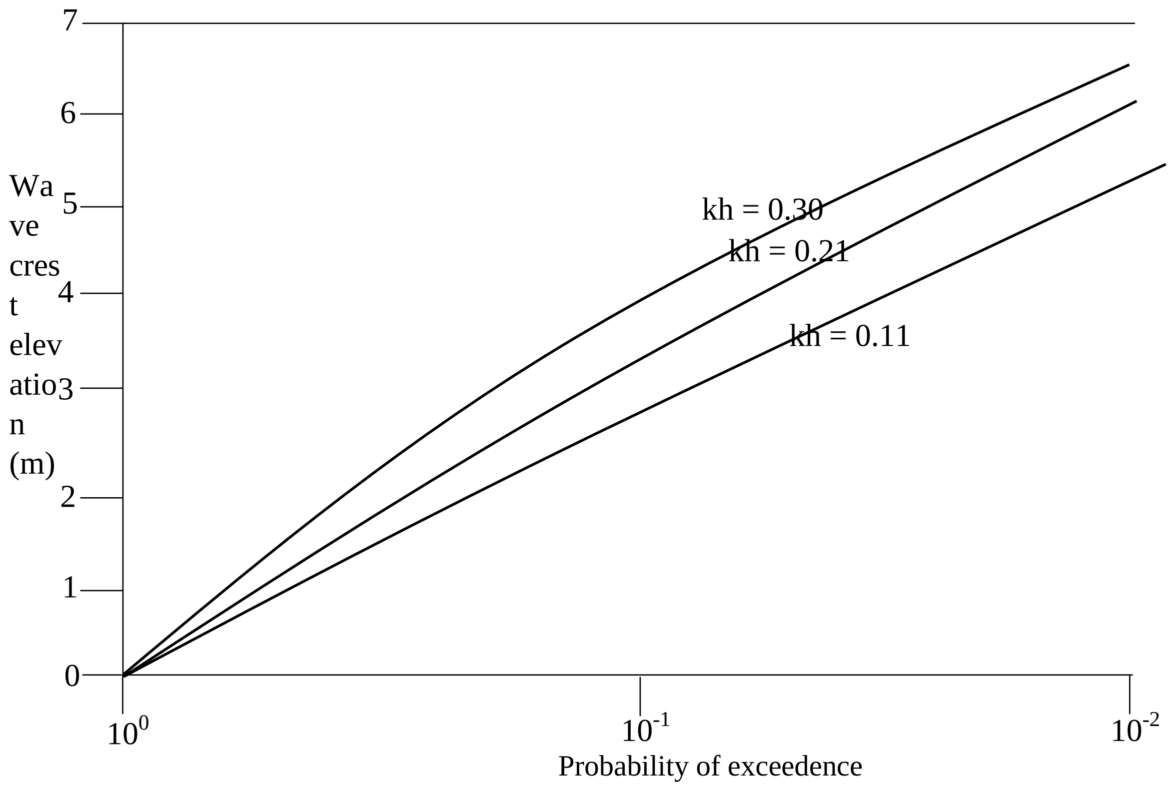


Fig. II Probability of exceedence for wave crest elevation (Period T=8secs, t = 5 mins)

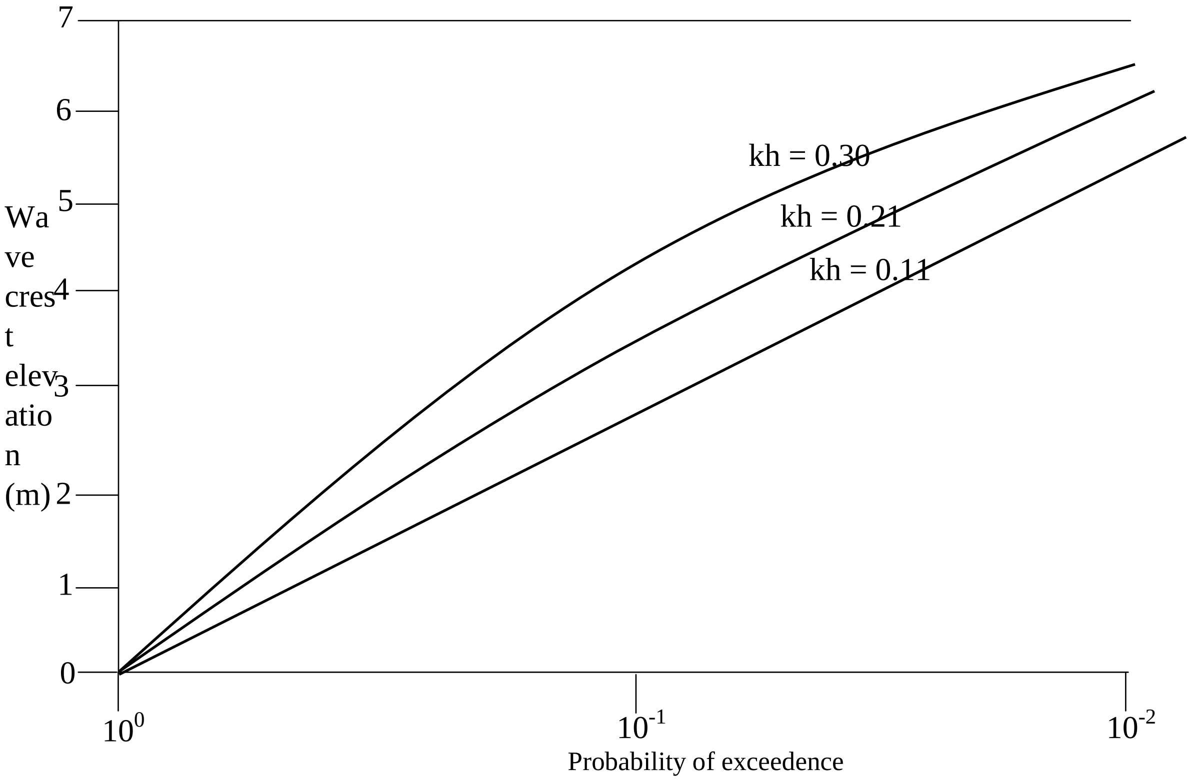


Fig. III Probability of exceedence for wave crest elevation. (Period T = 12 seconds, t = 5mins)

Fig II and Fig III depicit the probability of exceedence for wave crest elevations in the front of a vertical wall. The calculations were performed for various numerical values of the water depth parameter kh. The time t duration of the crest elevation for t = 7.5 mins, 5.5 mins, 2.5 mins, and the elevation corresponding to each time t duration is unchanged to a significant extent. For a fixed wave length, the exceedence is higher in deep water areas of the medium involved and this appears to represent dominant modes in the model of narrow banded spectrum [2,3].

2.0 Pressure distribution in the front of a vertical wall

The wave field interacting with off-shore vertical structure in assumed to be narrow banded. This is identical to the process leading to the stochastic function that models wave surface elevation. Thus, we have the following functional representation (eqn.1) and in this case;

$N_{1P} = N_{1P}(ky,kh) = (f_{1P}\mathbb{Z}_1 + f_{2P}\mathbb{Z}_2), \bar{N}_{2P} = \bar{N}_{2P}(ky,kh)$

$= [\bar{g}_{1P}(z^2{}_1 - z^2{}_2) + \bar{g}_{2P}z_1z_2], \qquad \bar{N}_{3P} = \bar{N}_{3P}(kh,ky) = 3(\bar{h}_{2P}z_2 - \bar{h}_{1P}z_1) + 4(\bar{h}_{2P}z_2{}^3 + \bar{h}_{1P}z_1{}^3)$

Similarly, the following Stokes expansion coefficients identically calculated as (4)

$$\bar{f}_{1P}(ky,kh) = \frac{coshk(y+h)}{coshkh}, \quad \bar{f}_{2P}(ky,kh) = \frac{sinhk(y+h)}{coshkh}$$

$$\bar{g}_{1P}(ky,kh) = \frac{cosh2k(y+h)}{coshkh + sinhkh}, \quad \bar{g}_{2P}(ky,kh) = \frac{cosh2k(y+h)}{cosh^2(2kh)}$$

$$\bar{h}_{1P} = \frac{cosh3k(y+h)}{coshkh + sinh2kh}, \quad \bar{h}_{2P} = \frac{sinh3k(y+h)}{cosh3kh - 2sinh2kh}$$

Subscript p implies related pressure fluctuation. Again, cosh (f(y)) = 1 and sinh (f(y)) = 0 for all f(y) that satisfies the constraint f(y) = o when y = o; thus, the parameters in eqn. (5) are integrable for $-h < y < \eta(x,kh,t)$. Consequently, the pressure fluctuation P(x, t, ky, kh) can be determined in identical stochastic Gaussian form as follows:

$P(x,t,ky,kh) = \rho g\sigma(N_{1P} + N_{2P} + N_{3P})$………………………………………. (8)

The form of eqn (8) involves stoke's expansion to third order nonlinearity as introduced in (1)

3.0 Wave pressure force on a vertical wall

This consideration is interestingly applicable in marine constructions offshore. The construction engineering needs a thorough understanding of wave activities in the locality especially wave pressure force on the structures offshore.

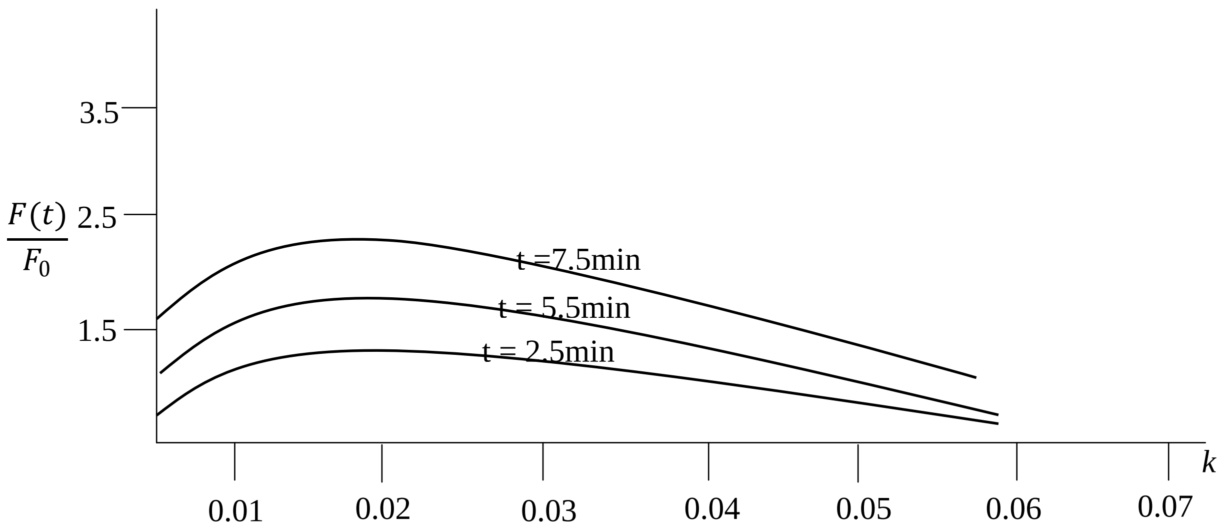
We now assume that the wall extends vertically from the sea-bed y=-h offshore to the level sufficiently above the surface and beyond the possible the wave crest elevation.

Thus, the pressure force associated with wave pressure on the offshore vertical wall is calculated by integrating (8) vertically from y = -h to y = $\eta(0, kh, t)$. That is $\int_{-h}^{\eta(0,kh,t)} P(0, kh, ky, t)dy = [R_1 Z_1 + \bar{R}_1 Z_2 + R_2 Z_1 + \bar{R}_2 Z_2 + R_3 Z_1 + \bar{R}_3 Z_2 +$ $R4Z1Z2\rho\sigma 2g+Fo=F(t)$ (9)

Where $kR_1 = \tanh kh$, $kR_1 = 1$, $2kR_2 = \frac{\sinh 2kh}{\cosh 2kh + \sinh kh}$

$2k\bar{R}_2 = \frac{3\cosh 2kh}{\cosh 2kh + \sinh kh}$, $3kR_3 = \frac{\sinh 3kh}{\cosh^2 kh + 1}$,

$3k\bar{R}_3 = \frac{\cosh 3kh}{\sinh^2 kh + 1}$, $2kR_4 = \tanh 2kh \operatorname{sech} 2kh$



k – (rad/min)

$F_0 = \rho g \sigma^2$

Fig IV. Fluctuating wave force in front of a vertical wall

Fig IV depicts the narrow band fluctuating wave force on a verticalwall. The significant increase in amplitude with time is evident. The peak amplitude occurring at k = 0.02 rad/min is also clearly evident.

The second order pressure force has symmetric behavior. However, in the present higher order nonlinearity considered in this study, the behavior is different being towards the range of long wave-length. The increasing force with time is as expected and is in agreement with wave tank experiment (1).

Conclusion

The consideration generalizes the stochastic family[5]. It involved hyperbolic ally behaved Stokes coefficients for the narrow band wave spectrum but with wave parameters randomly distributed following Rayleigh[4,1].

Following this considerations, the wave crest elevation and pressure forces on the vertical wall were calculated and sketched. The probability of exceedence which describes the wave crest elevation on the vertical wall are also calculated and sketched. These are in reasonable agreement with previous identical studies[2,3,1].

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